

RELATIVISTIC GEOMETRY AND ITS IMPLICATIONS FOR BLACK HOLES AND COSMOLOGY

Stephen J. Crothers

Queensland, Australia.

email: thenarmis@yahoo.com

The relativists have not understood the geometry of Einstein's gravitational field. They have failed to realise that the basic geometrical structure of spacetime is an intrinsic property of the spacetime metric and that particular geometrical features of the gravitational field are determined by the components of the metric tensor and the fixed geometrical relations between them, in accordance with the structure of the metric itself. The variable r , which appears in the metric for the gravitational field, bears no relation to the spherical symmetry associated with the metric and is, in general, nothing but a real-valued parameter for the true geometrical elements of spacetime. The consequences of this are significant: General Relativity does not in fact predict the existence of black holes or an expansion of the Universe, but, on the contrary, precludes them entirely.

1. Introduction

The specific characteristics of the geometry of Einstein's spherically symmetric gravitational field are entirely contained in the structure of the associated metric and the consequent intrinsic geometrical relations between the components of the metric tensor. These relations are definite and inviolable. The relativists have not understood this and have therefore failed to solve the problem of Einstein's gravitational field.

The alleged spherically symmetric solutions obtained by the relativists are all, either in part or in whole, invalid, owing to their transgression of the inviolable geometry of the metric.

2. The geometry of the gravitational field

Consider the standard Minkowski metric,

$$ds = dt^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

$$0 \leq r < \infty.$$

The spatial components of (1) describes a sphere of radius r centred at $r_0=0$. Compare it to the generalised metric,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2 \theta d\phi^2) \quad (2)$$

$$A, B, C > 0 \forall r \neq r_0.$$

where r_0 is some bound on the real variable r . On (2) I identify the radius of curvature R_c , the proper radius R_p , the real-valued r -parameter, the surface area A_s of the associated sphere, and the volume V of the said sphere, thus

$$R_c = \sqrt{C(r)},$$

$$R_p = \int_{r_0}^r \sqrt{B(r)} dr,$$

the real-valued r -parameter is just the variable r , (3)

$$A_s = C(r) \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi,$$

$$V = \int_{r_0}^r C(r) \sqrt{B(r)} dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi.$$

Now I remark that the geometrical relations between the components of the metric tensor of (1) are precisely the same as those between the components of the metric tensor of (2), so the radius of curvature is always the square root of the coefficient of the angular terms and the proper radius is always the integral of the square root of the term containing the square of the differential element of the radius of curvature. This is entirely a matter of geometry. Note that this is explicit in (1), but not in (2) which, although mathematically well-defined, is misleading. In (2) the radius of curvature is $\sqrt{C(r)}$, not r . I therefore write (2) in terms of only the radius of curvature on the metric, thus

$$ds = A^*(\sqrt{C(r)})dt^2 - B^*(\sqrt{C(r)})(d\sqrt{C(r)})^2 - \quad (4)$$

$$-C(r)(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$A^*, B^*, C > 0 \forall r \neq r_0,$$

It is now apparent that (2) is a mixed-term metric, i.e. it is written in terms of both the radius of curvature $\sqrt{C(r)}$ and its parameter r . Although (2) is mathematically valid, its form in (4) is preferred because (4) de-emphasizes the parameter r in favour of the intrinsic metric quantity, namely the radius of curvature, consistent with the form of (1).

In the terms of relations (3), it is easily seen that for (1),

$$\begin{aligned} R_c &= r, \\ R_p &= \int_0^r dr = r, \\ A_s &= r^2 \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi = 4\pi r^2 = 4\pi R_c^2 = 4\pi R_p^2, \\ V &= \int_0^r r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R_c^3 = \frac{4}{3}\pi R_p^3, \end{aligned}$$

so $R_c \cong R_p \cong r$, owing to the pseudo-Efclidean¹ nature of (1).

Next consider a transformation of (1), which I write as,

$$r = (r^{*3} + a^3)^{\frac{1}{3}}, \quad (5)$$

and following the incorrect practice of the relativists, I immediately drop the $*$, so that (1) becomes,

$$ds^2 = dt^2 - \frac{r^4}{(r^3 + a^3)^{\frac{4}{3}}} dr^2 - (r^3 + a^3)^{\frac{2}{3}} (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (6)$$

But, owing to (5),

$$-a \leq r^* < \infty,$$

i.e. on (6),

¹ For the geometry due to Efclideanes; usually erroneously and abominably rendered Euclid.

$$-a \leq r < \infty.$$

The relativists think that r in (6) is still a radius as defined on (1), and that $r = 0$ (i.e. $r^* = 0$) is the relevant "origin" on (6). This is not correct. The r (correctly r^*) in (6) is no longer the relevant radius, but is instead a real-valued parameter for the true radius on (6). Indeed,

$$\begin{aligned} R_c &= (r^3 + a^3)^{\frac{1}{3}}, \\ R_p &= \int_{-a}^r \frac{r^2}{(r^3 + a^3)^{\frac{2}{3}}} dr = (r^3 + a^3)^{\frac{1}{3}} \equiv R_c, \\ A_s &= (r^3 + a^3)^{\frac{2}{3}} \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi = 4\pi (r^3 + a^3)^{\frac{2}{3}} = 4\pi R_c^2 = 4\pi R_p^2, \\ V &= \int_{-a}^r r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}\pi (r^3 + a^3) = \frac{4}{3}\pi R_c^3 = \frac{4}{3}\pi R_p^3. \end{aligned}$$

Once again, $R_p \cong R_c$ owing to the pseudo-Efclidean nature of (6). Note however that $R_p \cong R_c \neq r$. The variable r in (6) is not a radial coordinate on (6), contrary to relativist claims. It is nothing more than a parameter for the determination of the true radial quantities R_c and R_p according to the geometrical relations between the components of the metric tensor, given in definitions (3).

In the case of (2), the relativists transgress the rules of mathematics in precisely the same fashion as I have illustrated in relation to (6). Here now is what they do.

Let

$$r^* = \sqrt{C(r)}. \quad (7)$$

They then transform (2) and immediately drop the $*$ to get,

$$ds^2 = M(r)dt^2 - N(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2),$$

and just assume (incorrectly [1]) that $0 \leq r < \infty$ from (1) still applies. They then solve this in the usual way to obtain,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (8)$$

which they incorrectly call the ‘‘Schwarzschild’’ solution. In truth, this is not Schwarzschild’s solution at all. Schwarzschild’s actual solution, which can be easily confirmed by reading his original paper [2], is,

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$R = (r^3 + \alpha^3)^{\frac{1}{3}}, 0 < r < \infty.$$

Schwarzschild’s original paper testifies to the little known fact that all the claims attributed to him by the orthodox relativists are completely false.

Equation (8) is actually due to J. Droste [3] (although its form appears in the solution by Schwarzschild), who emphatically maintained that on (8), $2m < r < \infty$. It was subsequently obtained by H. Weyl [4], who also emphatically maintained the same domain of definition on r as did Droste. Note that $\alpha < R < \infty$ for Schwarzschild’s original solution. Equation (8), obtained in the way described above, defined, without proof, on $0 < r < \infty$, was due to D. Hilbert. Unlike Hilbert, contemporary relativists maintain, without proof, (i. e. by mere invalid assumption) that there are two domains for r ,

$$0 < r < 2m, \quad 2m < r < \infty.$$

Notwithstanding the given inequalities, the orthodox relativists allow $r = 0$ and $r = 2m$ in practice, claiming that the former is the ‘‘physical’’ singularity of a black hole, and the latter a ‘‘co-ordinate’’ singularity due to a ‘‘pathology’’ of co-ordinates at $r = 2m$. The alleged ‘‘interior’’ interval, $0 < r < 2m$, gives rise to the nonsensical Kruskal-Szekeres extension, which incorrectly treats of r in (8) as a proper radius in the gravitational field, and is therefore false.

The allegations of Hilbert and the relativists are all demonstrably false. Their claims are derived from mere invalid assumption, not mathematical rigour. Any attempt to dismiss the issues as only of historical relevance is also

inadmissible, because a history of errors is still erroneous. Furthermore, the fact that Schwarzschild worked with Einstein’s penultimate version of the theory, requiring him to meet the condition $\det \|g\| = -1$, is quite irrelevant.

According to (7),

$$r_o^* = \sqrt{C(r_o)},$$

the value of which must be determined by a boundary condition. One cannot just assume on (8) that r (in place of r^*) denotes the ‘‘radius’’ in the gravitational field, and one cannot just assume that $r_o = 0$ therein, as Hilbert did, and as the relativists have done ever since. That transgresses the rules of mathematics.

In the general solution for the gravitational field of the fictitious point-mass, $R_p \neq R_c$ [5, 6], and r is merely a real-valued parameter for the determination of R_p and R_c , thus

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} d\sqrt{C(r)}^2 - \quad (9a)$$

$$-C(r)(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} \frac{[C'(r)]^2}{4C(r)} dr^2 - \quad (9b)$$

$$-C(r)(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$R_c = \sqrt{C(r)},$$

$$R_p = \int_{r_o}^r \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}} \frac{C'}{2C} dr = \int_{r_o}^r \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}} d\sqrt{C(r)},$$

$$A_s = C(r) \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi = 4\pi C(r),$$

$$V = \int_{\sqrt{C(r_0)}}^{\sqrt{C(r)}} \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}} C(r) d\sqrt{C(r)} \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi.$$

The admissible form of $R_c = \sqrt{C(r)}$ must be determined by the conditions of the gravitational field. Clearly, $R_p \neq R_c$, owing to the non-Efclidean nature of equations (9). I shall now generalise (1) so that the parametric origin is located at any arbitrary r_0 , in which case the relevant radius no longer takes the same value as the coordinate r . Consider a test particle fixed at an arbitrary $r_0 \neq 0$ in Minkowski space, and let a free test particle be located at any $r \neq r_0$. The distance between the test particles is $d = |r - r_0|$. Thus [7],

$$ds^2 = dt^2 - dr^2 - (r - r_0)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (10a)$$

$$= dt^2 - \frac{(r - r_0)^2}{|r - r_0|^2} dr^2 - |r - r_0|^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (10b)$$

$$= dt^2 - (d|r - r_0|)^2 - |r - r_0|^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (10c)$$

$$R_c = |r - r_0|,$$

$$R_p = \int_0^{|r-r_0|} d|r - r_0| = \int_{r_0}^r \frac{(r - r_0)}{|r - r_0|} dr = |r - r_0| = R_c,$$

$$A_s = |r - r_0|^2 \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi = 4\pi |r - r_0|^2 = 4\pi R_p^2 = 4\pi R_c^2,$$

$$\begin{aligned} V &= \int_0^{|r-r_0|} |r - r_0|^2 d|r - r_0| \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \\ &= \int_{r_0}^r |r - r_0|^2 \frac{(r - r_0)}{|r - r_0|} dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \\ &= \frac{4}{3} \pi |r - r_0|^3 = \frac{4}{3} \pi R_p^3 = \frac{4}{3} \pi R_c^3. \end{aligned}$$

These equations clearly render Efclidean forms, owing to the pseudo-Efclidean nature of equations (10). Note that $R_p \cong R_c$ but $R_c \neq r$, and that

$R_p(r_0) = R_c(r_0) = 0$, irrespective of the value of r_0 , so that the origin on the metric is always at $R_p(r_0) = 0^+$. The parametric origin for equations (10), i.e. the parametric location of the fixed test particle, is at the arbitrary r_0 , and r_0 can be approached from above or below. Furthermore, $r = 0$ is not the location of the fixed test particle unless $r_0 = 0$, in which case equations (10) reduce to equation (1), where $R_p \cong R_c \cong r$. There is nothing special about $r = 0$ that makes it always the relevant origin in parameter space. This amplifies the fact that only the distance between two points is important, not the particular value of a variable co-ordinate. It is also apparent that the location of the fixed test particle is the ‘‘origin of co-ordinates’’ for the problem concerning the interaction of the two test particles. Equation (1) is merely a special case of equations (10). The radius of the sphere associated with equations (10) must be determined by the geometrical relations (3), which are common to all forms (2).

Let the particle fixed at r_0 acquire mass. Then the radial distance between it and the free test particle is no longer given by $d = |r - r_0|$. This radial distance must be mapped into the corresponding radius of curvature and proper radius of the gravitational field. The mappings are achieved by the admissible form for $R_c(r)$ and the intrinsic geometrical relations between the components of the metric tensor.

Einstein’s gravitational field gives rise to a mapping of distances in Minkowski space into corresponding distances, the radius of curvature and the proper radius, of the gravitational field. Note that this mapping is one-to-two! Minkowski space is therefore a parameter space for the components of the metric tensor of Einstein’s gravitational field. Indeed, this is contained in the very generalisation of (1) to (2).

It is a rather trivial matter now to generalise (2), and therefore (9) [7]. One need only replace r there with $d = |r - r_0|$, and so the domain of the r -parameter becomes $\{r \mid r \in \mathfrak{R}, r \neq r_0\}$. Then r_0 can be approached from above or below, giving rise to a general mapping of an Efclidean distance in parameter space into a non-Efclidean distance in the gravitational field.

The relationship between the r -parameter and the proper radius is this: as $r \rightarrow r_0^\pm$, $R_p \rightarrow 0^+$. It follows from equations (9) that,

$$R_p = \sqrt{R_c(R_c - \alpha)} + \alpha \ln \left(\frac{R_c + \sqrt{R_c - \alpha}}{\sqrt{\alpha}} \right),$$

where $R_c \equiv \sqrt{C(r)}$ and

$$R_c(r_o) = \sqrt{C(r_o)} = \alpha \forall r_o.$$

The centre of mass of the source of the field is always located at $R_p = 0$ in the gravitational field, i.e. $R_p(r_o) = 0 \forall r_o$. The specific value of r_o is irrelevant.

In the case of the metric for the gravitational field for the simple “point-mass”, equations (9), the fact that $R_c(r_o) = \sqrt{C(r_o)} \equiv \alpha = 2m$ when $R_p(r_o) = 0$, i.e. when $R_p(r_o) \equiv 0$, is an inescapable consequence of Einstein’s geometry. There is nothing more “point-like” in Einstein’s gravitational field. The usual conception of a point in Minkowski space, manifest as $R_p(r_o) \equiv R_c(r_o) \equiv 0$, does not exist in Einstein’s gravitational field. In Einstein’s gravitational field distances are dependent upon the mass giving rise to the field. Furthermore, the “point-mass” is not a physical object, and is therefore fictitious, and rightly interpreted as the centre of mass of an extended body [8].

Assume, like most relativists, that it is possible for $0 < \sqrt{C(r)} < \alpha$. Then $R_p(r)$ is forced into complex values, which is nonsense. Furthermore, $\sqrt{C(r)} < \alpha$ causes the signs of g_{00} and g_{11} to change. This implies an interchange of time and length. The resulting line-element is,

$$ds^2 = - \left(\frac{\alpha}{\sqrt{C(r)}} - 1 \right) dt^2 + \left(\frac{\alpha}{\sqrt{C(r)}} - 1 \right)^{-1} d\sqrt{C(r)}^2 - C(r)(d\theta^2 + \sin^2 \theta d\phi^2).$$

Now let $r = \mathbb{F}$ and $t = \check{r}$ for the interchange of time and length. Then the foregoing line-element becomes,

$$ds^2 = \left(\frac{\alpha}{\sqrt{C}} - 1 \right) \frac{\dot{C}}{4C} d\mathbb{F}^2 - \left(\frac{\alpha}{\sqrt{C}} - 1 \right)^{-1} d\check{r}^2 - C(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$A(\mathbb{F}), B(\mathbb{F}), C(\mathbb{F}) > 0,$$

where the dot indicates $d/d\mathbb{F}$. This is a time-dependent metric and therefore has *nothing* to do with the original stated problem of a static field. Contra-hype! Thus, $0 < \sqrt{C(r)} < \alpha$ is again nonsense

3. The nature of spherical symmetry in spacetime

It is now clearly apparent that the spherical symmetry associated with the metric of Einstein’s gravitational field is not due to the variable r in any way whatsoever, contrary to the orthodox interpretation. I emphasize that the spherical symmetry is inherent in the structure of the metric, not in the variable r , since r is nothing more than a real-valued parameter. Moreover, since equations (9) are built upon the assumption of spherical symmetry in the form of equation (2), this means that the spherical symmetry is independent of the particular form of the analytic $C(r)$. Therefore, neither $C(r)$ nor r itself determine or contribute to the property of spherical symmetry.

Spherical symmetry is contained in the structural form of equations (9), and specific properties of the geometrical structure of spacetime are determined by the components of the metric tensor of equations (9) and the geometrical relations between them.

To amplify these facts I adduce the following example, which satisfies the spherically symmetric form of equations (2) and (9) and therefore also satisfies Einstein’s field equations, but is, in so far as a valid model of the gravitational field is concerned, utter nonsense. Let $C(r) = \sin r$. Paying no heed to any properties required of $C(r)$ to render a solution for the gravitational field other than its analyticity, equations (9) yield

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{\sin r}} \right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{\sin r}} \right)^{-1} d\sqrt{\sin r}^2 - \sin r(d\theta^2 + \sin^2 \theta d\phi^2). \quad (11)$$

The fact that $C(r)$ can take any analytic form whatsoever without disturbing the spherical symmetry of equations (9) has been noted by a number of authors [10, 11]. Consequently, satisfaction of the field equations is a necessary but insufficient property of a proffered solution for the gravitational field.

The actual admissible form of the function $C(r)$ in all configurations of mass, charge and angular momentum, is deduced in [6]. In the case of the simple (i.e. no charge, no rotation) “point-mass” of equations (9), it has the form, in relativistic units,

$$R_c^2 = C(r) = \left(|r - r_0|^n + \alpha^n \right)^{\frac{2}{n}}, \quad (12)$$

where r_0 and n are entirely arbitrary constants, and

$$r, r_0 \in \mathfrak{R}, \quad \alpha = 2m, \quad n \in \mathfrak{R}^+, \quad r \neq r_0.$$

In no case is there a “black hole”. Note that if $r_0 = \alpha$, $n = 1$, and $r > r_0$, then the usual metric used by the relativists, equation (8), is obtained, but the lower bound on $\sqrt{C(r)} = r$ for the metric (8) is $\sqrt{C(r_0)} \cong \alpha = r_0$, not $r_0 = 0$, as claimed by the relativists. If $r_0 = 0$, $n = 3$, and $r > r_0$ are selected, then Schwarzschild’s true solution results. Note that one can select a situation where $r < r_0$, and the resulting metric satisfies the intrinsic geometry of the metric and is a valid solution for Einstein’s gravitational field. For example, set $n = 1$, $r_0 = -2\alpha$, $r < r_0$. Then

$$R_c = (|r + 2\alpha| + \alpha) = (-r - 2\alpha + \alpha) = -(r + \alpha), \\ -\infty < r < -2\alpha,$$

and the associated metric is

$$ds^2 = \left(1 + \frac{\alpha}{r + \alpha} \right) dt^2 - \left(1 + \frac{\alpha}{r + \alpha} \right)^{-1} dr^2 - \\ -(r + \alpha)^2 (d\theta^2 + \sin^2 \theta d\phi^2), \\ -\infty < r < -2\alpha.$$

4. Measurable lengths in the gravitational field

Since r is an Efcleethean distance defined on (1), which is mapped into the proper radius $R_p(r)$ and the radius of curvature $R_c(r) = \sqrt{C(r)}$ of the gravitational field, which are non-Efcleethean distances, none of the quantities r , $R_p(r)$ and $R_c(r) = \sqrt{C(r)}$ are measurable in the gravitational field. In fact, the only measurable distance in the gravitational field is, in principle, the circumference χ of a great circle through some spacetime event. Therefore, the unique metric for the simple “point mass”, independent of the parameter r and the equivalent metrics (sets of co-ordinates) generated by (12), is

$$ds^2 = \left(1 - \frac{2\pi\alpha}{\chi} \right) dt^2 - \left(1 - \frac{2\pi\alpha}{\chi} \right)^{-1} \frac{d\chi^2}{4\pi^2} - \\ - \frac{\chi^2}{4\pi^2} (d\theta^2 + \sin^2 \theta d\phi^2), \quad (13)$$

where $2\pi\alpha < \chi < \infty$.

5. Consequences

The correction of the geometrical errors committed by Hilbert and the relativists leads directly to the following results in a very simple manner.

- (a) The Hilbert solution and its charged and rotating extensions such as the Reissner-Nordström, Kerr and Kerr-Newman solutions, do not contain black holes. The usual interpretations of these solutions are invalid [5, 6].
- (b) Schwarzschild’s true solution is a correct particular solution for the static vacuum field [5]. It does not admit of the Black Hole.
- (c) The Droste/Weyl solution is a correct particular solution for the static vacuum field [5]. It does not admit of the Black Hole.
- (d) Kepler’s laws are modified by General Relativity [2, 9].
- (e) Black holes have no theoretical basis whatsoever, since they are

inconsistent with General Relativity, and the Michell-Laplace Dark Body from Newton's theory has nothing to do with the Black Hole concept [13, 5, 6, 13].

- (f) The Kruskal-Szekeres extension is invalid [5, 6, 7].
 - (g) All solutions to Einstein's field equations purporting an expanding Universe are incorrect.
 - (h) Einstein's so-called "cylindrical universe" is spatially infinite for all time.
 - (i) de Sitter's so-called "spherical universe" is spatially infinite for all time.
 - (j) The conventional interpretations of the Hubble-Humason relation and the Cosmic Microwave Background are not consistent with General Relativity.
- Cosmological solutions to Einstein's field equations exist but do not predict expansion of the Universe. Consequently, the Big Bang hypothesis has no basis in theory whatsoever.
- (k) The differential element of coordinate length in the standard system of "isotropic co-ordinates" is misinterpreted [14].
 - (l) The Regge-Wheeler (tortoise) co-ordinates are inadmissible [15].
 - (m) There are no curvature-type singularities in Einstein's gravitational field. In particular, the Riemann Tensor scalar curvature (the Kretschmann scalar), $f = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, is finite everywhere [1, 5, 6, 7].

R E F E R E N C E S

- [1] Abrams L. S. Black holes: the legacy of Hilbert's error. *Can. J. Phys.*, 1989, v. 67, 919 (arXiv: gr-qc/0102055).
- [2] Schwarzschild, K. On the gravitational field of a mass point according to Einstein's theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 189, 1916,

- (www.geocities.com/theometria/schwarzschild.pdf).
- [3] Droste, J. The field of a single centre in Einstein's theory of gravitation, and the motion of a particle in that field. *Ned. Acad. Wet., S.A.*, 19, 197, 1917, (www.geocities.com/theometria/Droste.pdf).
- [4] Weyl, H. Zur Gravitationstheorie. *Ann. Phys. (Leipzig)*, v. 54, 117, 1917.
- [5] Crothers S. J. On the general solution to Einstein's vacuum field and its implications for relativistic degeneracy. *Progress in Physics*, v. 1, 68–73, 2005, (www.ptep-online.com).
- [6] Crothers S. J. On the ramifications of the Schwarzschild spacetime metric. *Progress in Physics*, v. 1, 74–80, 2005, (www.ptep-online.com).
- [7] Crothers S. J. On the geometry of the general solution for the vacuum field of the point-mass. *Progress in Physics*, v. 2, 3–14, 2005, (www.ptep-online.com).
- [8] Crothers S. J. On the vacuum field of a sphere of incompressible fluid. *Progress in Physics*, v. 2, 43–47, 2005, (www.ptep-online.com).
- [9] Crothers S. J. On the generalisation of Kepler's 3rd law for the vacuum field of the point-mass. *Progress in Physics*, v. 2, 37–42, 2005, (www.ptep-online.com).
- [10] Eddington A. S. *The mathematical theory of relativity*, Cambridge University Press, 2nd ed., 1923, s.43, p.95.
- [11] Loinger, A. On Black Holes and Gravitational Waves, La Goliardica Pavese, Pavia, 2002, p.34, ISBN 88-7830-371-2, (arXiv: physics/0107071 July 28, 2001).
- [12] McVittie G.C. Laplace's alleged "black hole". *The Observatory*, v. 98, 1978, 272, (www.geocities.com/theometria/McVittie.pdf).
- [13] Crothers S. J. A brief history of black holes. *Progress in Physics*, v. 2, 54–57, 2006, (www.ptep-online.com).
- [14] Crothers S. J. On isotropic coordinates and Einstein's gravitational field. *Progress in Physics*, v. 3, 7–12, 2006, (www.ptep-online.com).
- [15] Crothers S. J. On the Regge-Wheeler tortoise and the Kruskal-Szekeres coordinates. *Progress in Physics*, v. 3, 30-34, 2006, (www.ptep-online.com).