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3. Einstein's conceptions of the conservation and localisation
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4. The concept of black hole interactions is ill-conceived.

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Point-Mass Singularities, Energy and the Two-Body Problem in Einstein's Gravitational Field

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Abstract It is demonstrated herein that:-

1. The Theory of Relativity forbids the existence of point-mass singularities because they imply infinite energies (or equivalently, that a material body can acquire the speed of light in vacuo);
2. $\mathrm{Ric} = R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence' and so does not describe Einstein's gravitational field;
3. Einstein's conceptions of the conservation and localisation of gravitational energy are invalid;
4. The concept of black hole interactions is ill-conceived.

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1 Introduction

It is demonstrated herein that the Theory of Relativity does not permit the existence of point-mass singularities because they imply infinite energies (or equivalently that a material object can acquire the speed c of light in vacuo), and that $\mathrm{Ric} = 0$ violates Einstein's 'Principle of Equivalence' and so does not describe the gravitational field. Therefore, all solutions for $\mathrm{Ric} = 0$ have no physical significance. It immediately follows that Einstein's conceptions of the conservation and localisation of gravitational energy are erroneous and that the current search for Einstein's gravitational waves is ill-conceived. Finally, the concept of black hole interactions is also ill-conceived because the two-body problem has been neither correctly formulated nor solved by means of the General Theory of Relativity.

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2 The non-existence of point-mass singularities

According to Special Relativity, infinite densities are forbidden because their existence implies that a material object can acquire the speed of light c in vacuo (or equivalently, the existence of infinite energies), thereby violating the very basis of Special Relativity. Since General Relativity cannot violate Special Relativity, General Relativity must thereby also forbid infinite densities. Point-mass singularities are alleged to be infinitely dense objects. Therefore, point-mass singularities are forbidden by the Theory of Relativity.

Let a cuboid rest-mass m_0 have sides of length L_0. Let m_0 have a relative speed $v < c$ in the direction of one of three mutually orthogonal Cartesian axes. The mass m is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1}$$

and the volume V thereof is

$$V = L_0^3 \sqrt{1 - \frac{v^2}{c^2}}. \tag{2}$$

Thus, the density D is

$$D = \frac{m}{V} = \frac{m_0}{L_0^3 \left(1 - \frac{v^2}{c^2}\right)}, \tag{3}$$

and so $v \to c \;\Rightarrow\; D \to \infty$. Since by (1) no material object can acquire the speed c (this would require an infinite energy), infinite densities are forbidden by Special Relativity, and so point-mass singularities are forbidden. Since General Relativity cannot violate Special Relativity, it too must thereby forbid infinite densities and hence forbid point-mass singularities [1–5]. Point-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

3 Ric = 0 is inadmissible

According to Einstein [6], his 'Principle of Equivalence' requires that Special Relativity manifest in any freely falling inertial frame located in a sufficiently small region of the gravitational field. Now Special Relativity permits the presence of arbitrarily large (but not infinite) masses in spacetime, which are subject to the mass dilation relation (expression (1) above; and hence also to expressions (2) and (3) as well), and the definition of a relativistic inertial frame requires the *a priori* presence of two masses; the mass of the observer and the mass of the observed (to define relative motion of material bodies). In addition, at any instant the masses defining the freely falling inertial frame (and hence any other masses present therein) can have a speed up to but not including the speed of light in vacuo, by the action of the gravitational field. However, $R_{\mu\nu} = 0$ precludes, by definition, the presence of any masses and energies in the gravitational field because the energy-momentum tensor $T_{\mu\nu} = 0$ by hypothesis. Therefore, Special Relativity cannot manifest in any "freely falling" inertial frame in the spacetime of $R_{\mu\nu} = 0$. Indeed, a "freely falling" inertial frame cannot even be present since its very definition requires the presence of two masses which are, at any instant, subject to mass dilation under the action of the gravitational field. Thus, $R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence' and is therefore inadmissible – it does not describe Einstein's gravitational field. Matter can only be introduced into Einstein's gravitational field via the energy-momentum tensor since it alone is what specifies that which physically causes the curvature of spacetime (i.e. the gravitational field). Clearly, the standard *a posteriori* introduction of matter as the physical cause of spacetime curvature, into the so-called "Schwarzschild solution"[1] for $R_{\mu\nu} = 0$, violates the requirements of Einstein's theory because the energy-momentum tensor is set to zero in that case.

4 Gravitational energy cannot be localised

Since $R_{\mu\nu} = 0$ does not describe Einstein's gravitational field, the energy-momentum tensor can never be zero (i.e. if $T_{\mu\nu} = 0$ there is no gravitational field). Therefore, Einstein's field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

can be written as [9–11]

$$\frac{1}{\kappa} G_{\mu\nu} + T_{\mu\nu} = 0, \qquad (4)$$

wherein the $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Thus, $G_{\mu\nu}/\kappa$ and $T_{\mu\nu}$ *vanish identically*; the total energy is always zero; there is no localisation of gravitational energy (i.e. no Einstein gravitational waves).

It is of interest to note that Einstein's pseudo-tensor is frequently utilised as a basis for the localisation of gravitational energy [6,11–15]. From the foregoing it is evident that this cannot be correct. This is reaffirmed by the fact that Einstein's pseudo-tensor is mathematically (and hence also physically) meaningless, because it implies the existence of an invariant that has no mathematical existence [9]. Indeed, Einstein's pseudo-tensor, $\sqrt{-g}\, t_\nu^\mu$, is defined as [6, 9,11–15],

$$\sqrt{-g}\, t_\nu^\mu = \frac{1}{2}\left(\delta_\nu^\mu L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\nu}\right)$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^\gamma_{\alpha\kappa}\Gamma^\kappa_{\beta\gamma} - \Gamma^\gamma_{\alpha\beta}\Gamma^\kappa_{\gamma\kappa}\right).$$

Contracting the pseudo-tensor and applying Euler's theorem yields,

$$\sqrt{-g}\, t_\mu^\mu = L,$$

which is a 1st-order intrinsic differential invariant that depends only upon the components of the metric tensor and their 1st derivatives. However, the mathematicians Ricci and Levi-Civita [16] proved in 1900 that such invariants *do not exist*. Consequently, everything built upon Einstein's pseudo-tensor is invalid. Eddington's [15] other objections to the pseudo-tensor are therefore quite well-founded.

Similarly, Einstein's field equations cannot be linearised because linearisation implies the existence of a tensor that, except for the trivial case of being zero, *does not otherwise exist*, as proved by Hermann Weyl in 1944 [17].

Since it has already been proved elsewhere [18] that the so-called "cosmological constant" must be precisely zero, expression (4) can contain no other terms.

5 The two-body problem

Before one can talk of relativistic binary systems it must first be proved that the two-body system is theoretically well-defined by General Relativity. This can be done in only two ways:

(a) Derivation of an exact solution to Einstein's field equations for the two-body configuration of matter; or
(b) Proof of an existence theorem.

There are no known solutions to Einstein's field equations for the interaction of two (or more) comparable masses, so option (a) has never been fulfilled. No existence theorem has ever been proved, by which Einstein's field equations even admit of latent solutions for such configurations of matter, and so option (b) has never been fulfilled. The black hole

[1] Which, however, is *not* Schwarzschild's solution [1,2,4,5,7,8].

is allegedly obtained from a line-element satisfying $\text{Ric} = 0$. Ignoring for the moment that $\text{Ric} = 0$ violates Einstein's 'Principle of Equivalence', and, for the sake of argument, assuming that black holes are predicted by General Relativity, since $\text{Ric} = 0$ is a statement that there is no matter in the Universe, one cannot simply insert a second black hole into the spacetime of $\text{Ric} = 0$ of a given black hole so that the resulting two black holes (each obtained separately from $\text{Ric} = 0$) mutually interact in a mutual spacetime that *by definition contains no matter*. One cannot simply assert by an analogy with Newton's theory that two black holes can be components of binary systems, collide or merge [19, 20]. Moreover, General Relativity has to date been unable to account for the simple experimental fact that two fixed bodies will attract one another when released.

6 Recapitulation and conclusions

The Theory of Relativity forbids the existence of infinite densities. Therefore, the black hole, with its alleged point-mass singularity, and the Big Bang cosmological point-mass singularity are forbidden by the Theory of Relativity.

$\text{Ric} = 0$ violates Einstein's 'Principle of Equivalence' and therefore does not describe Einstein's gravitational field. Mass and energy cannot be introduced into Einstein's field equations in any way other than via the energy-momentum tensor. Therefore, any solution to the field equations cannot introduce, *a posteriori*, any mass or energy that is not contained in an associated energy-momentum tensor.

Einstein's conception of the conservation and localisation of gravitational energy are erroneous.

The current international search for Einstein's gravitational waves is destined to detect nothing.

The concepts of black hole binaries, collisions and mergers are invalid.

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