The Schwarzschild solution and its implications for gravitational waves

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The so-called 'Schwarzschild solution' is not Schwarzschild's solution, but a corruption of the Schwarzschild/Droste solutions. In the so-called 'Schwarzschild solution' the quantity m is alleged to be the mass of the source of a gravitational field and the quantity r is alleged to be able to go down to zero (although no valid proof of this claim has ever been advanced), so that there are two alleged 'singularities', one at $r = 2m$ and another at $r = 0$. It is routinely asserted that $r = 2m$ is a 'coordinate' or 'removable' singularity which denotes the so-called 'Schwarzschild radius' (event horizon) and that a 'physical' singularity is at $r = 0$. The quantity r in the 'Schwarzschild solution' has never been rightly identified by the physicists, who, although proposing many and varied concepts for what r therein denotes, effectively treat it as a radial distance from the claimed source of the gravitational field at the 'origin of coordinates'. The consequence of this is that the intrinsic geometry of the metric manifold has been violated. It is easily proven that the said quantity r is in fact the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of the 'Schwarzschild solution' and so does not in itself define any distance whatsoever in that manifold. Thus the 'Schwarzschild radius' is not a distance of any sort. With the correct identification of the associated Gaussian curvature it is also easily proven that there is only one singularity associated with all Schwarzschild metrics, of which there is an infinite number that are equivalent. Thus, the standard removal of the singularity at $r = 2m$ is erroneous, as the alleged singularity at $r = 0$ does not exist, very simply demonstrated herein. This has major implications for the localisation of gravitational energy, i.e. gravitational waves. It is demonstrated herein that Special Relativity forbids infinite density and in consequence of this General Relativity necessarily forbids infinite density, and so the infinitely dense point-mass singularity of the alleged black hole is forbidden by the Theory of Relativity. It is also shown that neither Einstein's Principle of Equivalence not his Laws of Special Relativity can manifest in a spacetime that by construction contains no matter, and therefore $Ric = 0$ violates the requirement that both the said Principle and Special Relativity manifest in Einstein's gravitational field. The immediate implication of this is that the total gravitational energy of Einstein's gravitational field is always zero, so that the energy-momentum tensor and the Einstein tensor must vanish identically. Attempts to preserve the usual conservation of energy and momentum by means of Einstein's pseudo-tensor are fatally flawed owing to the fact that the pseudo-tensor implies the existence of a first-order intrinsic differential invariant, dependent solely upon the components of the metric tensor and their first derivatives, an invariant which however does not exist, proven by the pure mathematicians G. Ricci-Curbastro and T. Levi-Civita, in 1900. Although it is standard method to utilise the Kretschmann scalar to justify infinite Schwarzschild spacetime curvature at the point-mass singularity, it is demonstrated that the Kretschmann scalar is not an independent curvature invariant, being in fact a function of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, and therefore constrained by the limitations set on the said Gaussian curvature by the geometric ground-form of the line-element itself. Since it is easily proven that the said Gaussian curvature cannot become unbounded in Schwarzschild spacetime, the Kretschmann scalar is necessarily finite everywhere in the Schwarzschild manifold.

I. Schwarzschild spacetime

It is reported almost invariably in the literature that Schwarzschild's solution for $Ric = R_{\mu\nu} = 0$ is (using $c = 1$, $G = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{1}$$

wherein it is asserted by inspection that r can go down to zero in some way, producing an infinitely dense point-mass singularity there, with an event horizon at the 'Schwarzschild radius' at $r = 2m$: a black hole. Contrast this metric with that actually obtained by K. Schwarzschild in 1915 (published January 1916),

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{2}$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty,$$

wherein α is an undetermined constant. There is only *one* singularity in Schwarzschild's solution, at $r = 0$, to which his solution is constructed. Contrary to the usual claims made by the astrophysical scientists, Schwarzschild did not set $\alpha = 2m$ where m is mass; he did not breathe a single word about the bizarre object that is called a black hole; he did not allege the so-called 'Schwarzschild radius'; he did not claim that there is an 'event horizon' (by any other name); and his solution clearly forbids the black hole because when Schwarzschild's $r = 0$, his $R = \alpha$, and so there is no possibility for his R to be less than α, let alone take the value $R = 0$. All this can be easily verified by simply reading Schwarzschild's original paper [1], in which he *constructs* his solution so that the singularity occurs at the *"origin"* of coordinates. Thus, eq. (1) for $0 < r < 2m$ is *inconsistent* with Schwarzschild's true solution, eq. (2). It is also inconsistent with the intrinsic geometry of the line-element, whereas eq. (2) is geometrically consistent, as demonstrated herein. Thus, eq. (1) is meaningless for $0 \leq r < 2m$.

In the usual interpretation of Hilbert's [2, 3, 4] version, eq. (1), of Schwarzschild's solution, the quantity r therein has **never** been properly identified. It has been variously and vaguely called a *"distance"* [5, 6], *"the radius"* [6–19,78,79], the *"radius of a 2-sphere"* [20], the *"coordinate radius"* [21], the *"radial coordinate"* [22–25,78,79], the *"radial space coordinate"* [26], the *"areal radius"* [21, 24, 27, 28], the *"reduced circumference"* [25], and even *"a gauge choice: it defines the coordinate r"* [29]. In the particular case of $r = 2m = 2GM/c^2$ it is almost invariably referred to as the *"Schwarzschild radius"* or the *"gravitational radius"*. However, none of these various and vague concepts of what r is are correct because the irrefutable geometrical fact is that r, in the spatial section of Hilbert's version of the Schwarzschild/Droste line-element, is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section [30, 31, 32], and as such it **does not** of itself determine the geodesic radial distance from the centre of spherical symmetry located at an arbitrary point in the related pseudo-Riemannian metric manifold. It does not of itself determine any distance at all in the spherically symmetric metric manifold. It is the radius of Gaussian curvature merely by virtue of its formal geometric relationship to the Gaussian curvature. It must also be emphasized that a geometry is completely determined by the **form** of its line-element [33].

Since r in eq. (1) can be replaced by *any* analytic function $R_c(r)$ [4, 30, 32, 34] without disturbing spherical symmetry and without violation of the field equations $R_{\mu\nu} = 0$, which is very easily verified, satisfaction of the field equations is a necessary but insufficient condition for a solution for Einstein's static vacuum 'gravitational' field. Moreover, the admissible *form* of $R_c(r)$ must be determined in such a way that an infinite number of equivalent metrics is generated thereby [32, 34]. In addition, the identification of the centre of spherical symmetry, origin of coordinates and the properties of points must also be clarified in relation to the non-Euclidean geometry of Einstein's gravitational field. In relation to eq. (1) it has been routinely presumed that geometric points in the spatial section (which is non-Euclidean) must have the very same properties of points in the spatial section (Euclidean) of Minkowski spacetime. However, it is easily proven that the non-Euclidean geometric points in the spatial section of Schwarzschild spacetime do not possess the same characteristics of the Euclidean geometric points in the spatial section of Minkowski spacetime [32, 35]. This should not be surprising, since the indefinite metric of Einstein's Theory of Relativity admits of other geometrical oddities, such as null vectors, i.e. non-zero vectors that have zero magnitude and which are orthogonal to themselves [36].

II. Spherical Symmetry

Recall that the squared differential element of arc of a curve in a surface is given by **the first fundamental quadratic form** for a surface,

$$ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2,$$

wherein u and v are curvilinear coordinates. If either u or v is constant the resulting line-elements are called *parametric curves* in the surface. The differential element of surface area is given by,

$$dA = \left| \sqrt{EG - F^2}\,du\,dv \right|.$$

An expression which depends only on E, F, G and their first and second derivatives is called a bending invariant. It is an intrinsic (or absolute) property of a surface. The Gaussian (or Total) curvature of a surface is an important intrinsic property of a surface.

The **'Theorema Egregium'** of **Gauss**

> *The Gaussian curvature K at any point P of a surface depends only on the values at P of the coefficients in the First Fundamental Form and their first and second derivatives.* [37, 38, 39]

And so,

> *"The Gaussian curvature of a surface is a bending invariant."* [38]

The plane has a constant Gaussian curvature of $K = 0$. *"A surface of positive constant Gaussian curvature is called a spherical surface."* [39]

Now a line-element, or squared differential element of arc-length, in spherical coordinates, for 3-dimensional Euclidean space is,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{3}$$

$$0 \leq r < \infty.$$

The scalar r can be construed, verified by calculation, as the magnitude of the radius vector $\vec{r}$ from the origin of the coordinate system, the said origin coincident with the centre of the associated sphere. All the components of the metric tensor are well-defined and related geometrical quantities are fixed by the form of the line-element. Indeed, the radius R_p of the associated sphere ($\theta = const., \varphi = const.$) is given by,

$$R_p = \int_0^r dr = r,$$

the length of the geodesic C_p (a parametric curve; $r = const.,\ \theta = \pi/2$) in an associated surface is given by,

$$C_p = r\int_0^{2\pi} d\varphi = 2\pi r,$$

the area A_p of an associated spherically symmetric surface ($r = const.$) is,

$$A_p = r^2\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi} d\varphi = 4\pi r^2,$$

and the volume V_p of the sphere is,

$$V_p = \int_0^r r^2 dr\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi} d\varphi = \frac{4}{3}\pi r^3.$$

Now the point at the centre of spherical symmetry for any problem at hand need not be coincident with the origin of the coordinate system. For example, the equation of a sphere of radius ρ centered at the point C located at the extremity of the fixed vector $\vec{r}_o$ in Euclidean 3-space, is given by

$$(\vec{r} - \vec{r}_o) \cdot (\vec{r} - \vec{r}_o) = \rho^2.$$

If $\vec{r}$ and $\vec{r}_o$ are collinear, the vector notation can be dropped, and this expression becomes,

$$|r - r_o| = \rho,$$

where $r = |\vec{r}|$ and $r_o = |\vec{r}_o|$, and the common direction of $\vec{r}$ and $\vec{r}_o$ becomes entirely immaterial. This scalar expression for a shift of the centre of spherical symmetry away from the origin of the coordinate system plays a significant rôle in the equivalent line-elements for Schwarzschild spacetime.

Consider now the generalisation of eq. (3) to a spherically symmetric metric manifold, by the line-element,

$$ds^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) = \Psi\left(R_c\right)dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{4}$$

$$R_c = R_c(r)$$

$$R_c(0) \leq R_c(r) < \infty,$$

where both $\Psi(R_c)$ and $R_c(r)$ are *a priori* unknown analytic functions. Since neither $\Psi(R_c)$ nor $R_c(r)$ are known, eq. (4) may or may not be well-defined at $R_c(0)$: one cannot know until $\Psi(R_c)$ and $R_c(r)$ are somehow specified. With this proviso, there is a one-to-one point-wise correspondence between the manifolds described by metrics (3) and (4), i.e. a mapping between the auxiliary Euclidean manifold described by metric (3) and the generalised non-Euclidean manifold described by metric (4), as the differential geometers have explained [30]. If R_c is constant, metric (4) reduces to a 2-dimensional spherically symmetric geodesic surface described by the first fundamental quadratic form,

$$ds^2 = R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{5}$$

If r is constant, eq. (3) reduces to the 2-dimensional spherically symmetric surface described by the first fundamental quadratic form,

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{6}$$

Although R_c and r are constants in equations (5) and (6) respectively, they share a definite geometric identity in their respective surfaces: but it is **not** that of being a radial quantity, or of a distance.

A surface is a manifold in its own right. It need not be considered in relation to an embedding space. Therefore, quantities appearing in its line-element must be identified in relation to the surface, not to any embedding space it might be in:

> "And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space." [40]

Note that eqs. (3) and (4) have the same metrical form and that eqs. (5) and (6) have the same metrical form. Metrics of the same form share the same fundamental relations between the components of their respective metric tensors. For example, consider eq. (4) in relation to eq. (3). For eq. (4), the radial geodesic distance (i.e. the proper radius) from the point at the centre of spherical symmetry ($\theta = const., \varphi = const.$) is,

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi(R_c(r))}dR_c(r) = \int_0^r \sqrt{\Psi(R_c(r))}\frac{dR_c(r)}{dr}dr,$$

the length of the geodesic C_p (a parametric curve; $R_c(r) = const., \theta = \pi/2$) in an associated surface is given by,

$$C_p = R_c(r)\int_0^{2\pi} d\varphi = 2\pi R_c(r),$$

the area A_p of an associated spherically symmetric geodesic surface ($R_c(r) = const.$) is,

$$A_p = R_c^2(r)\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\varphi = 4\pi R_c^2(r),$$

and the volume V_p of the geodesic sphere is,

$$V_p = \int_0^{R_p}R_c^2\left(r\right)dR_p\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\varphi = 4\pi\int_{R_c(0)}^{R_c(r)}\sqrt{\Psi\left(R_c\left(r\right)\right)}R_c^2(r)dR_c$$

$$= 4\pi \int_0^r \sqrt{\Psi\left(R_c\left(r\right)\right)} R_c^2(r) \frac{dR_c(r)}{dr} dr.$$

Remarkably, in relation to metric (1), Celotti, Miller and Sciama [11] make the following false assertion:

"The 'mean density' $\overline{\rho}$ of a black hole (its mass M divided by $\frac{4}{3}\pi r_s^3$) is proportional to $1/M^{2"}$

where r_s is the so-called "Schwarzschild radius". The volume they adduce for a black hole cannot be obtained from metric (1): it is a volume associated with the Euclidean 3-space described by metric (3).

Now in the case of the 2-dimensional metric manifold given by eq. (5) the Riemannian curvature associated with eq. (4) (which depends upon both position and direction) reduces to the Gaussian curvature K (which depends only upon position), and is given by [30,38,39,41–45],

$$K = \frac{R_{1212}}{g}, \tag{7}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$ (because the metric tensor of eq. (5) is diagonal). Gaussian curvature is an intrinsic geometric property of a surface (Theorema Egregium[1]); independent of any embedding space.

Now recall from elementary differential geometry and tensor analysis that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^\gamma_{.\ \nu\rho\varphi}$$

$$R^1_{.\ 212} = \frac{\partial \Gamma^1_{22}}{\partial x^1} - \frac{\partial \Gamma^1_{21}}{\partial x^2} + \Gamma^k_{22}\Gamma^1_{k1} - \Gamma^k_{21}\Gamma^1_{k2}$$

$$\Gamma^i_{ij} = \Gamma^i_{ji} = \frac{\partial\left(\frac{1}{2}\ln|g_{ii}|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i}, \quad (i \neq j) \tag{8}$$

and all other Γ^i_{jk} vanish. In the above, $i,j,k = 1,2$, $x^1 = \theta$, $x^2 = \varphi$. Applying expressions (7) and (8) to expression metric (5) gives,

$$K = \frac{1}{R_c^2} \tag{9}$$

so that $R_c(r)$ is the inverse square root of the Gaussian curvature, i.e. the radius of Gaussian curvature, and hence, in eq. (6) the quantity r therein is the radius of Gaussian curvature because this Gaussian curvature is intrinsic to all geometric surfaces having the form of eq. (5) [30], and a geometry is completely determined by the *form* of its line-element [33]. Note that according to eqs. (3), (6) and (7), the radius calculated for (3) gives the same value as the associated radius of Gaussian curvature of a spherically symmetric surface embedded in the space described by eq. (3). Thus, the Gaussian curvature (and hence the radius of Gaussian curvature) of the spherically symmetric surface embedded in the space of (3) can be associated with the radius calculated from eq. (3). This is a consequence of the Euclidean nature of the space described by metric (3), which also describes the spatial section of Minkowski spacetime. However, this is *not* a general relationship. The inverse square root of the Gaussian curvature (the radius of Gaussian curvature) is not a distance at all in Einstein's gravitational manifold but in fact determines the Gaussian curvature of the spherically symmetric geodesic surface through any point in the spatial section of the gravitational manifold, as proven by expression (9). Thus, the quantity r in eq. (1) is the inverse square root of the Gaussian curvature (i.e. the radius of Gaussian curvature) of a spherically symmetric geodesic surface in the spatial section, not the radial geodesic distance from the centre of spherical symmetry of the spatial section, or any other distance.

The platitudinous nature of the concepts *"reduced circumference"* and *"areal radius"* is now plainly evident - neither concept correctly identifies the geometric nature of the quantity r in metric (1). The geodesic C_p in the spherically symmetric geodesic surface in the spatial section of eq. (1) is a function of the curvilinear coordinate φ and the surface area A_p is a function of the curvilinear coordinates θ and φ where, in both cases, r is a constant.

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[1]i.e. Gauss' Most Excellent Theorem.

However, r therein has a clear and definite geometrical meaning: it is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. The Gaussian curvature K is a positive constant bending invariant of the surface, independent of the values of θ and φ. Thus, neither C_p nor A_p, or the infinite variations of them by means of the integrated values of θ and φ, rightly identify what r is in line-element (1). To illustrate further, when $\theta = constant$, the arc-length in the spherically symmetric geodesic surface is given by:

$$s = s(\varphi) = r \int_0^\varphi \sin \theta \, d\varphi = r \sin \theta \, \varphi, \quad 0 \le \varphi \le 2\pi,$$

where $r = constant$. This is the equation of a straight line, of gradient $ds/d\varphi = r \sin \theta$. If $\theta = const. = \frac{1}{2}\pi$ then $s = s(\varphi) = r\varphi$, which is the equation of a straight line of gradient $ds/d\varphi = r$. The maximum arc-length of the geodesic $\theta = const. = \frac{1}{2}\pi$ is therefore $s(2\pi) = 2\pi r = C_p$. Similarly the surface area is:

$$A = A(\varphi, \theta) = r^2 \int_0^\theta \int_0^\varphi \sin \theta \, d\theta \, d\varphi = r^2 \varphi \left(1 - \cos \theta\right),$$

$$0 \le \varphi \le 2\pi, \quad 0 \le \theta \le \pi, \quad r = constant.$$

The maximum area (i.e. the area of the entire surface) is $A(2\pi, \pi) = 4\pi r^2 = A_p$. Clearly, neither s nor A are functions of r, because r is a constant, not a variable. And since r appears in each expression (and so having the same value in each expression), neither s nor A rightly identify the geometrical significance of r in the 1st fundamental form for the spherically symmetric geodesic surface, $ds^2 = r^2 \left(d\theta^2 + \sin^2 \theta \, d\varphi^2\right)$, because r is **not** a distance in the surface and is **not** the "radius" of the surface. The geometrical significance of r is intrinsic to the surface and is determined from the components of the metric tensor and their derivatives (Gauss' Theorema Egregium): it is the inverse square root of the Gaussian curvature K of the spherically symmetric surface so described (the constant is $K = 1/r^2$). Thus, C_p and A_p are merely platitudinous expressions containing the constant r, and so the "*reduced circumference*" $r = C_p/2\pi$ and the "*areal radius*" $r = \sqrt{A_p/4\pi}$ do not identify the geometric nature of r in either metric (6) or metric (1), the former appearing in the latter. The claims by the astrophysical scientists that the "*areal radius*" and the "*reduced circumference*" each define [21, 25, 48] (in two different ways) the constant r in eq. (1) are entirely false. The "*reduced circumference*" and the "*areal radius*" are in fact one and the same, namely the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of eq. (1), as proven above. No proponent of black holes is aware of this simple geometrical fact, which completely subverts all claims made for black holes being predicted by General Relativity.

III. Derivation of Schwarzschild spacetime

The usual derivation begins with the following metric for Minkowski spacetime (using $c = 1$),

$$ds^2 = dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2\right), \tag{10}$$

$$0 \le r < \infty,$$

and proposes a generalisation thereof as, or equivalent to,

$$ds^2 = F(r)dt^2 - G(r)dr^2 - R^2(r) \left(d\theta^2 + \sin^2 \theta d\varphi^2\right), \tag{11}$$

where $F, G > 0$ and r is that which appears in the metric for Minkowski spacetime, making r in eq. (10) a parameter for the components of the metric tensor of eq. (11). The functions $F(r), G(r), R(r)$ are to be determined such that the signature of metric (10) is maintained in metric (11), at $(+, -, -, -)$. The substitution $r^* = R(r)$ is then usually made, to get,

$$ds^2 = W(r^*)dt^2 - M(r^*)dr^{*2} - r^{*2} \left(d\theta^2 + \sin^2 \theta d\varphi^2\right), \tag{11b}$$

Then the * is simply dropped, and with that it is just *assumed* that $0 \le r < \infty$ can be carried over from eq. (10), to get [5,8,9,21–23,26,30,33,34,36,47–55,79],

$$ds^2 = e^\lambda dt^2 - e^\beta dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2\right), \tag{12}$$

$$0 \le r < \infty,$$

the exponential functions being introduced for subsequent ease of mathematical manipulations. It is then required that $e^{\lambda(r)} > 0$ and $e^{\beta(r)} > 0$ be determined so as to satisfy $R_{\mu\nu} = 0$.

Now note that in going from eq. (11b) to eq. (12), it is merely *assumed* that $R(0) = 0$, making $0 \leq r^* < \infty$ (and hence in eq. (12), $0 \leq r < \infty$), since $r^* = R(r)$: but this cannot be known since $R(r)$ is *a priori* unknown [2, 3]. One simply cannot treat r^* in eq. (11b), and hence r in eqs. (12) and (1), as the r in eq. (10); contrary to the practice of the astrophysical scientists and their mathematician collaborators. Also note that eq. (12) not only retains the signature -2, but **also retains the signature** $(+,-,-,-)$, because $e^{\lambda} > 0$ and $e^{\beta} > 0$ **by construction**. Thus, neither e^{λ} nor e^{β} can change sign [5, 48, 55, 79]. This is a requirement since there is no possibility for Minkowski spacetime (eq. 10) to change signature from $(+,-,-,-)$ to, for example, $(-,+,-,-)$.

The Standard Analysis then obtains the solution given by eq. (1), wherein the constant m is claimed to be the mass generating the alleged associated gravitational field. Then by mere inspection of eq. (1) the Standard Analysis asserts that there are two singularities, one at $r = 2m$ and one at $r = 0$. It is claimed that $r = 2m$ is a removable coordinate singularity, and that $r = 0$ a physical singularity. It is also asserted that $r = 2m$ gives the event horizon (the 'Schwarzschild radius') of a black hole, from which the 'escape velocity' is that of light (in vacuo), and that $r = 0$ is the position of the infinitely dense point-mass singularity of the black hole, produced by irresistible gravitational collapse.

However, these claims cannot be true. First, the construction of eq. (12) to obtain eq. (1) in satisfaction of $R_{\mu\nu} = 0$ is such that neither e^{λ} nor e^{β} can change sign, because $e^{\lambda} > 0$ and $e^{\beta} > 0$. Therefore the claim that r in metric (1) can take values less than $2m$ is false; a contradiction by the very construction of the metric (12) leading to metric (1). Furthermore, since neither e^{λ} nor e^{β} can ever be zero, the claim that $r = 2m$ is a removable coordinate singularity is also false. In addition, the true nature of r in both eqs. (12) and (1) is entirely overlooked, and the geometric relations between the components of the metric tensor, fixed by the *form* of the line-element, are not applied, in consequence of which the Standard Analysis fatally falters.

In going from eq. (11) to eq. (12) the Standard Analysis has failed to realise that in eq. (11) all the components of the metric tensor are functions of r by virtue of the fact that all the components of the metric tensor are functions of $R(r)$. Indeed, to illuminate this, consider the metric,

$$ds^2 = B(R)dR^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$B(R) > 0.$$

This is the most general expression for the metric of a three-dimensional spherically symmetric metric-space [30]. Now if R is a function of some parameter σ, then the metric in terms of σ is,

$$ds^2 = B(R(\sigma))\left(\frac{dR}{d\sigma}\right)^2 d\sigma^2 + R^2(\sigma)(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$B(R(\sigma)) \equiv B(\sigma) > 0.$$

Relabelling the parameter σ with r gives precisely the generalisation of the spatial section of Minkowski spacetime. Now eq. (11) is given in terms of the parameter r of Minkowski spacetime, not in terms of the function $R(r)$. In eq. (11), set $G(r) = N(R(r))(dR/dr)^2$, then eq. (11) becomes,

$$ds^2 = F(R(r))dt^2 - N(R(r))\left(\frac{dR}{dr}\right)^2 dr^2 - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{11c}$$

or simply

$$ds^2 = F(R)dt^2 - N(R)dR^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{11d}$$

wherein $R = R(r)$. Similarly, working backwards from eq. (11b), using $r^* = R(r)$, eq. (11b) becomes,

$$ds^2 = W(R(r))dt^2 - M(R(r))dR^2(r) - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{11e}$$

or simply,

$$ds^2 = W(R)dt^2 - M(R)dR^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

wherein $R = R(r)$; and in terms of the parameter r of Minkowski spacetime, this becomes,

$$ds^2 = W(r)dt^2 - M(r)\left(\frac{dR}{dr}\right)^2 dr^2 - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$
(11f)

Writing $W(r) = F(r)$ and $G(r) = M(r)\left(dR/dr\right)^2$ gives,

$$ds^2 = F(r)dt^2 - G(r)dr^2 - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

which is eq. (11). So eq. (11) is a disguised form of eq. (11d), and so there is no need at all for the 'transformations' applied by the astrophysical scientists to get their eq. (12), from which they get their eq. (1). In other words, what the astrophysical scientists call r in their eq. (1) is actually $R(r)$, for which they have not given any definite admissible form in terms of the parameter r, and they incorrectly treat their $R(r)$, labelled r in eqs. (12) and (1), as the r in eq. (10), manifest in the miscarrying over of the range $0 \leq r < \infty$ from eq. (10).

Nonetheless, $R(r)$ is still an *a priori* unknown function, and so it cannot be arbitrarily asserted that $R(0) = 0$; contrary to the assertions of the astrophysical scientists. It is now quite plain that the 'transformations' used by the Standard Analysis in going from eq. (11) to eq. (12) are rather pointless, since all the relations are contained in eq. (11) already, and by its pointless procedure the Standard Analysis has confused matters and thereby introduced a major error concerning the range on the quantity r in its expression (12) and hence in its expression (1). One can of course, solve eq. (11d), subject to $R_{\mu\nu} = 0$, in terms of $R(r)$, without determining the admissible form of $R(r)$. However, the range of $R(r)$ **must** be ascertained by means of boundary conditions fixed by the very form of the line-element in which it appears. And if it is required that the parameter r appear explicitly in the solution, by means of a mapping between the manifolds described by eqs. (10) and (11), then the admissible form of $R(r)$ must also be ascertained, in which case r in Minkowski space is a parameter, and Minkowski space a parametric space, for the related quantities in Schwarzschild space. To highlight further, rewrite eq. (11) as,

$$ds^2 = A\left(R_c\right)dt^2 - B\left(R_c\right)dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
(13)

where $A\left(R_c\right), B\left(R_c\right), R_c\left(r\right) > 0$. The solution for $R_{\mu\nu} = 0$ then takes the form,

$$ds^2 = \left(1 + \frac{\kappa}{R_c}\right)dt^2 - \left(1 + \frac{\kappa}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$R_c = R_c(r),$$

where κ is a constant. There are two cases to consider; $\kappa > 0$ and $\kappa < 0$. In conformity with the astrophysical scientists take $\kappa < 0$, and so set $\kappa = -\alpha$, $\alpha > 0$. Then the solution takes the form,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
(14)

$$R_c = R_c(r),$$

where $\alpha > 0$ is a constant. It remains to determine the admissible form of $R_c\left(r\right)$, which, from **Section II**, is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the manifold associated with eq. (14), owing to the metrical *form* of eq. (14). From **Section II** herein the proper radius associated with metric (14) is,

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left[\sqrt{R_c} + \sqrt{R_c - \alpha}\right] + k,$$
(15)

where k is a constant. Now for some r_o, $R_p\left(r_o\right) = 0$. Then by (15) it is required that $R_c\left(r_o\right) = \alpha$ and $k = -\alpha \ln\sqrt{\alpha}$, so

$$R_p\left(r\right) = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left[\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right],$$
(16)

$$R_c = R_c(r).$$

It is thus also determined that the Gaussian curvature of the spherically symmetric geodesic surface of the spatial section ranges not from ∞ to 0, as it does for Euclidean 3-space, but from α^{-2} to 0. This is an inevitable consequence of the peculiar non-Euclidean geometry described by metric (14).

Schwarzschild's true solution, eq. (2), must be a particular case of the general expression sought for $R_c(r)$. Brillouin's solution [2, 35] must also be a particular case, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r+\alpha}\right) dt^2 - \left(1 - \frac{\alpha}{r+\alpha}\right)^{-1} dr^2 - (r+\alpha)^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (17)$$

$$0 < r < \infty,$$

and Droste's solution [46] must as well be a particular solution, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

$$\alpha < r < \infty. \qquad (18)$$

All these solutions must be particular cases in an infinite set of equivalent metrics [34]. The only admissible form for $R_c(r)$ is [32],

$$R_c(r) = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o, \qquad (19)$$

where r_o and n are entirely arbitrary constants. So the solution for $R_{\mu\nu} = 0$ is,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$R_c(r) = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o. \qquad (20)$$

Then if $r_o = 0$, $r > r_o$, $n = 1$, Brillouin's solution eq. (17) results. If $r_o = 0$, $r > r_o$, $n = 3$, then Schwarzschild's actual solution eq. (2) results. If $r_o = \alpha$, $r > r_o$, $n = 1$, then Droste's solution eq. (18) results, which is the correct solution in the particular metric of eq. (1). In addition the required infinite set of equivalent metrics is thereby obtained, all of which are asymptotically Minkowski spacetime. Furthermore, if the constant α is set to zero, eqs. (20) reduces to Minkowski spacetime, and if in addition r_o is set to zero, then the usual Minkowski metric of eq. (10) is obtained. The significance of the term $|r - r_o|$ was given in **Section II**: it is a shift of the location of the centre of spherical symmetry in the spatial section of the auxiliary manifold away from the origin of coordinates of the auxiliary manifold, along a radial line, to a point at distance r_o from the origin of coordinates. The point r_o in the auxiliary manifold is mapped into the point $R_p(r_o) = 0$ in Schwarzschild space, irrespective of the choice of the parametric point r_o in the auxiliary manifold. Minkowski spacetime is the auxiliary manifold for Schwarzschild spacetime. Strictly speaking, the parameter r of the auxiliary manifold need not be incorporated into metric (20), in which case the metric is defined only on $\alpha < R_c < \infty$. I have retained the quantity r to fully illustrate its rôle as a parameter and the part played by Minkowski spacetime as an auxiliary manifold.

It is clear from expressions (20) that there is only one singularity, at the arbitrary constant r_o, where $R_c(r_o) = \alpha \ \forall \ r_o \ \forall \ n$ and $R_p(r_o) = 0 \ \forall \ r_o \ \forall \ n$, and that **all** components of the metric tensor are affected by the constant α. Hence, the "removal" of the singularity at $r = 2m$ in eq. (1) is fallacious because it is clear from expressions (20), in accordance with the intrinsic geometry of the line-element as given in **Section II**, and the generalisation at eq. (13), that there is no singularity at $r = 0$ in eq. (1) and so $0 \leq r \leq 2m$ therein is meaningless [1–5,32,41,42,46,57,62]. The Standard claims for eq. (1) violate the geometry fixed by the *form* of its line-element and contradict the generalisations at eqs. (11) and (12) from which it has been obtained by the Standard method. There is therefore no black hole associated with eq. (1) since there is no black hole associated with eq. (2) and none with eq. (20),

of which Schwarzschild's actual solution, eq. (2), Brillouin's solution, eq. (17), and Droste's solution, eq. (18), are just particular equivalent cases.

In the case of $\kappa > 0$ the proper radius of the line-element is,

$$R_p = \int \frac{dR_c}{\sqrt{1 + \frac{\kappa}{R_c}}} = \sqrt{R_c \left(R_c + \kappa\right)} - \kappa \ln\left[\sqrt{R_c} + \sqrt{R_c + \kappa}\right] + k,$$

$$R_c = R_c(r),$$

where k is a constant. Now for some r_o, $R_p\left(r_o\right) = 0$, so it is required that $R_c\left(r_o\right) = 0$ and $k = \kappa \ln \sqrt{\kappa}$. The proper radius is then,

$$R_p(r) = \sqrt{R_c \left(R_c + \kappa\right)} - \kappa \ln\left[\frac{\sqrt{R_c} + \sqrt{R_c + \kappa}}{\sqrt{\kappa}}\right],$$

$$R_c = R_c(r).$$

The admissible form of $R_c(r)$ must now be determined. According to Einstein, the metric must be asymptotically Minkowski spacetime. Since $\kappa > 0$ by hypothesis, the application of the (spurious) argument for Newtonian approximation used by the astrophysical scientists cannot be applied here. There are no other boundary conditions that provide any means for determining the value of κ, and so it remains indeterminable. The only form that meets the condition $R_c\left(r_o\right) = 0$ and the requirement of asymptotic Minkowski spacetime is,

$$R_c(r) = |r - r_o| = \frac{1}{\sqrt{K}},$$

$$r \in \Re,$$

where r_o is entirely arbitrary. Then $R_p\left(r_o\right) = 0 \ \forall \ r_o$ and $R_c\left(r_o\right) = 0 \ \forall \ r_o$, and so, if explicit reference to the auxiliary manifold of Minkowski spacetime is not desired, $R_c(r)$ becomes superfluous and can be simply replaced by $R_c(r) = |r - r_o| = \rho$, $0 < \rho < \infty$. Thus, points in the spatial section of this spacetime have the very same properties of points in the spatial section of Minkowski spacetime. The line-element is again singular at only one point; $\rho = 0$ (i.e. in the case of explicit inclusion of the auxiliary manifold, only at the point $r = r_o$). The signature of this metric is always $(+, -, -, -)$. Clearly there is no possibility for a black hole in this case either.

The usual form of eq. (1) in isotropic coordinates is,

$$ds^2 = \frac{\left(1 - \frac{m}{2r}\right)^2}{\left(1 + \frac{m}{2r}\right)^2} dt^2 - \left(1 + \frac{m}{2r}\right)^4 \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right],$$

wherein it is again alleged that r can go down to zero. This expression has the very same metrical form as eq. (13) and so shares the very same geometrical character. Now the coefficient of dt^2 is zero when $r = m/2$, which, according to the astrophysical scientists, marks the 'radius' or 'event horizon' of a black hole, and where m is the alleged point-mass of the black hole singularity located at $r = 0$, just as in eq. (1). This further amplifies the fact that the quantity r appearing in both eq. (1) and its isotropic coordinate form is not a distance in the manifold described by these line-elements. Applying the intrinsic geometric relations detailed in **Section II** above it is clear that the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the isotropic coordinate line-element is given by,

$$R_c(r) = r \left(1 + \frac{m}{2r}\right)^2$$

and the proper radius is given by,

$$R_p(r) = r + m \ln\left(\frac{2r}{m}\right) - \frac{m^2}{2r} + \frac{m}{2}.$$

Hence, $R_c(m/2) = 2m$, and $R_p(m/2) = 0$, which are scalar invariants necessarily consistent with eq. (20). Furthermore, applying the same geometrical analysis leading to eq. (20), the generalised solution in isotropic coordinates is [57],

$$ds^2 = \frac{\left(1 - \frac{\alpha}{4h}\right)^2}{\left(1 + \frac{\alpha}{4h}\right)^2} dt^2 - \left(1 + \frac{\alpha}{4h}\right)^4 \left[dh^2 + h^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right],$$

$$h = h(r) = \left[|r - r_o|^n + \left(\frac{\alpha}{4} \right)^n \right]^{\frac{1}{n}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o,$$

wherein r_o and n are entirely arbitrary constants. Then,

$$R_c(r) = h(r) \left(1 + \frac{\alpha}{4h(r)} \right)^2 = \frac{1}{\sqrt{K(r)}},$$

$$R_p(r) = h(r) + \frac{\alpha}{2} \ln \left(\frac{4h(r)}{\alpha} \right) - \frac{\alpha^2}{8h(r)} + \frac{\alpha}{4},$$

and so

$$R_c(r_o) = \alpha, \quad R_p(r_o) = 0, \quad \forall \, r_o \, \forall \, n,$$

which are scalar invariants, in accordance with eq. (20). Clearly in these isotropic coordinate expressions r does not in itself denote any distance in the manifold, just as it does not in itself denote any distance in eq. (20) of which eqs. (1) and (2) are particular cases. It is a parameter for the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section and for the proper radius (i.e. the radial geodesic distance from the point at the centre of spherical symmetry of the spatial section). The 'interior' of the alleged Schwarzschild black hole does not form part of the solution space of the Schwarzschild manifold [2, 4, 5, 32, 41, 42, 57, 61, 62, 63].

In the same fashion it is easily proven [32, 61] that the general expression for the Kerr-Newman geometry is given by,

$$ds^2 = \frac{\Delta}{\rho^2} \left(dt - a \sin^2 \theta d\varphi^2 \right)^2 - \frac{\sin^2 \theta}{\rho^2} \left[\left(R^2 + a^2 \right) d\varphi - adt \right]^2 - \frac{\rho^2}{\Delta} dR^2 - \rho^2 d\theta^2$$

$$R = R(r) = \left(|r - r_o|^n + \beta^n \right)^{\frac{1}{n}}, \quad \beta = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - (q^2 + a^2 \cos^2 \theta)}, \quad a^2 + q^2 < \frac{\alpha^2}{4},$$

$$a = \frac{2L}{\alpha}, \quad \rho^2 = R^2 + a^2 \cos^2 \theta, \quad \Delta = R^2 - \alpha R + q^2 + a^2,$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o.$$

The Kruskal-Szekeres coordinates, the Eddington-Finkelstein coordinates, and the Regge-Wheeler coordinates do not take into account the rôle of Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of the Schwarzschild manifold [64], and so they thereby violate the geometric form of the line-element, making them invalid.

The foregoing amplifies the inadmissibility of the introduction of the Newtonian potential into Schwarzschild spacetime. The Newtonian potential is a **two-body** concept; it is defined as the work done per unit mass against the gravitational field. There is no meaning to a Newtonian potential for a single mass in an otherwise empty Universe. Newton's theory of gravitation is defined in terms of the interaction of two masses in a space for which the 'Principle of Superposition' applies. In Newton's theory there is no limit set to the number of masses that can be piled up in space, although the analytical relations for the gravitational interactions of many bodies upon one another quickly become intractable. In Einstein's theory matter cannot be piled up in a given spacetime because the matter itself determines the structure of the spacetime containing the matter. It is clearly impossible for Schwarzschild spacetime, which is alleged by the astrophysical scientists to contain one mass in an otherwise totally empty Universe, to reduce to or otherwise contain an expression that is defined in terms of the *a priori* interaction of two masses. This is illustrated even further by writing eq. (1) in terms of c and G explicitly,

$$ds^2 = \left(c^2 - \frac{2Gm}{r} \right) dt^2 - c^2 \left(c^2 - \frac{2Gm}{r} \right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right).$$

The term $2Gm/r$ is the square of the Newtonian escape velocity from a mass m. And so the astrophysical scientists assert that when the "escape velocity" is that of light in vacuum, there is an event horizon ("Schwarzschild radius") and hence a black hole. But escape velocity is a concept that involves **two bodies** - one body escapes from another

body. Even though one mass appears in the expression for escape velocity, it cannot be determined without recourse to a fundamental two-body gravitational interaction. Recall that Newton's Universal Law of Gravitation is,

$$F_g = -G\frac{mM}{r^2},$$

where G is the gravitational constant and r is the distance between the centre of mass of m and the centre of mass of M. A centre of mass is an infinitely dense point-mass, but it is not a physical object; merely a mathematical artifice. Newton's gravitation is clearly defined in terms of the interaction of two bodies. Newton's gravitational potential Φ is defined as,

$$\Phi = \lim_{\sigma \to \infty} \int_\sigma^r -\frac{F_g}{m} dr = -G\frac{M}{r},$$

which is the work done per unit mass in the gravitational field due to masses M and m. This is a two-body concept. The potential energy P of a mass m in the gravitational field due to masses M and m is therefore given by,

$$P = m\Phi = -G\frac{mM}{r},$$

which is clearly a two-body concept.

Similarly, the velocity required by a mass m to escape from the gravitational field due to masses M and m is determined by,

$$F_g = -G\frac{mM}{r^2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dr}.$$

Separating variables and integrating gives,

$$\int_v^0 mv\, dv = \lim_{r_f \to \infty} \int_R^{r_f} -GmM\, \frac{dr}{r^2},$$

whence

$$v = \sqrt{\frac{2GM}{R}},$$

where R is the radius of the mass M. Thus, escape velocity necessarily involves two bodies: m escapes from M. In terms of the conservation of kinetic and potential energies,

$$K_i + P_i = K_f + P_f$$

whence,

$$\frac{1}{2}mv^2 - G\frac{mM}{R} = \frac{1}{2}mv_f^2 - G\frac{mM}{r_f}.$$

Then as $r_f \to \infty$, $v_f \to 0$, and the escape velocity of m from M is,

$$v = \sqrt{\frac{2GM}{R}}.$$

Once again, the relation is derived from a two-body gravitational interaction.

The consequence of all this for black holes and their associated gravitational waves is that there can be no gravitational waves generated by black holes because the latter are fictitious.

IV. The prohibition of point-mass singularities

The black hole is alleged to contain an infinitely dense point-mass singularity, produced by irresistible gravitational collapse (see for example [17, 24, 77], for the typical claim). According to Hawking [80]:

> "The work that Roger Penrose and I did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole."

The singularity of the alleged Big Bang cosmology is, according to many proponents of the Big Bang, also infinitely dense. Yet according to Special Relativity, infinite densities are forbidden because their existence implies that a material object can acquire the speed of light c in vacuo (or equivalently, the existence of infinite energies), thereby violating the very basis of Special Relativity. Since General Relativity cannot violate Special Relativity, General Relativity must therefore also forbid infinite densities. Point-mass singularities are alleged to be infinitely dense objects. Therefore, point-mass singularities are forbidden by the Theory of Relativity.

Let a cuboid rest-mass m_0 have sides of length L_0. Let m_0 have a relative speed $v < c$ in the direction of one of three mutually orthogonal Cartesian axes attached to an observer of rest-mass M_0. According to the observer M_0, the moving mass m is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and the volume V thereof is

$$V = L_0^3 \sqrt{1 - \frac{v^2}{c^2}}.$$

Thus, the density D is

$$D = \frac{m}{V} = \frac{m_0}{L_0^3 \left(1 - \frac{v^2}{c^2}\right)},$$

and so $v \to c \;\Rightarrow\; D \to \infty$. Since, according to Special Relativity, no material object can acquire the speed c (this would require an infinite energy), infinite densities are forbidden by Special Relativity, and so point-mass singularities are forbidden. Since General Relativity cannot violate Special Relativity, it too must thereby forbid infinite densities and hence forbid point-mass singularities. It does not matter how it is alleged that a point-mass singularity is generated by General Relativity because the infinitely dense point-mass cannot be reconciled with Special Relativity. Point-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

It is nowadays routinely claimed that many black holes have been found. The signatures of the black hole are (a) an infinitely dense 'point-mass' singularity and (b) an event horizon. Nobody has ever found an infinitely dense 'point-mass' singularity and nobody has ever found an event horizon, so nobody has ever assuredly found a black hole. It takes an infinite amount of observer time to verify a black hole event horizon [24, 28, 36, 48, 54, 56, 71]. Nobody has been around and nobody will be around for an infinite amount of time and so no observer can ever verify the presence of an event horizon, and hence a black hole, in principle, and so the notion is irrelevant to physics. All reports of black holes being found are patently false; the product of wishful thinking.

V. Laplace's alleged black hole

It has been claimed by the astrophysical scientists that a black hole has an escape velocity c (or $\geq c$, the speed of light in vacuo) [6,12–14,16,18,19,24,28,76,78,80–82]. Chandrasekhar [24] remarked,

> "Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it.
>
> ... A trapped surface is one from which light cannot escape to infinity."

According to Hawking,

> "Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. Its boundary is called the event horizon. It coincides with the paths of the light rays that just fail to escape from the black hole."

However, according to the alleged properties of a black hole, nothing at all can even **leave** the black hole. In the very same paper Chandrasekhar made the following quite typical contradictory assertion propounded by the astrophysical scientists:

"The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge."

Hughes [28] reiterates,

"Things can go into the horizon (from $r > 2M$ to $r < 2M$), but they cannot get out; once inside, all causal trajectories (timelike or null) take us inexorably into the classical singularity at $r = 0$.

"The defining property of black holes is their event horizon. Rather than a true surface, black holes have a 'one-way membrane' through which stuff can go in but cannot come out."

Taylor and Wheeler [25] assert,

"... Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon ... and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter."

In the Dictionary of Geophysics, Astrophysics and Astronomy [78], one finds the following assertions:

*"**black hole** A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*

$$v_{esc} = \sqrt{\frac{2GM}{R}},$$

where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole. *The required radius R_{BH} follows from setting v_{esc} equal to c:*

$$R_{BH} = \frac{2GM}{c^2}.$$

... "In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression R_{BH} holds for the surface of a black hole. The surface of a black hole at R_{BH} is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon."

Now if its escape velocity is really that of light in vacuo, then by definition of escape velocity, light would escape from a black hole, and therefore be seen by all observers. If the escape velocity of the black hole is greater than that of light in vacuo, then light could emerge but not escape, and so there would always be a class of observers that could see it. Not only that, if the black hole had an escape velocity, then material objects with an initial velocity less than the alleged escape velocity, could leave the black hole, and therefore be seen by a class of observers, but not escape (just go out, come to a stop and then fall back), even if the escape velocity is $\geq c$. Escape velocity does not mean that objects cannot leave; it only means they cannot escape if they have an initial velocity less than the escape velocity. So on the one hand it is claimed that black holes have an escape velocity c, but on the other hand that nothing, not even light, can even leave the black hole. The claims are contradictory - nothing but a meaningless play on the words "escape velocity" [67, 68]. Furthermore, as demonstrated in **Section III**, escape velocity is a **two-body** concept, whereas the black hole is derived not from a two-body gravitational interaction, but from a one-body concept. The black hole has no escape velocity.

It is also routinely asserted that the theoretical Michell-Laplace (M-L) dark body of Newton's theory, which has an escape velocity $\geq c$, is a kind of black hole [6, 11, 14, 24, 78, 80] or that Newton's theory somehow predicts *"the radius of a black hole"* [25]. Hawking remarks,

"On this assumption a Cambridge don, John Michell, wrote a paper in 1783 in the Philosophical Transactions of the Royal Society of London. In it, he pointed out that a star that was sufficiently massive and compact would have such a strong gravitational field that light could not escape. Any light emitted from the surface of the star would be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be a large number of stars like this. Although we would not be able to see them because light from them would not reach us, we could still feel their gravitational attraction. Such objects are what we now call black holes, because that is what they are – black voids in space."

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But the M-L dark body is not a black hole. The M-L dark body possesses an escape velocity, whereas the black hole has no escape velocity; objects can leave the M-L dark body, but nothing can leave the black hole; it does not require irresistible gravitational collapse, whereas the black hole does; it has no infinitely dense point-mass singularity, whereas the black hole does; it has no event horizon, whereas the black hole does; there is always a class of observers that can see the M-L dark body [67, 68], but there is no class of observers that can see the black hole; the M-L dark body can persist in a space which contains other matter and interact with that matter, but the spacetime of the "Schwarzschild" black hole (and variants thereof) is devoid of matter by construction and so it cannot interact with anything. Thus the M-L dark body does not possess the characteristics of the alleged black hole and so it is not a black hole.

VI. Black hole interactions and gravitational collapse

The literature abounds with claims that black holes can interact in such situations as binary systems, mergers, collisions, and with surrounding matter generally. According to Chandrasekhar [24], for example, who also cites S. Hawking,

> "From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.

> "In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that **in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease**; it can at best remain unchanged (if the conditions are stationary).

> "Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass $\frac{1}{2}M$, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass $\overline{M}$. Then Hawking's theorem requires that

$$16\pi\overline{M}^2 \geq 16\pi \left[2 \left(\frac{1}{2}M \right)^2 \right] = 8\pi M^2$$

> or

$$\overline{M} \geq M/\sqrt{2}.$$

> Hence the maximum amount of energy that can be released in such a coalescence is

$$M \left(1 - 1/\sqrt{2} \right) c^2 = 0.293Mc^2."$$

Hawking [80] says,

> "Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes."

According to Schutz [48],

> "... Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can never bifurcate spontaneously into two smaller ones.

> "Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved."

Townsend [56] also arbitrarily applies the 'Principle of Superposition' to obtain charged black hole (Reissner-Nordström) interactions as follows:

"The extreme RN in isotropic coordinates is

$$ds^2 = V^{-2}dt^2 + V^2\left(d\rho^2 + \rho^2 d\Omega^2\right)$$

where

$$V = 1 + \frac{M}{\rho}$$

This is a special case of the multi black hole solution

$$ds^2 = V^{-2}dt^2 + V^2 d\vec{x} \cdot d\vec{x}$$

where $d\vec{x} \cdot d\vec{x}$ is the Euclidean 3-metric and V is any solution of $\nabla^2 V = 0$. In particular

$$V = 1 + \sum_{i=1}^{N} \frac{M_i}{\left|\vec{x} - \vec{\bar{x}}_i\right|}$$

yields the metric for N extreme black holes of masses M_i at positions $\bar{x}_i$.

Now Einstein's field equations are non-linear, so the 'Principle of Superposition' **does not apply** [51, 67, 79]. Therefore, before one can talk of black hole binary systems and the like it must first be proven that the two-body system is theoretically well-defined by General Relativity. This can be done in only two ways:

(a) Derivation of an exact solution to Einstein's field equations for the two-body configuration of matter; or

(b) Proof of an existence theorem.

There are no known solutions to Einstein's field equations for the interaction of two (or more) masses (charged or not), so option (a) has never been fulfilled. No existence theorem has ever been proven, by which Einstein's field equations can even be said to admit of latent solutions for such configurations of matter, and so option (b) has never been fulfilled. The "Schwarzschild" black hole is allegedly obtained from a line-element satisfying $Ric = 0$. For the sake of argument, assuming that black holes are predicted by General Relativity as alleged in relation to metric (1), since $Ric = 0$ is a statement that there is no matter in the Universe, one cannot simply insert a second black hole into the spacetime of $Ric = 0$ of a given black hole so that the resulting two black holes (each obtained separately from $Ric = 0$) mutually persist in and mutually interact in a mutual spacetime that **by construction contains no matter**! One cannot simply assert by an analogy with Newton's theory that two black holes can be components of binary systems, collide or merge [51, 67, 68], because the 'Principle of Superposition' does not apply in Einstein's theory. Moreover, General Relativity has to date been unable to account for the simple experimental fact that two fixed bodies will approach one another upon release. Thus, black hole binaries, collisions, mergers, black holes from supernovae, and other black hole interactions are all invalid concepts.

Much of the justification for the notion of irresistible gravitational collapse into an infinitely dense point-mass singularity, and hence the formation of a black hole, is given to the analysis due to Oppenheimer and Snyder [69]. Hughes [28] relates it as follows;

"In an idealized but illustrative calculation, Oppenheimer and Snyder ... showed in 1939 that a black hole in fact does form in the collapse of ordinary matter. They considered a 'star' constructed out of a pressureless 'dustball'. By Birkhof's Theorem, the entire exterior of this dustball is given by the Schwarzschild metric ... . Due to the self-gravity of this 'star', it immediately begins to collapse. Each mass element of the pressureless star follows a geodesic trajectory toward the star's center; as the collapse proceeds, the star's density increases and more of the spacetime is described by the Schwarzschild metric. Eventually, the surface passes through $r = 2M$. At this point, the Schwarzschild exterior includes an event horizon: A black hole has formed. Meanwhile, the matter which formerly constituted the star continues collapsing to ever smaller radii. In short order, all of the original matter reaches $r = 0$ and is compressed (classically!) into a singularity[4].

[4] *"Since all of the matter is squashed into a point of zero size, this classical singularity must be modified in a a complete, quantum description. However, since all the singular nastiness is hidden behind an event horizon where it is causally disconnected from us, we need not worry about it (at least for astrophysical black holes)."*

Note that the 'Principle of Superposition' has again been arbitrarily applied by Oppenheimer and Snyder, from the outset. They first **assume** a relativistic universe in which there are multiple mass elements present *a priori*, where the 'Principle of Superposition' however, does not apply, and despite there being no solution or existence theorem for such configurations of matter in General Relativity. Then all these mass elements "collapse" into a central point (zero volume; infinite density). Such a collapse has however not been given any specific general relativistic mechanism in this argument; it is simply asserted that the "collapse" is due to self-gravity. But the "collapse" cannot be due to Newtonian gravitation, given the resulting black hole, which does not occur in Newton's theory of gravitation. And a Newtonian universe cannot "collapse" into a non-Newtonian universe. Moreover, the black hole so formed is in an empty universe, since the "Schwarzschild black hole" relates to $Ric = 0$, a spacetime that by construction contains no matter. Nonetheless, Oppenheimer and Snyder permit, within the context of General Relativity, the presence of and the gravitational interaction of many mass elements, which coalesce and collapse into a point of zero volume to form an infinitely dense point-mass singularity, when there is no demonstrated general relativistic mechanism by which any of this can occur.

Furthermore, nobody has ever observed a celestial body undergo irresistible gravitational collapse and there is no laboratory evidence whatsoever for such a phenomenon.

VII. Further consequences for gravitational waves

The question of the localisation of gravitational energy is related to the validity of the field equations $R_{\mu\nu} = 0$, for according to Einstein, matter is the cause of the gravitational field and the causative matter is described in his theory by a mathematical object called the energy-momentum tensor, which is coupled to geometry (i.e. spacetime) by his field equations, so that matter causes spacetime curvature (his gravitational field). Einstein's field equations,

"... *couple the gravitational field (contained in the curvature of spacetime) with its sources.*" [36]

"*Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter.*" [53]

"*Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that* **the metrical field** *(or, in mathematical language, the tensor with components g_{ik})* **is related to the material filling the world.**" [5]

"... *we have, in following the ideas set out just above, to discover the* **invariant law of gravitation, according to which matter determines the components** $\Gamma^\alpha_{\beta\iota}$ **of the gravitational field**, *and which replaces the Newtonian law of attraction in Einstein's Theory.*"[5]

"*Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field.*" [51]

"*Clearly, the mass density, or equivalently, energy density $\varrho(\vec{x}, t)$ must play the role as a source. However, it is the 00 component of a tensor $T_{\mu\nu}(x)$, the mass-energy-momentum distribution of matter. So, this tensor must act as the source of the gravitational field.*" [10]

"*In general relativity, the stress-energy or energy-momentum tensor T^{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation*". [79]

Qualitatively Einstein's field equations are:

Spacetime geometry = -κ × *causative matter* (i.e. material sources)

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where *causative matter* is described by the energy-momentum tensor and κ is a constant. The spacetime geometry is described by a mathematical object called Einstein's tensor, $G_{\mu\nu}$, $(\mu, \nu = 0, 1, 2, 3)$ and the energy-momentum tensor is $T_{\mu\nu}$. So Einstein's full field equations are[2]:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa T_{\mu\nu}. \tag{21}$$

Einstein asserted that his 'Principle of Equivalence' and his laws of Special Relativity must hold in a sufficiently small region of his gravitational field. Here is what Einstein [52] himself said in 1954, the year before his death:

> *"Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K',uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For, according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K').*

> *"Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy."*

In their textbook, Foster and Nightingale [36] succinctly state the 'Principle of Equivalence' thus:

> *"We may incorporate these ideas into the principle of equivalence, which is this: In a freely falling (nonrotating) laboratory occupying a small region of spacetime, the laws of physics are the laws of special relativity."*

According to Pauli [53],

> *"We can think of the physical realization of the local coordinate system K_o in terms of a freely floating, sufficiently small, box which is not subjected to any external forces apart from gravity, and which is falling under the influence of the latter. ... "It is evidently natural to assume that the special theory of relativity should remain valid in K_o."*

Taylor and Wheeler state in their book [25],

> *"General Relativity requires more than one free-float frame."*

In the Dictionary of Geophysics, Astrophysics and Astronomy [78],

> *"Near every event in spacetime, in a sufficiently small neighborhood, in every freely falling reference frame all phenomena (including gravitational ones) are exactly as they are in the absence of external gravitational sources."*

Note that the 'Principle of Equivalence' involves the *a priori* presence of multiple arbitrarily large finite masses. Similarly, the laws of Special Relativity involve the *a priori* presence of at least two arbitrarily large finite masses; for otherwise relative motion between two bodies cannot manifest. The postulates of Special Relativity are themselves couched in terms of inertial systems, which are in turn defined in terms of mass via Newton's First Law of motion.

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[2]The so-called "cosmological constant" is not included.

In the space of Newton's theory of gravitation, one can simply put in as many masses as one pleases. Although solving for the gravitational interaction of these masses rapidly becomes beyond our capacity, there is nothing to prevent us inserting masses conceptually. This is essentially the 'Principle of Superposition'. However, one cannot do this in General Relativity, because Einstein's field equations are non-linear. In General Relativity, each and every configuration of matter must be described by a corresponding energy-momentum tensor and the field equations solved separately for each and every such configuration, because matter and geometry are coupled, as eq. (21) describes. Not so in Newton's theory where geometry is independent of matter. The 'Principle of Superposition' does not apply in General Relativity:

> *"In a gravitational field, the distribution and motion of the matter producing it cannot at all be assigned arbitrarily — on the contrary it must be determined (by solving the field equations for given initial conditions) simultaneously with the field produced by the same matter."* [51]

> *"An important characteristic of gravity within the framework of general relativity is that the theory is nonlinear. Mathematically, this means that if g_{ab} and γ_{ab} are two solutions of the field equations, then $ag_{ab} + b\gamma_{ab}$ (where a, b are scalars) may not be a solution. This fact manifests itself physically in two ways. First, since a linear combination may not be a solution, we cannot take the overall gravitational field of the two bodies to be the summation of the individual gravitational fields of each body."* [79]

Now Einstein and the relevant physicists assert that the gravitational field "outside" a mass contains no matter, and so they assert that $T_{\mu\nu} = 0$, and that there is only one mass in the whole Universe with this particular problem statement. But setting the energy-momentum tensor to zero means that there is no matter present by which the gravitational field can be caused! Nonetheless, it is so claimed, and it is also claimed that the field equations then reduce to the much simpler form,

$$R_{\mu\nu} = 0. \tag{22}$$

So this is a clear statement that spacetime is devoid of matter.

> *"Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time."* [78]

However, since this is a spacetime that **by construction** contains no matter, Einstein's 'Principle of Equivalence' and his laws of Special Relativity cannot manifest, thus violating the physical requirements of the gravitational field that Einstein himself laid down. It has never been proven that Einstein's 'Principle of Equivalence' and his laws of Special Relativity, both of which are defined in terms of the *a priori* presence of multiple arbitrary large finite masses, can manifest in a spacetime that by construction contains no matter. Indeed, it is a contradiction; so $R_{\mu\nu} = 0$ fails. Now eq. (1) relates to eq. (22). However, there is allegedly mass present, denoted by m in eq. (1). This mass is not described by an energy-momentum tensor. That m is actually responsible for the alleged gravitational field associated with eq. (1) is confirmed by the fact that if $m = 0$, eq. (1) reduces to Minkowski spacetime, and hence no gravitational field. So if not for the presence of the alleged mass m in eq. (1) there is no gravitational field. But this contradicts Einstein's relation between geometry and matter, since m is introduced into eq. (1) *post hoc*, not via an energy-momentum tensor describing the matter causing the associated gravitational field. The components of the metric tensor are functions of only one another, and reduce to functions of only one component of the metric tensor. None of the components of the metric tensor contain matter, because the energy-momentum tensor is zero. There is no transformation of matter in Minkowski spacetime into Schwarzschild spacetime, and so the laws of Special Relativity are not transformed into a gravitational field by $Ric = 0$. The transformation is merely from a pseudo-Euclidean geometry containing no matter into a pseudo-Riemannian (non-Euclidean) geometry containing no matter. Matter is introduced into the spacetime of $Ric = 0$ by means of a vicious circle, as follows. It is stated at the outset that $Ric = 0$ describes spacetime "outside a body". The words "outside a body" introduce matter, contrary to the energy-momentum tensor, $T_{\mu\nu} = 0$, that describes the causative matter as being absent. There is no matter involved in the transformation of Minkowski spacetime into Schwarzschild spacetime via $Ric = 0$, since the energy-momentum tensor is zero, making the components of the resulting metric tensor functions solely of one another, and reducible to functions of just one component of the metric tensor. To satisfy the initial claim that $Ric = 0$ describes spacetime "outside a body", so that the resulting spacetime is caused by the alleged mass present, the alleged causative mass is **inserted** into the resulting metric *ad hoc*, by means of a contrived

analogy with Newton's theory, thus closing the vicious circle. Here is how Chandrasekhar [24] presents the vicious circle:

> *"That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that $\frac{1}{2}v^2 > GM/R$; and it cannot escape if $v^2 < 2GM/R$. On the basis of this last inequality, Laplace concluded that if $R < 2GM/c^2 = R_s$ (say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*

> *"By a curious coincidence, the limit R_s discovered by Laplace is* exactly *the same that general relativity gives for the occurrence of the* trapped surface *around a spherical mass."*

But it is not surprising that general relativity gives the same R_s "*discovered by Laplace*" because the Newtonian expression for escape velocity is deliberately inserted *post hoc* by the astrophysical scientists, into the so-called "Schwarzschild solution" in order to make it so. Newton's escape velocity does not drop out of any of the calculations to Schwarzschild spacetime. Furthermore, although $Ric = 0$ is claimed to describe spacetime "outside a body", the resulting metric (1) is nonetheless used to describe the **interior** of a black hole, since the black hole begins at the alleged "event horizon", not at its infinitely dense point-mass singularity (said to be at $r = 0$ in eq. (1)).

In the case of a totally empty Universe, what would be the relevant energy-momentum tensor? It must be $T_{\mu\nu} = 0$. Indeed, it is also claimed that spacetimes can be intrinsically curved, i.e. that there are gravitational fields that have no material cause. An example is de Sitter's empty spherical Universe, based upon the following field equations [33, 34]:

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{23}$$

where λ is the so-called 'cosmological constant'. In the case of metric (1) the field equations are given by expression (22). On the one hand de Sitter's empty world is devoid of matter ($T_{\mu\nu} = 0$) and so has no material cause for the alleged associated gravitational field. On the other hand it is claimed that the spacetime described by eq. (22) has a material cause, *post hoc* as m in metric (1), even though $T_{\mu\nu} = 0$ there as well: a contradiction. This is amplified by the so-called "Schwarzschild-de Sitter" line-element,

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{24}$$

which is the standard solution for eq. (23). Once again, m is identified *post hoc* as mass at the centre of spherical symmetry of the manifold, said to be at $r = 0$. The completely empty universe of de Sitter [33, 34] can be obtained by setting $m = 0$ in eq. (24) to yield,

$$ds^2 = \left(1 - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{\lambda}{3}r^2\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{25}$$

Also, if $\lambda = 0$, eq. (23) reduces to eq. (22) and eq. (24) reduces to eq. (1). If both $\lambda = 0$ and $m = 0$, eqs. (24) and (25) reduce to Minkowski spacetime. Now in eq. (23) the term $\lambda g_{\mu\nu}$ is not an energy-momentum tensor, since according to the astrophysical scientists, expression (25) describes a world devoid of matter [33, 34]. The universe described by eq. (25), which also satisfies eq. (23), is completely empty and so its curvature has no material cause; in eq. (23), just as in eq. (22), $T_{\mu\nu} = 0$. So eq. (25) is alleged to describe a gravitational field that has no material cause. Furthermore, although in eq. (22), $T_{\mu\nu} = 0$, its usual solution, eq. (1), is said to contain a (*post hoc*) material cause, denoted by m therein. Thus for eq. (1) it is claimed that $T_{\mu\nu} = 0$ supports a material cause of a gravitational field, but at the same time, for eq. (25), $T_{\mu\nu} = 0$ is also claimed to preclude material cause of a gravitational field. So $T_{\mu\nu} = 0$ is claimed to include and to exclude material cause. This is not possible. The contradiction is due to the *post hoc* introduction of mass, as m, in eq. (1), by means of the Newtonian expression for gravitational potential. Furthermore, there is no experimental evidence to support the claim that a gravitational field can be generated without a material cause. Material cause is codified theoretically in eq. (21).

Since $R_{\mu\nu} = 0$ cannot describe Einstein's gravitational field, Einstein's field equations cannot reduce to $R_{\mu\nu} = 0$ when $T_{\mu\nu} = 0$. In other words, if $T_{\mu\nu} = 0$ (i.e. there is no matter present) then there is no gravitational field. Consequently Einstein's field equations must take the form [58, 59],

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0. \tag{26}$$

The $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Thus the total energy of Einstein's gravitational field is *always* zero; the $G_{\mu\nu}/\kappa$ and the $T_{\mu\nu}$ *must vanish identically*; there is *no possibility* for the localization of gravitational energy (i.e. there are no Einstein gravitational waves). This also means that Einstein's gravitational field violates the experimentally well-established usual conservation of energy and momentum [53]. Since there is no experimental evidence that the usual conservation of energy and momentum is invalid, Einstein's General Theory of Relativity violates the experimental evidence, and so it is invalid.

In an attempt to circumvent the foregoing conservation problem, Einstein invented his gravitational pseudo-tensor, the components of which he says are '*the "energy components" of the gravitational field*' [60]. His invention had a two-fold purpose (a) to bring his theory into line with the usual conservation of energy and momentum, (b) to enable him to get gravitational waves that propagate with speed c. First, Einstein's gravitational pseudo-tensor is not a tensor, and is therefore not in keeping with his theory that all equations be tensorial. Second, he constructed his pseudo-tensor in such a way that it behaves like a tensor in one particular situation, that in which he could get gravitational waves with speed c. Now Einstein's pseudo-tensor is claimed to represent the energy and momentum of the gravitational field and it is routinely applied in relation to the localisation of gravitational energy, the conservation of energy and the flow of energy and momentum.

Dirac [54] pointed out that,

> "It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy within a definite (three dimensional) region at a certain time is independent of the coordinate system. Thus, in general, gravitational energy cannot be localized. The best we can do is to use the pseudo-tensor, which satisfies condition (i) but not condition (ii). It gives us approximate information about gravitational energy, which in some special cases can be accurate."

On gravitational waves Dirac [54] remarked,

> "Let us consider the energy of these waves. Owing to the pseudo-tensor not being a real tensor, we do not get, in general, a clear result independent of the coordinate system. But there is one special case in which we do get a clear result; namely, when the waves are all moving in the same direction."

About the propagation of gravitational waves Eddington [34] remarked ($g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$),

$$\frac{\partial^2 h_{\mu\nu}}{\partial t^2} - \frac{\partial^2 h_{\mu\nu}}{\partial x^2} - \frac{\partial^2 h_{\mu\nu}}{\partial y^2} - \frac{\partial^2 h_{\mu\nu}}{\partial z^2} = 0,$$

> "... showing that the deviations of the gravitational potentials are propagated as waves with unit velocity, i.e. the velocity of light. But it must be remembered that this representation of the propagation, though always permissible, is not unique. ... All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials $g_{\mu\nu}$ pertain not only to the gravitational influence which is objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with the speed of thought, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of the $g_{\mu\nu}$.

> "The statement that in the relativity theory gravitational waves are propagated with the speed of light has, I believe, been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle."

Now Einstein's pseudo-tensor, $\sqrt{-g}\, t_\nu^\mu$, is defined by [23, 33, 34, 53, 54, 58, 60],

$$\sqrt{-g}\, t_\nu^\mu = \frac{1}{2} \left(\delta_\nu^\mu L - \frac{\partial L}{\partial g_{,\mu}^{\sigma\rho}}\, g_{,\nu}^{\sigma\rho} \right), \tag{27}$$

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wherein L is given by

$$L = -g^{\alpha\beta} \left(\Gamma^{\gamma}_{\alpha\kappa} \Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta} \Gamma^{\kappa}_{\gamma\kappa} \right). \tag{28}$$

According to Einstein [60] his pseudo-tensor *"expresses the law of conservation of momentum and of energy for the gravitational field."*

In a remarkable paper published in 1917, T. Levi-Civita [58] provided a clear and rigorous proof that Einstein's pseudo-tensor is meaningless, and therefore any argument relying upon it is fallacious. I repeat Levi-Civita's proof. Contracting eq. (27) produces a linear invariant, thus

$$\sqrt{-g}\ t^{\mu}_{\mu} = \frac{1}{2} \left(4L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}\ g^{\sigma\rho}_{,\mu} \right). \tag{29}$$

Since L is, according to eq. (28), quadratic and homogeneous with respect to the Riemann-Christoffel symbols, and therefore also with respect to $g^{\sigma\rho}_{,\mu}$, one can apply Euler's theorem to obtain (also see [34]),

$$\frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}\ g^{\sigma\rho}_{,\mu} = 2L. \tag{30}$$

Substituting expression (30) into expression (29) yields the linear invariant at L. This is a first-order, intrinsic differential invariant that depends only on the components of the metric tensor and their first derivatives (see expression (28) for L). However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita [65] proved, in 1900, that such invariants **do not exist**. This is sufficient to render Einstein's pseudo-tensor entirely meaningless, and hence all arguments relying on it false. Einstein's conception of the conservation of energy in the gravitational field is erroneous.

Linearisation of Einstein's field equations and associated perturbations have been popular. *"The existence of exact solutions corresponding to a solution to the linearised equations must be investigated before perturbation analysis can be applied with any reliability"* [21]. Unfortunately, the astrophysical scientists have not properly investigated. Indeed, linearisation of the field equations is inadmissible, even though the astrophysical scientists write down linearised equations and proceed as though they are valid, because linearisation of the field equations implies the existence of a tensor which, except for the trivial case of being precisely zero, **does not exist**; proven by Hermann Weyl [66] in 1944.

VIII. Other Violations

In writing eq. (1) the Standard Model incorrectly asserts that only the components g_{00} and g_{11} are modified by $R_{\mu\nu} = 0$. However, it is plain by expressions (20) that this is false. All components of the metric tensor are modified by the constant α appearing in eqs. (20), of which metric (1) is but a particular case.

The Standard Model asserts in relation to metric (1) that a 'true' singularity must occur where the Riemann tensor scalar curvature invariant (i.e. the Kretschmann scalar) is unbounded [21, 23, 50]. However, it has never been proven that Einstein's field equations require such a curvature condition to be fulfilled: in fact, it is not required by General Relativity. Since the Kretschmann scalar is finite at $r = 2m$ in metric (1), it is also claimed that $r = 2m$ marks a "coordinate singularity" or "removable singularity". However, these assertions violate the intrinsic geometry of the manifold described by metric (1). The Kretschmann scalar depends upon all the components of the metric tensor and all the components of the metric tensor are functions of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, owing to the form of the line-element. The Kretschmann scalar is not therefore an independent curvature invariant. Einstein's gravitational field is manifest in the curvature of spacetime, a curvature induced by the presence of matter. It should not therefore be unexpected that the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the gravitational manifold might also be modified from that of ordinary Euclidean space, and this is indeed the case for eq. (1). Metric (20) gives the modification of the Gaussian curvature fixed by the intrinsic geometry of the line-element and the required boundary conditions specified by Einstein and the astrophysical scientists, in consequence of which the Kretschmann scalar is constrained by the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. Recall that the Kretschmann scalar f is,

$$f = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}.$$

Using metric (20) gives,

$$f = 12\alpha^2 K^3 = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{(|r - r_o|^n + \alpha^n)^{\frac{6}{n}}},$$

then

$$f(r_o) = \frac{12}{\alpha^4} \quad \forall \, r_o \, \forall \, n,$$

which is a scalar invariant that corresponds to the scalar invariants $R_p(r_o) = 0$, $R_c(r_o) = \alpha$, $K(r_o) = \alpha^{-2}$.

Doughty [70] has shown that the radial geodesic acceleration a of a point in a manifold described by a line-element with the form of eq. (13) is given by,

$$a = \frac{\sqrt{-g_{11}} \, (-g^{11}) \, |g_{00,1}|}{2g_{00}}.$$

Using metric (20) once again gives,

$$a = \frac{\alpha}{R_c^{\frac{3}{2}}(r) \sqrt{R_c(r) - \alpha}}.$$

Now,

$$\lim_{r \to r_o^{\pm}} R_p(r) = 0, \qquad \lim_{r \to r_o^{\pm}} R_c(r) = \alpha,$$

and so

$$r \to r_o^{\pm} \Rightarrow a \to \infty \quad \forall \, r_o \, \forall \, n.$$

According to metric (20) there is no possibility for $R_c \leq \alpha$.

In the case of eq. (1), for which $r_o = \alpha = 2m$, $n = 1$, $r > \alpha$, the acceleration is,

$$a = \frac{2m}{r^{\frac{3}{2}} \sqrt{r - 2m}}.$$

which is infinite at $r = 2m$. But the usual unproven (and invalid) assumption that r in eq. (1) can go down to zero means that there is an infinite acceleration at $r = 2m$ where, according to the Standard Model, **there is no matter!** However, r can't take the values $0 \leq r \leq r_o = 2m$ in eq. (1), as eq. (20) shows, by virtue of the nature of the Gaussian curvature of spherically symmetric geodesic surfaces in the spatial section associated with the gravitational manifold, and the intrinsic geometry of the line-element.

The proponents of the Standard Model admit that if $0 < r < 2m$ in eq. (1), the rôles of t and r are interchanged. But this violates their construction at eq. (12), which has the fixed signature $(+,-,-,-)$, and is therefore inadmissible. To further illustrate this violation, when $2m < r < \infty$ the signature of eq. (1) is $(+,-,-,-)$; but if $0 < r < 2m$ in eq. (1), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) \text{ is negative, and } \quad g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1} \text{ is positive.}$$

So the signature of metric (1) changes to $(-,+,-,-)$. Thus the rôles of t and r are interchanged. According to Misner, Thorne and Wheeler, who use the spacetime signature $(-,+,+,+)$,

> "The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial/\partial t$, is spacelike ($g_{tt} > 0$) and $\partial/\partial r$, is timelike ($g_{rr} < 0$).

> "What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at $r = 2M$ always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside $r = 2M$. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back

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time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate $r = 2M$ to the later time coordinate $r = 0$. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty, and with never one halt along the way, r drops from $2M$ to 0.

"At $r = 2M$, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite."

Chandrasekhar [24] has expounded the same claim as follows,

'There is no alternative to the matter collapsing to an infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: "all the King's horses and all the King's men" cannot prevent it.'

Carroll [50] also says,

"This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for $r < 2GM$, t becomes spacelike and r becomes timelike.) Thus you can no more stop moving toward the singularity than you can stop getting older."

Vladmimirov, Mitskiévich and Horský [71] assert,

"For $r < 2GM/c^2$, however, the component g_{oo} becomes negative, and g_{rr}, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t.

To amplify this, set $t = r^*$ and $r = t^*$, and so for $0 < r < 2m$, eq. (1) becomes,

$$ds^2 = \left(1 - \frac{2m}{t^*}\right) dr^{*2} - \left(1 - \frac{2m}{t^*}\right)^{-1} dt^{*2} - t^{*2} \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$0 < t^* < 2m.$$

It now becomes quite clear that this is a **time-dependent metric** since all the components of the metric tensor are functions of the timelike t^*, and so this metric **bears no relationship to the original time-independent problem to be solved** [35, 46]. In other words, this metric is a **non-static solution to a static problem**: *contra hyp*! Thus, in eq. (1), $0 < r < 2m$ is meaningless, as eqs. (20) demonstrate.

Furthermore, if the signature of "Schwarzschild" spacetime is permitted to change from $(+, -, -, -)$ to $(-, +, -, -)$ in the fashion claimed for black holes, then there must be for the latter signature a corresponding generalisation of the Minkowski metric, taking the fundamental form

$$ds^2 = -e^\lambda dt^2 + e^\beta dr^2 - R^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

where λ, β and R are all unknown real-valued functions of only the real variable r, and where $e^\lambda > 0$ and $e^\beta > 0$. But this is impossible because the Minkowski spacetime metric has the fixed signature $(+, -, -, -)$, since the spatial section of Minkowski spacetime is a positive definite quadratic form; and so the foregoing generalised metric is not a generalistion of Minkowski spacetime at all.

Also of importance is the fact that Hagihara [72] proved, in 1931, that all geodesics that do not run into the boundary of the "Schwarzschild" metric at $r = 2m$ (i.e. at $R_p(r_o = 2m) = 0$) are complete.

Nobody has ever found a black hole anywhere because nobody has found an infinitely dense point-mass singularity and nobody has found an event horizon.

"Unambiguous observational evidence for the existence of astrophysical black holes has not yet been established. [28]

All claims for detection of black holes are patently false.

It has recently been admitted by astronomers [73] at the Max Planck Institute for Extraterrestrial Physics that,

(a) Nobody has ever found a black hole, despite the numerous claims for their discovery;

(b) The infinitely dense point-mass singularity of the alleged black hole is nonsense;

(c) The alleged black hole has no escape velocity, despite the claims of the astrophysical scientists;

(d) They were until very recently informed, unaware of Schwarzschild's actual solution.

The LIGO project and its international counterparts have not detected gravitational waves [74]. They are destined to detect nothing. Furthermore, the Lense-Thirring or 'frame dragging' effect was not detected by the Gravity Probe B and NASA has terminated further funding of that project [75].

IX. Three-Dimensional Spherically Symmetric Metric Manifolds - First Principles

To complete the purely mathematical foundations of this paper, the differential geometry expounded in the foregoing is now developed from first principles.

Following the method suggested by Palatini, and developed by Levi-Civita [30], denote ordinary Euclidean 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point $O \in \mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation T of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric $d\ell'^2$ unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of $d\ell'^2$ of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$, except possibly at an arbitrary point, the centre of spherical symmetry $O' \in \mathbf{M}^3$.

Let a ray i emanate from an arbitrary point $O \in \mathbf{E}^3$. There is then a corresponding geodesic $i' \in \mathbf{M}^3$ issuing from the corresponding point $O' \in \mathbf{M}^3$. Let P be any point on i other than O. There corresponds a point P' on $i' \in \mathbf{M}^3$ different to O'. Let g' be a geodesic in $\mathbf{M}^3$ that is tangential to i' at P'.

Taking i as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on i' unchanged. If g' is distinct from i', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about i would cause g' to occupy an infinity of positions in $\mathbf{M}^3$ wherein g' has for each position the property of being tangential to i' at P' in the same direction, which is impossible. Hence, g' coincides with i'.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point $O \in \mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' in $\mathbf{M}^3$ with centre of symmetry at the corresponding point $O' \in \mathbf{M}^3$.

Let Q be a point in $\Sigma \in \mathbf{E}^3$ and Q' the corresponding point in $\Sigma' \in \mathbf{M}^3$. Let $d\sigma^2$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma'^2 \in \Sigma'$ issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates r, θ, φ. Then

$$d\sigma^2 = r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.1}$$

$$r = |\overline{OQ}|.$$

Clearly, if r, θ, φ are known, Q is determined and hence also Q' in Σ'. Therefore, θ and φ can be considered to be curvilinear coordinates for Q' in Σ' and the line element $d\sigma' \in \Sigma'$ will also be represented by a quadratic form similar to (1.1). To determine $d\sigma'$, consider two elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma'_1$ and $d\sigma'_2$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis $\overline{OQ}$ in $\mathbf{E}^3$, and so $d\sigma'_1$ and $d\sigma'_2$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $\frac{d\sigma'}{d\sigma}$ is the same for the two different directions irrespective of $d\theta$ and $d\varphi$, and so the foregoing ratio is a function of position, i.e. of

r, θ, φ. But Q is an arbitrary point in Σ, and so $\frac{d\sigma'}{d\sigma}$ must have the same ratio for any corresponding points Q and Q'. Therefore, $\frac{d\sigma'}{d\sigma}$ is a function of r alone, thus

$$\frac{d\sigma'}{d\sigma} = H(r),$$

and so

$$d\sigma'^2 = H^2(r)d\sigma^2 = H^2(r)r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.2}$$

where $H(r)$ is *a priori* unknown. For convenience set $R_c = R_c(r) = H(r)r$, so that (1.2) becomes

$$d\sigma'^2 = R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.3}$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing (1.3) with (1.1) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point Q', i.e. in terms of the relation $K = \frac{1}{R_c^2}$ since the Gaussian curvature of (1.1) is $K = \frac{1}{r^2}$. This is an intrinsic property of all line elements of the form (1.3) [30]. Accordingly, R_c, the inverse square root of the Gaussian curvature, can be regarded as the radius of Gaussian curvature. Therefore, in (1.1) the radius of Gaussian curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilics.

Let the element of radial distance from $O \in \mathbf{E}^3$ be dr. Clearly, the radial lines issuing from O cut the surface Σ orthogonally. Combining this with (1.1) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$

$$d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1.4}$$

Let the corresponding radial geodesic from the point $O' \in \mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from O' cut the geodesic surface Σ' orthogonally. Combining this with (1.3) by the theorem of Pythagoras gives the line element in $\mathbf{M}^3$ as,

$$d\ell'^2 = dR_p^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.5}$$

where dR_p is, by spherical symmetry, also a function only of R_c. Set $dR_p = \sqrt{B(R_c)}dR_c$, so that (1.5) becomes

$$d\ell'^2 = B(R_c)dR_c^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.6}$$

where $B(R_c)$ is an *a priori* unknown function.

Expression (1.6) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point $O' \in \mathbf{M}^3$.

Considering (1.4), the distance $R_p = |\overline{OQ}|$ from the point at the centre of spherical symmetry O to a point $Q \in \Sigma$, is given by

$$R_p = \int_0^r dr = r = R_c.$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature of the spherically symmetric geodesic surface containing any point in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point $O \in \mathbf{E}^3$ and the point $Q \in \Sigma$. Thus, in this case, $K = \frac{1}{R_c^2} = \frac{1}{R_p^2} = \frac{1}{r^2}$. However, this is not a general relation, since according to (1.5) and (1.6), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point $O' \in \mathbf{M}^3$ is not the same as the radius of Gaussian curvature of the associated spherically symmetric geodesic surface, but is given by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c(r) = \int_0^r \sqrt{B(R_c(r))}\, \frac{dR_c(r)}{dr}dr,$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \leq r < \infty$ in (1.4) that it must follow that in (1.5) and (1.6) $0 \leq R_c(r) < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from (1.5) and (1.6) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so

from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the point at the centre of spherical symmetry to some other point in a metric manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the radius of Gaussian curvature of an associated spherically symmetric surface and the radial distance from the point at the centre of spherical symmetry, owing to the fact that R_p and R_c are identical in that special case.

It should also be noted that in writing expressions (1.4) and (1.5) it is implicit that $O \in \mathbf{E}^3$ is defined as being located at the origin of the coordinate system of (1.4), i.e. O is located where $r = 0$, and by correspondence O' is defined as being located at the origin of the coordinate system of (1.5) and of (1.6); $O' \in \mathbf{M}^3$ is located where $R_p = 0$. Furthermore, since it is well known that a geometry is completely determined by the form of the line element describing it [33], expressions (1.4), (1.5) and (1.6) share the very same fundamental geometry because they are line elements of the same metrical groundform.

Expression (1.6) plays an important rôle in Einstein's alleged gravitational field.

X. Conclusions

"Schwarzschild's solution" is not Schwarzschild's solution. Schwarzschild's actual solution forbids black holes. The quantity 'r' appearing in the so-called "Schwarzschild solution" is not a distance of any kind in the associated manifold - it is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. This simple fact completely subverts all claims for black holes.

The generalisation of Minkowski spacetime to Schwarzschild spacetime, via $Ric = 0$, a spacetime that by construction contains no matter, is not a generalisation of Special Relativity. Neither Einstein's 'Principle of Equivalence' nor his laws of Special Relativity can manifest in a spacetime that by construction contains no matter.

Despite claims for discovery of black holes, nobody has ever found a black hole; no infinitely dense point-mass singularity and no event horizon have ever been found. There is no physical evidence for the existence of infinitely dense point-masses. The black hole is fictitious. The international search for black holes is destined to find none.

The Michell-Laplace dark body is not a black hole. Newton's theory of gravitation does not predict black holes.

Curved spacetimes without material cause violate the physical principles of General Relativity. There is no experimental evidence supporting the notion of gravitational fields generated without material cause.

No celestial body has ever been observed to undergo irresistible gravitational collapse. There is no laboratory evidence for irresistible gravitational collapse. Infinitely dense point-mass singularities howsoever formed cannot be reconciled with Special Relativity, i.e. they violate Special Relativity, and therefore violate General Relativity.

The Riemann tensor scalar curvature invariant (the Kretschmann scalar) is not an independent curvature invariant - it is a function of the intrinsic Gaussian curvature of the spherically symmetric geodesic surface in the spatial section.

General Relativity cannot account for the simple experimental fact that two fixed bodies will approach one another upon release. There are no known solutions to Einstein's field equations for two or more masses and there is no existence theorem by which it can even be asserted that his field equations contain latent solutions for such configurations of matter. All claims for black hole interactions are invalid.

Einstein's gravitational waves are fictitious; Einstein's gravitational energy cannot be localised; so the international search for Einstein's gravitational waves is destined to detect nothing. No gravitational waves have been detected. Einstein's pseudo-tensor is meaningless and linearisation of Einstein's field equations inadmissible. And the Lense-Thirring effect was not detected by the Gravity Probe B.

Einstein's field equations violate the experimentally well-established usual conservation of energy and momentum, and therefore violate the experimental evidence.

Acknowledgment

Dedication

I dedicate this paper to my late brother,

Paul Raymond Crothers

12th May 1968 − 25th December 2008.

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