Dear Professor MacCallum,

I am new to theoretical physics. I have taken up the subject in my middleaged spare time. Consequently, I approach the arguments for and against the conventional interpretation of the Hilbert solution without any academic bias, in terms of the analytical skills I have acquired in my other academic work and from my long experience as an erstwhile private detective. My motivation has been to find out for myself whether or not Einstein's theory is as esoteric and incomprehensible as so often claimed. I feel that I have verified my initial suspicions that the theory is not beyond the cognitive powers of anyone interested in the subject, willing to think hard about it.

I am certainly not as well-read on the subject as the experts so I am not sure if what I have to say will simply be a repeat of the arguments you have warned me about, and therefore I suppose I run the risk of trying your patience.

In my study I have been surprised to find so many inaccuracies and contrary claims that my experience as a private detective leads me to believe that all is not as it should be. Perhaps my former professional requirements for accuracy in argument and evidence influence me unduly in matters of science, but science and detective work have the common requirement of logic and substantiation; and in mathematical sciences mathematical rigour is paramount.

The first thing that struck me when I encountered the Hilbert solution and the subsequent conventional interpretation thereof, aside of the inaccuracies in the claims attributed to Schwarzschild, and the fact that the solution is not due to Schwarzschild at all, is that two assumptions are made which I do not feel are mathematically justified, viz.,

a) The regions 0 < r < 2m and $2m < r < \infty$ are valid regions. b) The parameter r is a radius in the gravitational field - that r is a measurable quantity.

As I see it, one cannot talk about extensions into the region 0 < r < 2m or division into R and T regions until it has been rigorously established that the said regions are valid to begin with. Mere assumption is not, in my view, permissible. Similarly, one cannot treat the r-parameter as a radius and measurable quantity in the gravitational field without first demonstrating that it is such. My perception is that none of this has been done in the conventional analysis. Therefore, my investigations first considered the answers to these issues.

I note that it is required that the metric of Special Relativity is to be recovered in the absence of matter, and in the far-field. The said metric is

$$ds^{2} = dt^{2} - dr^{2} - r^{2} \left(d\theta^{2} + \sin^{2} \theta d\varphi^{2} \right).$$
(1)

Obviously, the spatial component of this metric describes a sphere, centred at the point $r_c = 0$. Consider two concentric spherical surfaces at r_o and r, $r_o < r$ (owing to the isotrophy of space there is no loss of generality in the restriction $r \ge r_o$). The distance between these surfaces is an interval along a radial line through r_c, r_o and r. Consequently, one can consider r_o and r as points, just any

points, that are on the radial line cutting the surfaces at r_o and r. These points lie at the ends of that interval, which is orthogonal to the spherical surfaces.

The geometry of (1) is such that that the distance between r_o and r is given by,

$$d = \int_{r_o}^{r} dr = r - r_o.$$
 (2)

d is a Euclidean distance. If $r_o = 0$ then $d \equiv r$.

According to (1) the circumference χ of a great circle is given by,

$$\chi = 2\pi r. \tag{3}$$

 χ is Euclidean; the usual equation for the circumference of a circle. Furthermore, the circumference of a great circle centred at r_o and reaching to r is,

$$\chi = 2\pi d,\tag{4}$$

where d is given by (2). Indeed, d may be considered the radius of a sphere centred on r_o .

Now introduce a test particle at each of r_o and r. Let the particle located at r_o acquire mass. As I see it, the coordinates r_o and r do not change, however the distance between r_o and r will no longer be given by (2) and the circumference of a great circle centred at r_o and reaching to r will no longer be given by (4). The static generalisation of (1) can be written as,

$$ds^{2} = A(r)dt^{2} - B(r)dr^{2} - C(r)\left(d\theta^{2} + \sin^{2}\theta d\varphi^{2}\right), \qquad (5)$$
$$A, B, C > 0.$$

The solution of (5) for the gravitational field will yield a mapping of the Euclidean distance $d = r - r_o$ into a non-Euclidean distance $R_P(r)$ in the pseudo-Riemannian manifold locally generated by the presence of matter at r_o . I seek this mapping.

Transform (5) by setting,

$$r^* = \sqrt{C(r)}.\tag{6}$$

Then (5) becomes,

$$ds^{2} = A^{*}(r^{*})dt^{2} - B^{*}(r^{*})dr^{*2} - r^{*2}\left(d\theta^{2} + \sin^{2}\theta d\varphi^{2}\right).$$
 (7)

In the usual way one obtains the solution to (7) as,

$$ds^{2} = \left(\frac{r^{*} - \alpha}{r^{*}}\right) dt^{2} - \left(\frac{r^{*}}{r^{*} - \alpha}\right) dr^{*2} - r^{*2} (d\theta^{2} + \sin^{2}\theta d\varphi^{2}), \qquad (8)$$
$$\alpha = 2m,$$

which by using (6) becomes,

$$ds^{2} = \left(\frac{\sqrt{C} - \alpha}{\sqrt{C}}\right) dt^{2} - \left(\frac{\sqrt{C}}{\sqrt{C} - \alpha}\right) \frac{C^{2}}{4C} dr^{2} - C(d\theta^{2} + \sin^{2}\theta d\varphi^{2}).$$
(9)

Alternatively one could drop the * in (8) to obtain the familiar Droste/Weyl/(Hilbert) line-element,

$$ds^{2} = \left(\frac{r-\alpha}{r}\right)dt^{2} - \left(\frac{r}{r-\alpha}\right)dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\varphi^{2}), \qquad (10)$$

and then noting, as did J. Droste and A. Eddington, that r^2 can be replaced by a general function of r without destroying the spherical symmetry of (10). Let that function be C(r), and so equation (9) is again obtained. Equation (10) is a particular solution, but equation (9) provides a means by which the form of C(r) might be determined to obtain a means by which all particular solutions, in terms of an infinite sequence, may be constructed, according to the general prescription of Eddington. Clearly, the correct form of C(r) must naturally yield the Droste/Weyl/(Hilbert) solution, as well as the Schwarzschild solution, and the Brillouin solution, amongst the infinitude of particular solutions that the field equations admit.

I have made no assumptions as to the range on the parameter r. The only assumption about r that I make is that the point-mass is to be located somewhere, and that somewhere is r_o , the value of which must be obtained rigorously from the geometry of equation (9).

The geometrical relationships between the components of the metric tensor of (1) must be precisely the same in (9), and in (10). Therefore, the circumference χ of a great circle on (9) is given by,

$$\chi = 2\pi \sqrt{C(r)},\tag{11}$$

and the proper distance (proper radius) $R_p(r)$ is,

$$R_p(r) = \int \sqrt{B(r)} dr.$$
 (12)

I call $\sqrt{C(r)}$ the curvature radius.

Taking B(r) from (9) gives,

= 1

$$R_{p}(r) = \int \sqrt{\frac{\sqrt{C}}{\sqrt{C} - \alpha}} \frac{C'}{2\sqrt{C}} dr, \qquad (13)$$
$$\sqrt{\sqrt{C(r)} \left(\sqrt{C(r)} - \alpha\right)} + \alpha \ln \left| \frac{\sqrt{\sqrt{C(r)}} + \sqrt{\sqrt{C(r)} - \alpha}}{K} \right|,$$
$$K = const.$$

The relationship between r and R_p is,

as
$$r \to r_o, R_p(r) \to 0$$

where r_o is the location of the point-mass. Clearly $0 \le R_p < \infty$.

From (13),

$$R_p(r_o) = 0 = \sqrt{\sqrt{C(r_o)} \left(\sqrt{C(r_o)} - \alpha\right)} +$$

$$+ \alpha \ln \left| \frac{\sqrt{\sqrt{C(r_o)}} + \sqrt{\sqrt{C(r_o)} - \alpha}}{K} \right|,$$
(14)

and so, from (14),

$$\sqrt{C(r_o)} = \alpha, \ K = \sqrt{\alpha}.$$
 (15)

Therefore (14) becomes

$$R_p(r) = \sqrt{\sqrt{C(r)} \left(\sqrt{C(r)} - \alpha\right)} + \alpha \ln \left| \frac{\sqrt{\sqrt{C(r)}} + \sqrt{\sqrt{C(r)}} - \alpha}{\sqrt{\alpha}} \right|, \quad (16)$$
$$r_o < r < \infty.$$

Equation (16) is the required mapping. One can see that r_o cannot be determined: in other words, r_o is entirely arbitrary. One also notes that (9) is singular only when $r = r_o$ in which case $g_{oo} = 0$. There is no value of r that makes $g_{11} = 0$.

Clearly, r does not determine the geometry of the gravitational field directly. It is not a radial coordinate in the gravitational field. $R_p(r)$ is the non-Euclidean radial coordinate in the pseudo-Riemannian manifold of the gravitational field around the point r_o .

Now in addition to the established fact that $\sqrt{C(r_o)} = \alpha$, C(r) must also satisfy the no matter condition,

$$C(r) \equiv r^2$$
 when $\alpha = 0$,

and the far-field condition,

$$\lim_{r \to \infty} \frac{C(r)}{r^2} \to 1.$$
(17)

Furthermore, C(r) is a strictly monotonically increasing function of r, i.e. $C'(r) > 0 \forall r > r_o$. The only general form for C(r), from which an infinite sequence of particular solutions satisfying all the required conditions, is,

$$C_n(r) = \left[(r - r_o)^n + \alpha^n \right]^{\frac{2}{n}},$$
$$n \in \Re^+, \ r_o = b\alpha, \ b \in \Re,$$

where n and b are arbitrary. Owing to the isotrophy of space there is no loss of generality in setting the restriction $b \ge 0$, so that the admissible $C_n(r)$ are given by,

$$C_n(r) = \left[(r - r_o)^n + \alpha^n \right]^{\frac{2}{n}},$$

$$n \in \Re^+, \ r_o = b\alpha, \ b \in \left(\Re - \Re^-\right).$$
(18)

According to (18), when b = 0 and n is taken in integers, the following infinite sequence of particular solutions obtains,

$$\begin{split} C_1(r) &= (r+\alpha)^2 \quad \text{(Brillouin's solution)} \\ C_2(r) &= r^2 + \alpha^2 \\ C_3(r) &= (r^3 + \alpha^3)^{\frac{2}{3}} \quad \text{(Schwarzschild's original solution)} \\ C_4(r) &= (r^4 + \alpha^4)^{\frac{1}{2}}, \quad \text{etc.} \end{split}$$

When b = 1 and n is taken in integers, the following infinite sequence of particular solutions obtains,

$$C_{1}(r) = r^{2} \text{ [Droste/Weyl/(Hilbert) solution]} C_{2}(r) = (r - \alpha)^{2} + \alpha^{2} C_{3}(r) = [(r - \alpha)^{3} + \alpha^{3}]^{\frac{2}{3}} C_{4}(r) = [(r - \alpha)^{4} + \alpha^{4}]^{\frac{1}{2}}, \text{ etc.}$$

The form (18) satisfies Eddington's prescription for a general solution.

By (9) and (18) the circumference χ of a great circle in the gravitational field is,

$$\chi = 2\pi \sqrt{C_n(r)} = 2\pi [(r - r_o)^n + \alpha^n]^{\frac{1}{n}},$$
(19)

and the proper radius $R_p(r)$ is,

$$R_{p}(r) = \sqrt{[(r - r_{o})^{n} + \alpha^{n}]^{\frac{1}{n}} \left([(r - r_{o})^{n} + \alpha^{n}]^{\frac{1}{n}} - \alpha \right)} +$$
(20)
+ $\alpha \ln \left| \frac{[(r - r_{o})^{n} + \alpha^{n}]^{\frac{1}{2n}} + \sqrt{[(r - r_{o})^{n} + \alpha^{n}]^{\frac{1}{n}} - \alpha}}{\sqrt{\alpha}} \right|,$
 $r_{o} < r < \infty.$

According to (19), $\chi = 2\pi\alpha$ is a scalar invariant. By (19) and (20),

$$\lim_{r \to r_o} \frac{\chi(r)}{R_p(r)} \to \infty,$$

irrespective of the values of n and b, so that when $R_p = 0$, at $r = r_o$, there is a quasiregular singularity, and therefore inextendible. The Kretschmann scalar $f = R_{ijkm}R^{ijkm}$ for (5) with (18) is,

$$f = \frac{12\alpha^2}{[C_n(r)]^3} = \frac{12\alpha^2}{[(r-r_o)^n + \alpha^n]^{\frac{6}{n}}}.$$
(21)

Then $f(r_o) = \frac{12}{\alpha^4}$ is a scalar invariant, irrespective of the values of n and b. The Kruskal-Szekeres form has no meaning since the r-parameter is not the

radial coordinate in the gravitational field at all. Furthermore, the value of r_o is entirely arbitrary.

If $\alpha = 0$, then (9) with (18) reduces to the metric of Special Relativity.

The value of the *r*-parameter of a spacetime event depends upon the coordinate system chosen. However, the proper radius R_p and the curvature radius $\sqrt{C_n(r)}$ of that event are independent of the coordinate system. This is easily seen as follows. Consider a great circle centred at the point-mass and passing through a spacetime event. Its circumference is measured at χ . Divide χ by 2π ,

$$\frac{\chi}{2\pi} = \sqrt{C_n(r)}.$$

Thus, the coordinate system is determined by χ . Putting $\frac{\chi}{2\pi} = \sqrt{C_n(r)}$ into (14) gives the proper radius of the spacetime event,

$$R_p(r) = \sqrt{\frac{\chi}{2\pi} \left(\frac{\chi}{2\pi} - \alpha\right)} + \alpha \ln \left| \frac{\sqrt{\frac{\chi}{2\pi}} + \sqrt{\frac{\chi}{2\pi} - \alpha}}{\sqrt{\alpha}} \right|,$$

which is independent of the coordinate system chosen. To find the r-parameter in terms of a particular coordinate system set,

$$\frac{\chi}{2\pi} = \sqrt{C_n(r)} = [(r - r_o)^n + \alpha^n]^{\frac{1}{n}},$$

 \mathbf{SO}

$$r = r_o + \left[\left(\frac{\chi}{2\pi}\right)^n - \alpha^n \right]^{\frac{1}{n}} = b\alpha + \left[\left(\frac{\chi}{2\pi}\right)^n - \alpha^n \right]^{\frac{1}{n}}.$$

Thus r depends upon the arbitrary values n and b, which establish a coordinate system. Then when $R_p = 0$, $\chi = 2\pi\alpha$, and so $r = r_o$ irrespective of the values of n and b. Indeed, $\frac{\chi}{2\pi}$ is independent of the coordinate system. A truly coordinate independent description of spacetime events has been attained.

The foregoing analysis can be easily generalised to treat of the point-charge, the rotating point-mass, and the rotating point-charge. The overall solution I obtain (when $\Lambda = 0$) is,

$$ds^{2} = \frac{\Delta}{\rho^{2}} \left(dt - a \sin^{2} \theta d\varphi \right)^{2} - \frac{\sin^{2} \theta}{\rho^{2}} \left[\left(C_{n} + a^{2} \right) d\varphi - a dt \right]^{2} - \frac{\rho^{2}}{\Delta} \frac{{C_{n}}^{\prime 2}}{4C_{n}} dr^{2} - \rho^{2} d\theta^{2},$$

$$C_{n}(r) = \left[(r - r_{o})^{n} + \beta^{n} \right]^{\frac{2}{n}}, r_{o} = b\beta, \ b \in \left(\Re - \Re^{-} \right), \ n \in \Re^{+},$$

$$a = \frac{L}{m}, \ \rho^{2} = C_{n} + a^{2} \cos^{2} \theta, \ \Delta = C_{n} - \alpha \sqrt{C_{n}} + q^{2} + a^{2}.$$

$$r_{b}(\theta) = \left[\left(m + \sqrt{m^{2} - q^{2} - a^{2} \cos^{2} \theta} \right)^{n} - \beta^{n} \right]^{\frac{1}{n}} + r_{o},$$

$$\beta = m + \sqrt{m^{2} - (q^{2} + a^{2})}, \ a^{2} + q^{2} < m^{2},$$

$$r_{o} < r < \infty,$$

$$(22)$$

where L is the angular momentum, q the charge. It is easily seen that (22) reduces to (1) for appropriate choice of parameters. The solutions for the point-charge and the rotating point-mass are similarly extracted. In no case is there a black hole. In (22) $r_h \equiv r_o$.

From (22) the general solution for the point-charge in relativistic units is,

$$ds^{2} = \left(1 - \frac{\alpha}{\sqrt{C_{n}}} + \frac{q^{2}}{C_{n}}\right) dt^{2} - \left(1 - \frac{\alpha}{\sqrt{C_{n}}} + \frac{q^{2}}{C_{n}}\right)^{-1} \frac{{C'_{n}}^{2}}{4C_{n}} dr^{2} - C_{n} (d\theta^{2} + \sin^{2}\theta d\varphi^{2})$$
(23)
$$C_{n}(r) = \left[(r - r_{o})^{n} + \beta^{n}\right]^{\frac{2}{n}},$$
$$n \in \Re^{+}, \ r_{o} = b\beta, \ b \in \left(\Re - \Re^{-}\right), \ \beta = m + \sqrt{m^{2} - q^{2}}, \ q^{2} < m^{2},$$
$$r_{o} < r < \infty.$$

It can be shown that the angular velocity ω of a test particle is,

$$\omega^{2} = \left(\frac{\alpha}{2C_{n}^{\frac{3}{2}}} - \frac{q^{2}}{C_{n}^{2}}\right) = \left[\frac{\alpha}{2\left[\left(r - r_{o}\right)^{n} + \beta^{n}\right]^{\frac{3}{n}}} - \frac{q^{2}}{\left[\left(r - r_{o}\right)^{n} + \beta^{n}\right]^{\frac{4}{n}}}\right].$$
 (24)

Then,

$$\lim_{r \to r_o} \omega = \sqrt{\frac{\alpha}{2\beta^3} - \frac{q^2}{\beta^4}}.$$
(25)

Equation (24) is Kepler's 3rd Law for the point-charge. It obtains the finite limit given in (25), which is a scalar invariant for the point-charge. When q = 0, equations (24) and (25) reduce to those for the simple point-mass. In the case of the simple point-mass, Kepler's 3rd law is,

$$\omega^{2} = \frac{\alpha}{2C_{n}^{\frac{3}{2}}} = \frac{\alpha}{2\left[(r-r_{o})^{n} + \alpha^{n}\right]^{\frac{3}{n}}},$$
(26)

Then,

$$\lim_{r \to r_o} \omega = \sqrt{\frac{\alpha}{2\alpha^3}} = \frac{1}{\alpha\sqrt{2}}.$$
(27)

(27) is a scalar invariant for the simple point-mass, and is precisely that obtained by Schwarzschild in his 1916 paper, and consistent with the form for Kepler derived by Droste, in 1916.

How is $r_c = 0$ to be interpreted? I regard it as simply the arbitrary location of an external observer. However, such an observer can be located anywhere, so that $r_c = r_o = 0$ can always be defined as the arbitrary location of the pointmass and $r = r_{ob} > 0$ the arbitrary location of an external observer. In this case $C_n(r)$ takes the more restricted form,

$$C_n(r) = (r^n + \alpha^n)^{\frac{2}{n}},$$

and so the Droste/Weyl/(Hilbert) solution and its associated infinite sequence of particular solutions would actually be excluded, but Schwarzschild's original solution and the solution due to Brillouin are not - because Schwarzschild defined $r_c = r_o = 0$ as the location of the point-mass at the outset of his analysis and constructed his solution to it. However, owing to the limitations set upon him

by the penultimate version of Einstein's theory, which he was working with, he could only obtain a particular solution, not a general solution. Also, this result is unaffected if the observer and the point-mass are one and the same at $r_o = 0$.

The local acceleration of a test particle approaching the point-mass along a radial path has been determined by N. Doughty (Am. J. Phys. **49**, 720, 1981) at,

$$a = \frac{\sqrt{-g_{rr}} \left(-g^{rr}\right) \left|g_{tt,r}\right|}{2g_{tt}}$$

By (9), using (18), the acceleration is,

$$a = \frac{\alpha}{2C_n^{\frac{3}{4}} \left(C_n^{\frac{1}{2}} - \alpha\right)^{\frac{1}{2}}}.$$

Then,

$$\lim_{r \to r_o} a = \infty,$$

since $C_n(r_o) = \alpha^2$.

Y. Hagihara (Jpn. J. Astron. Geophys. 8, 67, 1931) has shown that all those geodesics which do not run into the Hilbert boundary at r = 2m are complete. According to my arguments r = 2m is a point at which the point-mass is located (and an arbitrary one at that). However, as I have also argued, the curvature invariant is finite there. I conclude that no curvature singularity can arise in the vacuum field.

I would now like to make some comment on specific remarks you made in your email.

I agree that the modern relativists do not interpret the Hilbert solution over $0 < r < \infty$ as Hilbert did, instead making a distinction between 0 < r < 2m and $2m < r < \infty$. I have been previously told by a modern relativist that one is then entitled to 'choose' a region. However, I do not see that this is admissible because, as I have argued, the validity of the regions must be rigorously determined before such claims can be made. One cannot just look at the Hilbert metric and assume, tacitly or otherwise, that these regions are valid ranges on the *r*-parameter. Furthermore, it is also tacitly assumed by the mainstream analysis that the *r*-parameter is a radius in the gravitational field. Again, I view this as inadmissible, and that the *r*-parameter must be rigorously established as a radius in the gravitational field before one can use it as such. J. L. Synge makes the same mathematically unjustified assumptions on the Hilbert line-element. He remarks, (Proc. Roy. Irish Acd. **53**, 6, 1950, pp. 83),

'This line-element is usually regarded as having a singularity at $r = \alpha$, and appears to be valid only for $r > \alpha$. This limitation is not commonly regarded as serious, and certainly is not so if the general theory of relativity is thought of solely as a macroscopic theory to be applied to astronomical problems, for then the singularity $r = \alpha$ is buried inside the body, i.e. outside the domain of the field equations

 $R_{mn} = 0$. But if we accord to these equations an importance comparable to that which we attach to Laplace's equation, we can hardly remain satisfied by an appeal to the known sizes of astronomical bodies. We have a right to ask whether the general theory of relativity actually denies the existence of a gravitating particle, or whether the form (1.1) may not in fact lead to the field of a particle in spite of the apparent singularity at $r = \alpha$.'

M. Kruskal (Phys. Rev. 116, 1743, 1960) remarks of his proposed extension,

'That this extension is possible was already indicated by the fact that the curvature invariants of the Schwarzschild metric are perfectly finite and well behaved at r = 2m*.'

which betrays the very same unproven assumptions. Moreover, his claim about the finitude of the curvature invariant implies incorrectly that the singularity cannot occur where the curvature is finite.

Szekeres (Math. Debreca, 7, 285, 1960) says of the Hilbert line-element,

'... it consists of two disjoint regions, 0 < r < 2m, and r > 2m, separated by the singular hypercylinder r = 2m.'

which again betrays the same unproven assumptions.

I draw your attention to the following additional problems with the K-S form.

a). Applying Doughty's acceleration formula to the K-S form, it is easily found that,

$$\lim_{x \to 2m} a = \infty$$

But according to K-S there is no singularity at r = 2m, i.e. no matter there - contra-hype. This is a direct result of the conventional incorrect assumptions about the *r*-parameter.

b). As $r \to 0, u^2 - v^2 \to -1$. These loci are spacelike, and therefore cannot describe any configuration of matter or energy.

Either of these features alone proves the K-S form inadmissible, in my opinion.

I am of the view that correct geometrical analysis excludes the interior Hilbert region on the grounds that it is not a region at all, and invalidates the assumption that the *r*-parameter is a radius in the gravitational field. Consequently, the Kruskal-Szekeres formulation is meaningless, since it does not even deal with the correct radial parameter associated directly with the gravitational field. In addition, the so-called 'Schwarzschild radius' is also meaningless - it is not a radius in the gravitational field, as far as I can see. Furthermore, I maintain that Hilbert's r = 2m is indeed a point.

I agree with you that the *form* of the Hilbert line-element is given by Schwarzschild in his equation (14), where it occurs in terms of the parameter R, however Schwarzschild also includes there $R = (r^3 + \alpha^3)^{\frac{1}{3}}$, having previously established that $0 < r < \infty$. Consequently, Schwarzschild's R has

the lower bound $\alpha = 2m$. Since Schwarzschild's R and Hilbert's r can be replaced with any appropriate analytic function C(r), the range on r will depend upon the function chosen. In Schwarzschild's case that is $0 < r < \infty$ (since $r_o = 0, C_3(r) = (r^3 + \alpha^3)^{\frac{2}{3}}$) and in Hilbert's case $2m < r < \infty$ (since $r_o = 2m, C_1(r) = r^2$), according to the geometrical relationships between the components of the metric tensor, as far as I can see. Therefore, I agree with you that the geometry and the invariants are the important properties, but it seems to me that the conventional analysis has erred in its geometrical analysis and identification of the invariants, as a direct consequence of its initial unvalidated assumptions about the r-parameter.

I my opinion, the only reason that the Hilbert r-parameter conventionally breaks down at r = 2m is because of the initial arbitrary and incorrect assumptions that the inner region is also valid and that r is a radius in the gravitational field. I do not see how these assumptions can possibly be admitted.

There is indeed no doubt that the Kruskal-Szekeres form is a solution, as you say, of the Einstein vacuum field equations, however that does not guarantee that it is a solution to the problem. There exists an infinite number of solutions to the vacuum field equations which do not yield a solution for the gravitational field of the point-mass. Satisfaction of the field equations is a necessary but insufficient condition for a potential solution to the problem. It is evident to me that the conventional conditions that must be met are inadequate, viz.,

- 1. be analytic;
- 2. be Lorentz signature;
- 3. be a solution to Einstein's free-space field equations;
- 4. be invariant under time translations;
- 5. be invariant under spatial rotations;
- 6. be (spatially) asymptotically flat;
- 7. be inextendible to a wordline L;
- 8. be invariant under spatial reflections;
- 9. be invariant under time reflection;
- 10. have a global time coordinate.

I am of the view that this list must be augmented by a boundary condition at the location of the point-mass, which is, in my formulation of the solution, $r \to r_o \Rightarrow R_p(r) \to 0$. Schwarzschild actually applied a form of this boundary condition in his analysis. Marcel Brillouin also pointed out the necessity of such a boundary condition in 1923. The condition has been disregarded or gone unrecognised by the mainstream authorities.

I agree with you that any constants appearing in a valid solution must appear in an invariant derived from the solution. The solution I obtain meets this

condition in the invariance, at $r = r_o$, of the scalar curvature, of the circumference of a great circle, of Kepler's 3rd law, of $C(r_o) = \alpha^2$, of $R_p(r_o) = 0$, and not only in the case of the point-mass, but also in all the relevant configurations, with or without charge.

The fact that the circumference of a great circle approaches the finite value $2\pi\alpha$ does not seem to me more odd than the conventional oddity of the change in the arrow of time in the interior Hilbert region. Indeed, I regard the latter as an even more violent oddity. The finite limit of the said circumference does however, according to my analysis, appear to be consistent with the geometry resulting from Einstein's gravitational tensor. The variations of θ and φ displace the proper radius vector, which has zero length at $r = r_o$, over the spherical surface of finite area $4\pi\alpha^2$. Einstein's theory admits nothing more pointlike, so perhaps the point-mass is a misnomer. Whether or not such an oddity is permissible is another question entirely, one bound up, I suspect, with the formulation of the gravitational tensor itself. Objections to Einstein's formulation of the gravitational tensor were raised as long ago as 1917, by T. Levi-Civita, on the grounds that, from the mathematical standpoint, it lacks the invariant character actually required of General Relativity, and further, produces an unacceptable consequence concerning gravitational waves. At this time, I have no definite opinion on this matter, but it is not pertinent to the issue of whether or not the black hole is consistent with the theory as it currently stands on Einstein's gravitational tensor.

Your arguments about cutting a sphere out of flat space are based, I believe, upon the unjustified assumption that the r-coordinate is a radial coordinate in the gravitational field. I do not see how this assumption can be maintained. I regard r = 2m as a point. I do not agree therefore, with your view that Schwarzschild's original coordinates implicitly identify a sphere topologically with a point. I have argued that r is not the radial coordinate of a test particle in the gravitational field. Only $R_p(r)$ can claim that character in the gravitational field.

I would now like to make a few general remarks.

It is not uncommon for experts (e.g. C. Misner, K. Thorne and J. Wheeler, S. Hawking and G. Ellis, S. Chandrasekhar, amongst others) to proclaim that the Michell-Laplace dark body is a primitive black hole. This claim is utterly false since it can easily be shown that there is always a class of observers which can see the Michell-Laplace dark body. This claim is of course based upon a confusion about the meaning of an escape velocity. It is astonishing that any experts at all make this claim.

It is also frequently claimed by many experts (e.g. Misner, Thorne and Wheeler, Hawking and Ellis, I. Novikov, amongst others) that black holes can be members of binary systems and that black holes can collide. Even if it is assumed that black holes exist, for the sake of the argument, it makes no sense to talk about such situations since all the known solutions to the field equations involve a lone gravitating body and a single test particle. There are no known solutions involving two or more comparable masses. It is not even known if Einstein's field equations admit of such configurations. Therefore, without at least an existence theorem rigorously establishing the possibility of these configurations it is not possible to justify them theoretically.

I attempted to obtain the paper by I. Novikov that you referred to, but it cannot be read or downloaded from the GRG website - the links are inactive. I have not read this paper previously. Nonetheless I suspect that Novikov will have made the same unjustified assumptions about the *r*-coordinate.

There seems to me to be a very disturbing situation in modern physics in that free and open discussion is not encouraged, and further, that the questioning of established authority is deliberately gagged.

There appears to be a cult-like following of Einstein that does not permit his theory to be seriously questioned. This is very bad. Physics is a science and so no theory is absolute. Perhaps only in pure mathematics can we achieve the 'absolute'. Although I have not questioned the foundations of Einstein's theory in my analysis of the problem of the point-mass, I do not object to others doing so. Furthermore, I am open to alternative theories of gravitation, and any other physical phenomena, provided they are based upon sound interpretation of valid physical data and consistent theoretical considerations. Free and open discussion alone can lead to progress, but this is, sadly, apparently no longer the case in physics.

I would welcome your comments, especially if you can demonstrate to me that my thinking is awry. However, if you maintain that assumption about the *r*-coordinate is legitimate I fear that we will be unlikely to come to an agreement.

If you feel disinclined to correspond further then please let me know. I would prefer to hear from you to that effect rather than be left to figure it out for myself. In any event I would like to thank you once again for bothering to write in the first place.

I am, Yours faithfully, Stephen J. Crothers.