supermassive black hole at Sagittarius A* Inbox | X

from Stephen Crothers <thenarmis@gmail.com> hide details 12/19/08
to Stefste@mpe.mpg.de,
genzel@mpe.mpg.de,
eisenhau@mpe.mpg.de

date Fri, Dec 19, 2008 at 12:20 AM
subject supermassive black hole at Sagittarius A*
mailed-by gmail.com

Stefan Gillessen, Reinhard Genzel, Frank Eisenhauer,

Dear Sirs,

I write in response to this report on your claims in relation to a supermassive black hole at Sagittarius A*:

http://www.space.com/scienceastronomy/081209-blackhole-stars.html#comments

It has been claimed by the astrophysical scientists that a black hole has an escape velocity >= c (speed of light in vacuo). However, according to the alleged properties of a black hole, nothing, not even light, can leave the black hole, so the BH has no escape velocity. If its minimum escape velocity is really that of light, then by definition of escape velocity, light would escape in that case. Not only that, if the BH had an escape velocity then material objects with an initial velocity less than that alleged escape velocity could leave the black hole but not escape (just go out, come to a stop and then fall back). Escape velocity does not mean that objects cannot leave; it only means they cannot escape if they have an initial velocity less than the escape velocity. So on the one hand it is claimed that BH's have an escape velocity >=c, but on the other hand that nothing, not even light, can even leave the BH. The claims are contradictory - nothing but a meaningless play on the words "escape velocity".

The so-called Schwarzschild BH comes from a solution to the field equations Ric = 0, which is a spacetime that by construction contains no matter. Now the Principle of Superposition does not apply in General Relativity (in other words, one cannot insert into any given spacetime, by an analogy with Newton's theory, any number of objects into that given spacetime. The Principle of Superposition does apply in Newton's theory). Since the spacetime of the so-called Schwarzschild BH is by construction devoid of matter, one cannot insert another Schwarzschild BH (obtained separately from Ric = 0) into the spacetime of the first Schwarzschild BH, so that the two BH's persist in and mutually interact in a mutual spacetime that by construction contains no matter! But that is precisely what the proponents of black holes do, all the time. One cannot talk of black hole interactions until it has been proven that such configurations of matter are well-defined in General Relativity. This can be done in only two possible ways: (a) derivation of an exact solution for two (or more) bodies, or (b) prove an existence theorem. But there are no known solutions to Einstein's field equations for two or more masses and no existence theorem has been proven by which it can even be asserted that his field equations contain latent solutions for such configurations of matter. Therefore, all talk of the presence of multiple black holes or black holes interacting with anything, is just wishful thinking. Upon what solution to Einstein's field equations do you rely for the presence of numerous black holes, or even one black hole interacting with other matter, such as that alleged at Sagittarius A*?

The alleged signatures of the black hole are (1) an infinitely dense point-mass singularity and (b) an event horizon. Nobody has ever found an infinitely dense point-mass singularity and nobody has ever found an event horizon, so nobody has ever found a black hole, despite the now daily claims of the astrophysical scientists that they have found them all over the place, in great numbers. All reports of black holes being found are also just wishful thinking - patently false – unless you can provide the coordinates of a verified infinitely dense point-mass singularity and a verified event horizon. But there are of course, as you know, no such coordinates, because no black holes have been found.

Since you advocate existence of black holes from General Relativity you are invited to provide answers to the following questions.

1. In the so-called "Schwarzschild solution" what does the quantity 'r' represent? Provide a proof of what your conception of 'r' therein is.

2. What do you say is the range on 'r' in (1) above? Provide a proof of your claimed range on 'r'.

3. Can you prove that the so-called "Schwarzschild solution" is Schwarzschild's solution? If so, provide the proof.

Resorting to mere citation of the usual "authorities" will not do. I request that you provide arguments and adduce proofs in explanation and justification of your answers. After all, my questions are not complex.

Yours faithfully,

Stephen J. Crothers.

from **Stefan Gillessen** <ste@mpe.mpg.de> hide details 12/21/08
to Stephen Crothers <thenarmis@gmail.com>
date Sun, Dec 21, 2008 at 5:20 AM
subject Re: supermassive black hole at Sagittarius A*
mailed-by mpe.mpg.de

Dear Mr Crothers,

let me briefly answer your points.

It has been claimed by the astrophysical scientists that a black hole has an escape velocity >= c (speed of light in vacuo). However, according to the alleged properties of a black hole, nothing, not even light, can leave the black hole, so the BH has no escape velocity. If its minimum escape velocity is really that of light, then by definition of escape velocity, light would escape in that case. Not only that, if the BH had an escape velocity then material objects with an initial velocity less than that alleged escape velocity could leave the black hole but not escape (just go out, come to a stop and then fall back). Escape velocity does not mean that objects cannot leave; it only means they cannot escape if they have an initial velocity less than the escape velocity. So on the one hand it is claimed that BH's have an escape velocity >=c, but on the other hand that nothing, not even light, can even leave the BH. The claims are contradictory - nothing but a meaningless play on the words "escape velocity".

Of course you are right that nothing can escape from a black hole and therefore it is meaningless to talk of an escape velocity. The concept arises from the attempts to explain what a black hole is. If you make a bodier heavier and heavier, the escape velocity increases, and at the point where it would exceed c the body is a black hole. But this is only
a way to explain a black hole, e.g. to the general public. The attempt is fair, it catches the idea. But of course it would be more correct to talk in terms of a solution to the Einstein equations.

The so-called Schwarzschild BH comes from a solution to the field equations Ric = 0, which is a spacetime that by construction contains no matter. Now the Principle of Superposition does not apply in General Relativity (in other words, one cannot insert into any given spacetime, by an analogy with Newton's theory, any number of objects into that given spacetime. The Principle of Superposition does apply in Newton's theory). Since the spacetime of the so-called Schwarzschild BH is by construction devoid of matter, one cannot insert another Schwarzschild BH (obtained separately from Ric = 0) into the spacetime of the first Schwarzschild BH, so that the two BH's persist in and mutually interact in a mutual spacetime that by construction contains no matter! But that is precisely what the proponents of black holes do, all the time. One cannot talk of black hole interactions until it has been proven that such configurations of matter are well-defined in General Relativity. This can be done in only two possible ways: (a) derivation of an exact solution for two (or more) bodies, or (b) prove an existence theorem. But there are no known solutions to Einstein's field equations for two or more masses and no existence theorem has been proven by which it can even be asserted that his field equations contain latent solutions for such configurations of matter. Therefore, all talk of the presence of multiple black holes or black holes interacting with anything, is just wishful thinking. Upon what solution to Einstein's field equations do you rely for the presence of numerous black holes, or even one black hole interacting with other matter, such as that alleged at Sagittarius A*?

I disagree here. A Schwarzschild BH is a solution that is asymptotically flat, i.e. at infinity it reaches flat space time. So two black hole, sufficiently far away from each other, should look very similar to two independent Schwarzschild solutions (but not exactly). So the situation is not as bad as you describe it, one can describe one as the perturbation to the other.
The point where your argument really gets wrong is that there is more ways than your (a) and (b). Nowadays, theoreticians can solve equations without having to write a solution down - it is all done numerically inside a computer. So, even if you cannot find an analytical way to write the solution down, you can get a numerical solution (that is a very large, finely sampled table of numbers). There is quite some impressive work in this field. Several groups for example are able to calculate in this way what happens if two black holes collide. These are really solutions to the Einstein equations!
So your argument is proven wrong by the fact that the solutions are found. And maybe not the way you had hoped for (some formula), but the solutions are completely valid.

The alleged signatures of the black hole are (1) an infinitely dense point-mass singularity and (b) an event horizon. Nobody has ever found an infinitely dense point-mass singularity and nobody has ever found an event horizon, so nobody has ever found a black hole, despite the now daily claims of the astrophysical scientists that they have found them all over the place, in great numbers. All reports of black holes being found are also just wishful thinking - patently false – unless you can provide the coordinates of a verified infinitely dense point-mass singularity and a verified event horizon. But there are of course, as you know, no such coordinates, because no black holes have been found.

You are right. We find a very high mass in a small volume. That's proven experimentally. Whether it has an event horizon and a singularity is another issue. Usually, the singularity is not considered a real option in theory. It rather tells you, that the Einstein equations are not yet the final laws of nature. The hope is, that a quantum theory of gravity would fix that.
For the event horizon, we have one more additional observational fact: The object that we observe is dark. If you calculate how much of the surrounding mass falls on it and then assume that it falls on some surface, you can estimate how bright the object should be. This is done for example here:
http://arxiv.org/abs/astro-ph/0512211

For the following questions I ignore the slightly aggressive undertone and answer what common sense

among physicists is. For the proofs, I would ask you to look them up in Wheeler's book on gravitation:
http://www.amazon.com/Gravitation-Physics-Charles-W-Misner/dp/0716703440/ref=pd_bbs_sr_2?ie=UTF8&s=books&qid=1229796945&sr=8-2

1. In the so-called "Schwarzschild solution" what does the quantity 'r' represent? Provide a proof of what your conception of 'r' therein is.

I think the question shows that you are not familiar with the mathematics of general relativity. What coordinate system do you refer to? I assume you refer to the Schwarzschild coordinates, but there are others in which the meaning of r changes.
In Schwarzschild coordinates r is the radial coordinate (circumference of a circle centered on the object divided by 2 pi). Only for very large r it has the meaning of distance.

2. What do you say is the range on 'r' in (1) above? Provide a proof of your claimed range on 'r'.

r goes from 0 to infinity in the above.

3. Can you prove that the so-called "Schwarzschild solution" is Schwarzschild's solution? If so, provide the proof.

I do not know what the question means. Whether it was really Schwarzschild who has found it - better ask an historicist.

Resorting to mere citation of the usual "authorities" will not do. I request that you provide arguments and adduce proofs in explanation and justification of your answers. After all, my questions are not complex.

I hope I could help you a bit in the world of black holes. Please note that as a scientist I can only try to help you understand what the concepts are. If on the other hand you don't want to believe, I am not the right person to talk to.

Regards,
Stefan Gillessen

from **Stephen Crothers** <thenarmis@gmail.com> hide details 12/23/08
to ste@mpe.mpg.de
cc genzel@mpe.mpg.de,
eisenhau@mpe.mpg.de

Dear Dr. Gillessen,

Thanks for your reply.

I have attached my response, since mathematical symbols cannot be rendered adequately in email.

Yours faithfully,
Stephen J. Crothers.

Dear Dr. Gillessen,

First, thankyou for your reply.

I offer you now the following comments in response to your remarks (yours in bold).

1) "**Of course you are right that nothing can escape from a black hole and therefore it is meaningless to talk of an escape velocity. The concept arises from the attempts to explain what a black hole is. If you make a bodier heavier and heavier, the escape velocity increases, and at the point where it would exceed c the body is a black hole. But this is only a way to explain a black hole, e.g. to the general public. The attempt is fair, it catches the idea. But of course it would be more correct to talk in terms of a solution to the Einstein equations.**"

I note that you concede that the concept of escape velocity is meaningless for the alleged black hole and so black holes have no escape velocity. Notwithstanding your arguments, the notion of escape velocity does not catch the idea of a black hole, and it is in fact routinely alleged in popular writings for the general readership, in scientific papers by many astrophysical scientists, and in textbooks for students, that black holes have an escape velocity, and that the escape velocity is $\geq c$ (c the speed of light in vacuo). It is a thoroughly misleading and patently false claim. The general public and students alike have not been told the simple truth that the alleged black hole has no escape velocity. It is not difficult to explain to anybody that black holes have no escape velocity. I explained it in one short paragraph which you have conceded is accurate. Nonetheless, owing to misinformation, students and layman alike have been misled on a simple matter of physics. It is astonishing that physics students and physics professors have not worked this out for themselves, instead, passively accepting a falsehood that is easily proved false.

What you have described in your comments is a Michell-Laplace dark body, which is a theoretical object associated with Newtonian gravitation. The M-L dark body is not a black hole. Despite this fact, it is very frequently claimed by astrophysical scientists that Newton's theory describes or anticipates a kind of black hole. That too is patently false. The M-L dark body possesses an escape velocity, whereas the black hole has no escape velocity; objects can leave the M-L dark body, but nothing can leave the black hole; it does not require irresistible gravitational collapse, whereas the black hole does; it has no infinitely dense point-mass singularity, whereas the black hole does; it has no event horizon, whereas the black hole does; there is always a class of observers that can see the M-L dark body, but there is no class of observers that can see the black hole; the M-L dark body can persist in a space which contains other matter and interact with that matter, but the spacetime of the 'Schwarzschild' black hole (and variants thereof) is devoid of matter by construction and so it cannot interact with anything. Thus the M-L dark body does not possess the characteristics of the alleged black hole and so it is not a black hole.

2) "**I disagree here. A Schwarzschild BH is a solution that is asymptotically flat, i.e. at infinity it reaches flat space time. So two black hole, sufficiently far away from**

each other, should look very similar to two independent Schwarzschild solutions (but not exactly). So the situation is not as bad as you describe it, one can describe one as the perturbation to the other. The point where your argument really gets wrong is that there is more ways than your (a) and (b). Nowadays, theoreticians can solve equations without having to write a solution down - it is all done numerically inside a computer. So, even if you cannot find an analytical way to write the solution down, you can get a numerical solution (that is a very large, finely sampled table of numbers). There is quite some impressive work in this field. Several groups for example are able to calculate in this way what happens if two black holes collide. These are really solutions to the Einstein equations! So your argument is proven wrong by the fact that the solutions are found. And maybe not the way you had hoped for (some formula), but the solutions are completely valid."

First, the fact that Schwarzschild spacetime is asymptotically flat does not nullify my arguments concerning Ric = 0. Schwarzschild spacetime is asymptotically Minkowski spacetime, it is not asymptotically Special Relativity and it is not asymptotically Newtonian gravitation. Matter does not suddenly appear asymptotically in a universe that is by construction devoid of matter. Minkowski spacetime is independent of the presence of matter, and Ric = 0 is a spacetime that by construction contains no matter, so the asymptotically flat Minkowski spacetime for Schwarzschild spacetime is devoid of matter too, by the very same construction (Ric = 0). Furthermore, the Principle of Superposition does not apply in General Relativity, and so matter cannot be arbitrarily inserted into the empty spacetime of Ric = 0 described by Schwarzschild spacetime. In addition, the spacetime around the alleged black hole is not flat. It does not matter how far you place your second Schwarzschild black hole from some given Schwarzschild black hole, the spacetime becomes curved from the perspective of each black hole, not asymptotically flat, owing to the alleged presence of two black holes. And still, the insertion of a second black hole into the spacetime of Ric = 0 for some given black hole contradicts the fact that Ric = 0 is a spacetime that by construction contains no matter. It is also an application of the Principle of Superposition, which does not apply in General Relativity. So the situation is indeed that which I advanced. There are no known solutions to Einstein's field equations for two or more bodies and there is no existence theorem by which it can even be asserted that his field equations contain latent solutions for the gravitational interaction of such configurations of matter. I note that you did not answer my question as to what solution to the field equations you and your co-authors reply upon for the alleged black hole at Sagittarius A*, where it allegedly interacts with all sorts of matter, as your joint paper claims. Upon what solution to the field equations do you and your co-authors rely for the alleged black hole at Sagittarius A*?

The numerical analysis and perturbations you describe do not have any validity because they do not relate to any well-defined problem within GR. Furthermore, your assertions that my argument is "**proven wrong by the fact that the solutions are found**" and that "**These really are solutions to the Einstein equations**" are not correct. Without at least a proven existence theorem for such configurations of matter, one cannot claim that the numerical analysis relates to a well-defined problem in GR. Moreover, satisfaction of the field equations is no guarantee of anything. It is a necessary but insufficient condition for a description of Einstein's gravitational field. For instance, one can replace the quantity *r* in the so-called "Schwarzschild solution" with *any* analytic function of *r* without disturbing the spherical symmetry and without violation of Ric = 0. But any analytic function of *r* will not do for a description of Einstein's associated gravitational field. Replace *r* in the so-called "Schwarzschild solution" with exp(*r*); the resulting metric is spherically symmetric and satisfies Ric = 0, but it does not describe Einstein's gravitational field for the simple fact that it is not asymptotically Minkowski spacetime; the latter condition being required by Einstein and his followers. Thus, your assertion that I am wrong because numerical solutions can be found (to an erroneous problem statement) that satisfy Einstein's field equations, is quite spurious.

3) "**You are right. We find a very high mass in a small volume. That's proven experimentally. Whether it has an event horizon and a singularity is another issue. Usually, the singularity is not considered a real option in theory. It rather tells you, that the Einstein equations are not yet the final laws of nature. The hope is, that a quantum theory of gravity would fix that. For the event horizon, we have one more additional observational fact: The object that we observe is dark. If you calculate how much of the surrounding mass falls on it and then assume that it falls on some surface, you can estimate how bright the object should be. This is done for example here:** http://arxiv.org/abs/astro-ph/0512211"

I note that you concede that nobody has ever found a black hole. Nonetheless it is claimed now almost daily by the astrophysical scientists that black holes not only exist, but have been found all over the place. Your joint paper on the alleged black hole at Sagittarius A* is a typical example. The claims are, as you concede, completely false, since nobody has ever found the tell-tale signatures of the alleged black hole; (a) an infinitely dense point-mass singularity and (b) an event horizon. The paper you have cited does nothing to alter these facts. You also claim that "**Usually, the singularity is not considered a real option in theory**." This is not true. The books and papers of the astrophysical scientists are replete with descriptions of objects and astronauts and cosmonautical twins and other things, falling into a black hole, being stretched into spaghetti-like strands, and coming to ultimate grief by collision and coalescence with the black hole's infinitely dense point-mass singularity produced by irresistible gravitational collapse, and thereby increasing the mass and angular momentum (and even the charge) of the singularity (the "no hair concept"). It is also claimed by the same scientific community at large that the infinitely dense point-mass singularity of the black hole is a real object, not a mathematical artifact. These widespread claims are often obfuscated by application of the same argument that you adduce – that the singularity tells us that "**the Einstein equations are not yet the final laws of nature**." Indeed, but this has not stopped the astrophysical scientists from making endless claims for irresistible gravitational collapse, infinitely dense point-mass singularities and event horizons (and that they are real entities), despite the fact that there is no experimental or other observational evidence for the existence of such alleged phenomena. Not a single instance of gravitational collapse has been astronomically observed or produced in the laboratory; not a single singularity has been found; not a single event horizon has been found; and so, not a single black hole has been found. All claims for the discovery of black holes are wishful thinking on a phantasm. Similarly the common invocation of complete unknowns such as "**The hope is, that a quantum theory of gravity would fix that"** means nothing, and so does not justify the demonstrably false claims of the astrophysical scientists.

The observational reports you adduce do not substantiate black holes. The M-L dark body satisfies the said observations just as well, although no M-L dark body has been identified either. And despite your admissions as to black holes, you and your co-authors have claimed a black hole at Sagittarius A*, amongst others, in published papers. Such are the simple facts.

4) "**For the following questions I ignore the slightly aggressive undertone and answer what common sense among physicists is. For the proofs, I would ask you to look them up in Wheeler's book on gravitation:**
http://www.amazon.com/Gravitation-Physics-Charles-W-Misner/dp/0716703440/ref=pd_bbs_sr_2?ie=UTF8&s=books&qid=1229796945&sr=8-2"

I am perplexed by this opening remark. I don't see anything aggressive in my email to you. My questions and arguments were clear and sober, without emotion, rendered in a business-like and honest manner. And if you truly intended to ignore what you somehow took to be "**slightly aggressive undertone",** then why did you even mention it?

I now address your answers to my three questions concerning the so-called "Schwarzschild solution", which I reiterate below. I had asked you to provide proofs of your answers, but you did not do so, citing instead the book by Misner, Thorne and Wheeler for your proofs. So I take it that you are an advocate of their "proofs".

i) In the so-called "Schwarzschild solution" what does the quantity 'r' represent? Provide a proof of what your conception of 'r' therein is.

"**I think the question shows that you are not familiar with the mathematics of general relativity. What coordinate system do you refer to? I assume you refer to the Schwarzschild coordinates, but there are others in which the meaning of r changes. In Schwarzschild coordinates r is the radial coordinate (circumference of a circle centered on the object divided by 2 pi). Only for very large r it has the meaning of distance**."

Your opening remark is unscientific. There is nothing in my simple question by which you could so hastily draw conclusions as to my knowledge of and expertise in the mathematics of General Relativity. The line-element I refer to is commonly known, and you have rightly identified it by its commonality. I write it now for reference:

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (1)$$

Now you assert that "**r is the radial coordinate (circumference of a circle centered on the object divided by 2 pi)".** This is a very widespread inaccuracy. Moreover, in similar fashion, many astrophysical scientists call the said r the areal radius (the square root of the quotient of the calculated surface area and 4π, centred on the object). Thus, they have already adduced two different concepts of what this r is. They go much further however, calling this r such other things as *the radius of a sphere*; *the radius of a 2-sphere*; *the coordinate radius*; *the radial space coordinate*; *the reduced circumference*; and even a *gauge choice: it defines the coordinate r*. In the particular case of $r = 2m$ they invariably call it *the Schwarzschild radius* or *the gravitational radius*. None of these vague notions are correct. Although is is true that the r in eq. (1) is not of itself a distance in the manifold, let alone a radial distance, except asymptotically at infinity, as you have noted, it is nonetheless irrefutable that the r in eq. (1) is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section; but the astrophysical scientists do not know this elementary mathematical fact. The r in eq. (1) is no more a "**radial coordinate**" in eq. (1) than it is in the isotropic form of eq. (1), which is

$$ds^2 = \frac{\left(1-\frac{m}{2r}\right)^2}{\left(1+\frac{m}{2r}\right)^2}dt^2 - \left(1+\frac{m}{2r}\right)^4\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] \tag{2}$$

The foregoing simple fact is sufficient to completely subvert all claims for black holes from General Relativity. In other words, black holes are not in fact predicted by General Relativity. Here now are the proofs.

Your definition of r in eq. (1) is platitudinous, and so quite meaningless, even though it is commonplace. Consider the metric

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{3}$$

This is the first fundamental form for a spherically symmetric surface. It appears in eq. (1). The only curvilinear coordinates are θ and φ. The quantity r here is not a coordinate. Since eq. (3) is a surface it has no intrinsic radius. The quantity ds^2 is the squared differential element of arc-length in the surface. If θ or φ are constant the metric reduces to a parametric curve in the surface. In particular, if $\theta = \pi/2$, then (3) reduces to

$$ds^2 = r^2\, d\varphi^2 \tag{4}$$

which is not just a parametric curve, but also a geodesic. The length of this geodesic is

$$s = \int_0^{2\pi} r\, d\varphi = 2\pi r \tag{5}$$

This does not identify what r is; it only determines a relationship between the length of the geodesic in the surface and r. This is the relationship you have used to identify r. The length of the geodesic is calculated from eq. (5), and then you divide the result by 2π to get $r = s/2\pi$, by which you "define" r. The argument is platitudinous, and hence quite meaningless. The first fundamental form for a surface is

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2 \tag{6}$$

where u and v are curvilinear coordinates, and from which the element of surface area is defined as

$$dA = |\sqrt{(EG - F^2)}dudv| \tag{7}$$

In the case of the surface by eq. (3), $E = r^2$, $F = 0$, $G = r^2\sin^2\theta$, $u = \theta$, $v = \varphi$, and so

$$A = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta \, d\varphi = 4\pi r^2 \tag{8}$$

This does not determine the identity of r; it only determines a relationship between the surface area and the quantity r. This is the relationship that many astrophysical scientists use to "identify" the quantity r. The surface area is calculated from (8), and then the result is divided by 4π, and then reduced to get $r = \sqrt{(A/4\pi)}$, by which they "define" r. The argument is platitudinous, and hence quite meaningless.

An important intrinsic property of a surface is its Gaussian curvature; intrinsic because it is determined solely from the components of the metric tensor and their derivatives. The Gaussian curvature K of a surface is given by

$$K = \frac{R_{1212}}{g}, \tag{9}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and g the determinant of the metric tensor. Recall that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\gamma}_{.\nu\rho\sigma}$$

$$R^{1}_{.212} = \frac{\partial \Gamma^{1}_{22}}{\partial x^1} - \frac{\partial \Gamma^{1}_{21}}{\partial x^2} + \Gamma^{k}_{22}\Gamma^{1}_{k1} - \Gamma^{k}_{21}\Gamma^{1}_{k2}$$

$$\Gamma^{i}_{ij} = \Gamma^{i}_{ji} = \frac{\partial\left(\frac{1}{2}\ln|g_{ii}|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i}, \quad (i \neq j) \tag{10}$$

and all other Γ^i_{jk} vanish. In the above, $i,j,k = 1,2$; $x^1 = \theta$, $x^2 = \varphi$. Applying expressions (9) and (10) to expression (3) gives,

$$K = \frac{1}{r^2} \tag{11}$$

and so r in eq. (3) is the inverse square root of the Gaussian curvature of the surface, and hence r in eq. (1) is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section. It can be called "the radius of Gaussian curvature" by virtue of its relationship to the Gaussian curvature, but it is not a distance in the manifolds described by either eq. (3) or eq. (1). This simple fact repudiates all claims made for black holes from General Relativity, and amplifies the inadequacy of your "definition"of r in eq. (1).

Now consider Minkowski spacetime in the form

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{12}$$

The spherically symmetric surface in the spatial section of eq. (12) is precisely eq. (3), which appears in eq. (1).

ii)What do you say is the range on 'r' in (1) above? Provide a proof of your claimed range on 'r'.

"**r goes from 0 to infinity in the above.**"

Once again I take it that you advocate the "proofs" given by Misner Thorne and Wheeler. MTW actually give no proof that the range on r in eq. (1) is 0 to ∞. Indeed, there is no proof given in any textbook on the subject. What is generally given is an assertion by mere inspection of eq. (1). Attempts to substantiate this unproven assertion rely upon a demonstrably false (circular) argument involving the Riemann tensor scalar curvature invariant (the Kretschmann scalar). It is claimed (also without proof) that since the Kretschmann scalar is finite at $r = 2m$, the latter is a *coordinate singularity* or *removable singularity*. But it has never been proven that Einstein's theory requires a singularity where the Kretschmann scalar is unbounded. In fact, it is not required. Furthermore, the Kretschmann scalar is not an independent curvature invariant. Although the Kretschmann scalar depends upon the components of the metric tensor, all the components of the metric tensor are functions of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, owing to the *form* of the line-element, in consequence of which the Kretschmann scalar is constrained by the intrinsic Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. Recall that the Kretschmann scalar f is,

$$f = R_{\alpha\beta\gamma\delta}\, R^{\alpha\beta\gamma\delta}.$$

Then by eq. (1),

$$f = 48m^2K^3 = \frac{48m^2}{r^6} \tag{13}$$

Simply by inspection of eq. (1) it is asserted by the astrophysical scientists, but not proven, that $0 \le r < \infty$. Then it is asserted, again without proof, that eq. (13) must be unbounded at a *physical singularity*, and so $r = 0$ is chosen to make it so, to "prove" that $0 \le r < \infty$ in eq. (1). Thus a vicious circle is closed. Note also that eq. (13) contains the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the Schwarzschild manifold. The astrophysical scientists are entirely ignorant of this fact. I now prove that the usual related assertions are false. The spatial section of Minkowski spacetime is

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{14}$$

If both θ and φ are constant eq. (14) reduces to

$$ds^2 = dr^2$$

which gives the arc-length or radial distance from the point at the centre of spherical symmetry, thus

$$s = \int dr = r + k$$

where k is a constant. The arc-length s must be zero for some $r = r_o$, and so $k = -r_o$. If it is chosen so that $r_o = 0$ in accordance with eq. (14), then $k = 0$, and calling the radial arc-length R_p (which is a real number), it obtains that

$$R_p = s = r$$

Denoting the inverse square root of the Gaussian curvature (the radius of Gaussian curvature) of the spherically symmetric geodesic surface in the spatial section, by R_c, it obtains from eq. (14) that

$$R_c = r = R_p$$

and so in the case of eq. (14) the geodesic radial distance and the radius of Gaussian curvature can be mutually construed. Now in the case of eq. (1), if both θ and φ are constant, the radial geodesic arc-length is

$$R_p = \int\left(1 - \frac{2m}{r}\right)^{-1/2} dr = \sqrt{r(r-2m)} + 2m\ln\left(\sqrt{r} + \sqrt{r-2m}\right) + k \tag{15}$$

where k is a constant. At the point at the centre of spherical symmetry $R_p = 0$; so the arc-length must be zero for some $r = r_o$. Hence in eq. (15) the minimum $r_o = 2m$ and so $k = -2m \ln\sqrt{(2m)}$, and therefore for eq. (1),

$$R_p = R_p(r) = \sqrt{r(r-2m)} + 2m\ln\left(\frac{\sqrt{r}+\sqrt{r-2m}}{\sqrt{2m}}\right) \qquad (16)$$

$$2m \le r < \infty.$$

Therefore, for eq. (1)

$$R_c = r \ne R_p$$

in general. By eq. (1), $2m < r < \infty$, since eq. (1) is singular at $r = 2m$. Thus $R_p(2m) = 0$, where $K(2m) = 1/4m^2$, and $f(2m) = 3/4m^4 = 12K^2$, which are all scalar invariants. Thus, the usual claims, as quite typically adduced in the book by Misner, Thorne and Wheeler, are proved false.

iii) Can you prove that the so-called "Schwarzschild solution" is Schwarzschild's solution? If so, provide the proof.

Resorting to mere citation of the usual "authorities" will not do. I request that you provide arguments and adduce proofs in explanation and justification of your answers. After all, my questions are not complex.

"**I do not know what the question means. Whether it was really Schwarzschild who has found it - better ask an historicist.**

"**I hope I could help you a bit in the world of black holes. Please note that as a scientist I can only try to help you understand what the concepts are. If on the other hand you don't want to believe, I am not the right person to talk to.**"

Your remark about historiography is unscientific, and so I am left wondering as to your motive for it. This is not a question of history or belief but one of fact involving principles of mathematics. Gauss' Theorema Egregium for instance is no less valid for its age. Similarly Schwarzschild's solution is no less valid for its age. The fact is now plain that you are unaware of Schwarzschild's solution. Most astrophysical scientists do not know Schwarzschild's solution. Here is Schwarzschild's solution,

$$ds^2 = \left(1-\frac{\alpha}{R}\right)dt^2 - \left(1-\frac{\alpha}{R}\right)^{-1}dR^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (17)$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{1/3}, 0 < r < \infty,$$

Schwarzschild's solution describes a non-Euclidean metric manifold that is in one-to-one correspondence with Minkowski spacetime. Note that when $r = 0$ in eq. (17), $R = \alpha$. There is no value of r that can make $R < \alpha$. There is only one singularity, at $r = 0$, where $R_p(0) = 0$, $K(0) = 1/\alpha^2$, and $f(0) = 12/\alpha^4 = 12K^2$. These are scalar invariants.

Now it is easily proven that the generalised metric

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^{\ 2} - R_c^{\ 2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{18}$$

$$R_c = R_c(r)$$

is satisfied by

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{1/n} = \frac{1}{\sqrt{K(r)}}, \tag{19}$$

$$r \in \mathbf{R}, \quad n \in \mathbf{R}^+, \quad r \neq r_o, \quad \alpha = constant,$$

where r_o and n are entirely arbitrary constants. Expression (19) produces an infinite number of equivalents metrics that are spherically symmetric, satisfy Ric = 0, and are asymptotically Minkowski spacetime. It is immediately apparent that the quantity r in eq. (19) plays the role of a parameter for the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section and for the radial geodesic distance from the point at the centre of spherical symmetry of the spatial section of Schwarzschild spacetime. The parameter r is located in the spatial section of Minkowski space. The selection of the arbitrary constant r_o is an arbitrary selection of the parametric point (in Minkowski space) at the centre of spherical symmetry for the problem at hand, and corresponds to the position $R_p(r_o) = 0$, irrespective of the choice of r_o, required by the fact that $R_p(r_o) = 0$ is a scalar invariant. That the selection of r_o is an arbitrary selection of the parametric point at the centre of spherical symmetry in the spatial section of Mankowski spacetime for the problem at hand is easily verified by the simple fact that in Euclidean 3-space, which is precisely the spatial section of Minkowski spacetime, the equation of a sphere of radius ρ and centre C located at the point at the extremity of the fixed vector $\mathbf{r}_o$, may be written

$$[\mathbf{r} - \mathbf{r}_o] \bullet [\mathbf{r} - \mathbf{r}_o] = \rho^2$$

and if $\mathbf{r}$ and $\mathbf{r}_o$ are collinear, $\rho = |r - r_o|$, which is precisely the term appearing in eq. (19). Hence, if in eq. (19) it is chosen that $r_o = \alpha$, this amounts to a shift, for the problem at hand, of the parametric point at the centre of spherical symmetry of the spherically symmetric spatial section of Minkowski, away from the origin of the coordinate system for the spatial section of Minkowski space (at $r = 0$). This point $r_o = \alpha$ is mapped into the scalar invariant $R_p(r_o) = 0$ in Schwarzschild space, where $K(r_o) = 1/\alpha^2$, which is also a scalar invariant, and hence $f(r_o) = 12K^2$ which is also a scalar invariant (dependent upon K).

Examples: in eq. (19) choose $r_o = 0$, $n = 3$, $r > r_o$, then

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^{\ 2} - R_c^{\ 2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$R_c = R_c(r) = \left(r^3 + \alpha^3\right)^{1/3}, \quad 0 < r < \infty,$$

which is Schwarzschild's solution.

Choose $r_o = 0$, $n = 1$, $r > r_o$, then

$$ds^2 = \left(1 - \frac{\alpha}{r+\alpha}\right)dt^2 - \left(1 - \frac{\alpha}{r+\alpha}\right)^{-1} dr^2 - (r+\alpha)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$0 < r < \infty,$$

which is Brillouin's solution.

Choose $r_o = \alpha$, $n = 1$, $r > r_o$, then $R_c(r) = r - \alpha + \alpha = r$, and so

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$\alpha < r < \infty,$$

which is Droste's solution (the correct form of eq. (1) above).

Choose $r_o = 0$, $n = 1$, then

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^{\ 2} - R_c^{\ 2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$R_c = R_c(r) = |r| + \alpha, \quad r \neq 0$$

which is a metric defined on the whole real line except for the single point $r = 0$. This example in particular amplifies the fact that there is no value of r that can make $g_{00} < 0$ for any of the infinite number of equivalent metrics for Schwarzschild space. Thus, the signature of Schwarzschild spacetime is (+,-,-,-) or (-,+,+,+), just as it is for Minkowski spacetime. It is not possible to have a signature of (-,+,-,-) or (+,-,+,+) respectively for Schwarzschild spacetime just as it is not possible for Minkowski spacetime. To amplify this fact even further, when $2m < r < \infty$, the signature of eq. (1) is (+,–,–,–). But if $0 < r < 2m$ in eq. (1), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) \text{ is negative, and}$$

$$g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1} \text{ is positive.}$$

So the signature of eq. (1) changes to (–,+,–,–). Thus the rôles of t and r are interchanged (which the astrophysical scientists actually admit). To clarify this, set $t = -r^*$ and $r = -t^*$, so that for $0 < r < 2m$, eq. (1) becomes,

$$ds^2 = -\left(1 + \frac{2m}{t^*}\right)^{-1} dt^{*2} + \left(1 + \frac{2m}{t^*}\right) dr^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$-2m < t^* < 0.$$

But this is now a *time-dependent* metric since all the components of the metric tensor are functions of the time t^*, and so this metric bears *no relationship* to the original *time-independent problem* to be solved. In other words, this metric is a *non-static solution to a static problem:-* contra-hype! Thus, in eq. (1), $0 \le r \le 2m$ is, again, proven entirely meaningless.

In similar fashion it is easily shown that in the case of Schwarzschild spacetime in isotropic coordinates,

$$ds^2 = \frac{\left(1 - \frac{\alpha}{4h}\right)^2}{\left(1 + \frac{\alpha}{4h}\right)^2} dt^2 - \left(1 + \frac{\alpha}{4h}\right)^4 \left[dh^2 + h^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right],$$

$$h = h(r) = \left[\left|r - r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n},$$

$$r \in \mathbf{R}, \quad n \in \mathbf{R}^+, \quad r \neq r_o.$$

wherein r_o and n are entirely arbitrary constants. Then,

$$R_c(r) = h(r)\left(1 + \frac{\alpha}{4h(r)}\right)^2 = \frac{1}{\sqrt{K(r)}},$$

$$R_p(r) = h(r) + \frac{\alpha}{2}\ln\left(\frac{4h(r)}{\alpha}\right) - \frac{\alpha^2}{8h(r)} + \frac{\alpha}{4}.$$

and so

$$R_c(r_o) = \alpha, \quad R_p(r_o) = 0, \quad \forall r_o \ \forall n$$

which are scalar invariants.

So the Kruskal-Szekeres "coordinates" are quite meaningless. Black holes, as well as not having not been found, are entirely fictitious.

SPHERICALLY SYMMETRIC METRIC MANIFOLDS

In the interest of completeness of mathematical exposition, I now give a detailed description of spherically symmetric metric manifolds, of which the spatial section of Schwarzschild spacetime is a particular case.

Following the method suggested by Palatini, and developed by T. Levi-Civita, denote ordinary Euclidean 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point O be in $\mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation **T** of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric dl'^2 unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of dl'^2 of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$ except possibly at an arbitrary point, the centre of spherical symmetry O' in $\mathbf{M}^3$.

Let a ray i emanate from an arbitrary point O in $\mathbf{E}^3$. There is then a corresponding geodesic i' in $\mathbf{M}^3$ issuing from the corresponding point O' in $\mathbf{M}^3$. Let P be any point on i other than O. There corresponds a point P' on i' in $\mathbf{M}^3$ different to O'. Let g' be a geodesic in $\mathbf{M}^3$ that is tangential to i' at P'.

Taking i as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on i' unchanged. If g' is distinct from i', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about i would cause g' to occupy an infinity of positions in $\mathbf{M}^3$ wherein g' has for each position the property of being tangential to i' at P' in the same direction, which is impossible. Hence, g' coincides with i'.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point O in $\mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' in $\mathbf{M}^3$ with centre of spherical symmetry at the corresponding point O' in $\mathbf{M}^3$.

Let Q be a point in Σ in $\mathbf{E}^3$ and Q' the corresponding point in Σ' in $\mathbf{M}^3$. Let $d\sigma^2$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma'^2$ in Σ' issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates r, θ, φ. Then

$$d\sigma^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{20}$$

$$r = |OQ|.$$

Clearly, if r, θ, φ are known, Q is determined and hence also Q' in Σ'. Therefore, θ and φ can be considered to be curvilinear coordinates for Q' in Σ' and the line element $d\sigma'^2$ in Σ' will also be represented by a quadratic form similar to eq. (20). To determine $d\sigma'^2$, consider two elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma'_1$ and $d\sigma'_2$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis |OQ| in $\mathbf{E}^3$, and so $d\sigma'_1$ and $d\sigma'_2$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $d\sigma'/d\sigma$ is the same for the two different directions irrespective of θ and φ, and so the foregoing ratio is a function of position, i.e. of r, θ, φ. But Q is an arbitrary point in Σ, and so $d\sigma'/d\sigma$ must have the same ratio for any corresponding points Q and Q'. Therefore, $d\sigma'/d\sigma$ is a function of r alone, thus

$$\frac{d\sigma'}{d\sigma} = H(r),$$

and so

$$d\sigma'^2 = H^2(r)\,d\sigma^2 = H^2(r)\,r^2\left(d\theta^2 + \sin^2\theta\, d\varphi\;\right), \tag{21}$$

where $H(r)$ is *a priori* unknown. Set $R_c = R_c(r) = H(r)r$, so that eq. (21) becomes

$$d\sigma'^2 = R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi\;\right), \tag{22}$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing eq. (22) with eq. (20) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point Q', i.e. in terms of the relation $K = 1/R_c^2$ since the Gaussian curvature of eq. (20) is $K = 1/r^2$. This is an intrinsic property of all line elements of the form of eq. (22). Accordingly, R_c, the inverse square root of the Gaussian curvature, can be regarded as the radius of Gaussian curvature. Therefore, in eq. (20) the radius of Gaussian curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilics.

Let the element of radial distance from O in $\mathbf{E}^3$ be dr. Clearly, the radial lines issuing from O cut the surface Σ orthogonally. Combining this with eq. (20) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$,

$$dl^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{23}$$

Let the corresponding radial geodesic from the point O' in $\mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from O' cut the geodesic surface Σ' orthogonally. Combining this with eq. (22) by the theorem of Pythagoras gives the corresponding line element in $\mathbf{M}^3$ as,

$$dl^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (24)$$

where dR_p is, also by spherical symmetry, a function only of R_c. Set $dR_p = \sqrt{B(R_c)}\, dR_c$, so that eq. (24) becomes

$$dl^2 = B(R_c)dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (25)$$

where $B(R_c)$ is an *a priori* unknown function.

Expression (25) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point O' in $\mathbf{M}^3$.

Considering eq. (23), the distance $R_p = |OQ|$ from the point at the centre of spherical symmetry O to a point Q in Σ is given by

$$R_p = \int_0^r dr = r = R_c.$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature of a spherically symmetric surface in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point O in $\mathbf{E}^3$ and the point Q in Σ. Thus, in this case, $K = 1/R_c^2 = 1/R_p^2 = 1/r^2$. However, this is not a general relation, since according to eqs. (24) and (25), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point O' in $\mathbf{M}^3$ is not the same as the radius of Gaussian curvature of the spherically symmetric geodesic surface in $\mathbf{M}^3$, but is given by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c(r) = \int_0^r \sqrt{B(R_c(r))}\frac{dR_c(r)}{dr}dr$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \le r < \infty$ in eq. (23) that it must follow in eqs. (24) and (25) that $0 \le R_c(r) < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from eqs. (24) and (25) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the centre of spherical symmetry to some other point in a metric manifold such as $\mathbf{M}^3$, or indeed of itself any distance at all in the associated manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the radius of Gaussian curvature of the spherically symmetric surface in $\mathbf{E}^3$ and the radial distance from the point at the centre of spherical symmetry of $\mathbf{E}^3$, owing to the fact that R_p and R_c are identical in that special case, as determined from the line-element.

It should also be noted that in writing expressions (23) and (24) it is implicit that O in $\mathbf{E}^3$ is defined as being located at the origin of the coordinate system of eq. (23), i.e. O is located where $R_p = 0$, and by correspondence O' is defined as being located at the origin of the coordinate system of eq. (24) and of eq. (25), i.e. O' in $\mathbf{M}^3$ is located where $R_p = 0$. Furthermore, since it is well known that a geometry is completely determined by the form of the line-element describing it, expressions (23), (24) and (25) share the very same fundamental geometry because they are line-elements of the same form.

I await your comments for continued scientific discussion of these very important matters.

Yours faithfully,
Stephen J. Crothers.
23 December 2008

from **Stefan Gillessen** <ste@mpe.mpg.de> hide details 12/25/08
to Stephen Crothers <thenarmis@gmail.com>
date Thu, Dec 25, 2008 at 12:14 AM
subject Re: supermassive black hole at Sagittarius A*
mailed-by mpe.mpg.de

Dear Mr Crothers,

please note that I am an observational astronomer. So I am perhaps not the best person to discuss the theoretical concept of a black hole. But apparently you wish to discuss that with me, and I try to do my best.

My first reply would to your text is:
I am an observer. Tell me what observations you would consider sufficient to conclude on the existence of black holes; i.e. what observation of some celestial body would make you give up your opinion.
(Note: any person not being fundamentalist should always know what fact could make him/her change mind.)
Future instruments might be able to test far more than what we can dream of today. Tell me what you would like to see, and we can try!
Also note that natural sciences are based on observations, these are the basis of all theory. So if observations contradict theory, that means one has to give up the theory.

My second point is:
The undoubted facts of SgrA* are:
- It is a radio source with a measured (!) size comparable to the Jupiter orbit.
- The radio source has to have 4 x 10^5 solar masses at least, since it moves so straight.

- To within 2mas of the radio source, the orbits of stars show that a mass of 4×10^6 solar masses resides.
- The radio/submm emission (including polarimetry) of SgrA* are well-describes by accretion models, so-called RIAFs. The existence of a hard surface in such models would lead to a detectable emission, which has not been seen however.

--> Please name any object that could fulfill these requirements; at least it is clear something is there, so what is it? (Assuming GR is correct, and assuming that the object be stable for some time further constrains it, this together will exclude any M-L dark body.)

Now let me try to answer some of your points:

1)

- For the word escape velocity: We only disagree on whether the use of a semi-classical picture is fair or not. In public talks I actually give a 3-fold explanation to shed some light on what a BH is: a) escape velocity, b) closed light trajectories and c) infinite gravitational redshift.
- For the M-L dark body: I think if you assume Newtonian gravity and special relativity then you can anticipate black holes. Newton's laws alone of course only can predict a dark body, but not an event horizon.

2)

- In principle I agree, a Schwarzschild BH is highly artificial since it really is a whole universe with just one BH and nothing else.

But also note, that for weak gravitational fields, the superposition principle holds. (In the linear order of the equations.) Otherwise the fly-by of spacecraft on some planet would not work (3-body interaction of planet, Sun, spacecraft). So, the concept might be questionable, but it is correct to the observable order. That's at least something!

- For what we see in SgrA*-observations nowadays, we only need a Newtonian 1/r potential. So we are not yet at the stage that we could tell one GR solution from another. (But we would like to do that, of course.)

Again: We observe a mass enclosed in a small radius, the most conservative explanation for that is a BH in the eyes of the astrophysical community.

- I strongly disagree with your rejection of numerical solutions. Of course, all solutions that are constructed in that way are asymptotically flat. If you a) believe GR, b) find a solution that fulfills the equations and c) also satisfies the boundary conditions, you have truly found a description of the Einstein gravitational field. Note that these solutions can be calculated to arbitrary precision, and that they can be verified by plugging them into the field equations.

-- Also note that a proof of existence can be given by giving an example. (If I show you a black swan, I have proven the sentence 'Black swans do exist'.)

-- Being able to write something down as a formula is not a necessary condition for being a solution to some equation.

3)

- One cannot believe in singularities, since it would require knowing the physics at the Planck mass scale. I doubt that astrophysicists in general believe in the reality of the singularity.
- But I think an event horizon can exist and might really be an option in nature. (It is actually not so difficult to form a BH, the density need not be high, just the mass has to be big. Since Rs is prop to M, the density is prop $1/M^2$.)

4) Equations (1) to (11):

- You (correctly) state: $s = 2\pi r$, $A = 4\pi^2 r^2$, $K = 1/r^2$. Each is a relation between some quantity and r. But only the latter you accept to be a valid definition of r. Why? The structure of all 3 relations is the same.
- The first ($s=2\pi r$) in my eyes is the best, since it gives a clear prescription how to measure r: You count how many rulers you need along the coordinate line and divide by 2π. The second is more difficult to execute. But for a spherical symmetric situation it is equivalent to the first. (since you are free to orient the angles as you wish). The third also gives a measurement prescription, one needs to measure curvature, i.e. by looking at triangles and checking the angle sum.

5) Eq. (16): Now I looked it up in a text book: Your argument is correct, but it is not complete. Initially r is confined to $2m \le r < \infty$. But then by going to ingoing Eddington-Finkelstein coordinates one realizes that

one can analytically continue the range of r to the whole positive axis. Why are you not familiar with the argument? (Of course, r < 2m exchanges the role of r and t, you notice that later in the text)
- Also, it is wrong that the solution is not static for r < 2m. Just the light cones are tilted by more than 45deg and no test particle can move at constant r. But the metric itself remains static (your t*=-r is not the time coordinate)

6) For Schwarzschild's solution: Indeed I was not aware that he found a slightly different solution. Thanks for letting me know. (But so what?)

Regards,
Stefan Gillessen

from **Stephen Crothers** <thenarmis@gmail.com> hide details 12/30/08
to Stefan Gillessen <ste@mpe.mpg.de>
cc genzel@mpe.mpg.de,
eisenhau@mpe.mpg.de

Dear Sir,

My reply is attached.

Stephen J. Crothers.

Dear Dr. Gillessen,

Thankyou for your reply. I appreciate your willingness to discuss these matters. My comments follow (yours in bold type).

> **Dear Mr Crothers,**
>
> **please note that I am an observational astronomer. So I am perhaps not the best person to discuss the theoretical concept of a black hole. But apparently you wish to discuss that with me, and I try to do my best.**
>
> **My first reply would to your text is:**
>
> **I am an observer. Tell me what observations you would consider sufficient to conclude on the existence of black holes; i.e. what observation of some celestial body would make you give up your opinion. (Note: any person not being fundamentalist should always know what fact could make him/her change mind.) Future instruments might be able to test far more than what we can dream of today. Tell me what you would like to see, and we can try!**
>
> **Also note that natural sciences are based on observations, these are the basis of all theory. So if observations contradict theory, that means one has to give up the theory.**

I understand that you are an observational astronomer. I am not an observational astronomer. My interest was geometry, and on that account I investigated Einstein's General Theory of Relativity. I disagree that you are not "**the best person**" to discuss these matters with. The theoreticians are wedded to their dogmas and quite unable to reason objectively any more.

As to what observations would satisfy me that black holes exist, they must be the tell-tale signatures alleged from theory; (a) an infinitely dense point-mass singularity and (b) an event horizon. I have noted your remarks on singularities and horizons, but it is nonetheless true that most astrophysical scientists assert that both are features of a black hole. As far as I can tell, black holes were not conceived of from observational data, but were spawned entirely by theory. It is true "**that natural sciences are based on observations, these are the basis of all theory. So if observations contradict theory, that means one has to give up the theory.**" We are in no dispute over this. What is problematic is the theory, upon which observations have subsequently been interpreted. First, the theory alleges, as we previously discussed, that black holes have an escape velocity $\geq c$ (c the speed of light in vacuo). We are agreed that is false, since the very definition of escape velocity means that black holes have no escape velocity, bearing in mind that black hole theory says that nothing can even leave a black hole, let alone escape. Second, the alleged signatures of a black hole have never been found, so no black hole has ever been found. We are agreed on this too. Third, there are no astronomical observations of gravitational collapse and there is no laboratory evidence for such a phenomenon. Fourth, there is no astronomical evidence of infinitely dense point-masses and no laboratory evidence for such objects. Fifth, when we talk in theory of point-masses what is meant is the mathematical abstraction we call the centre of mass, which is not a physical object. Sixth, there are now almost daily claims that black holes have been found everywhere, but this is not true, as we have agreed, since the tell-tale signatures have never been found.

> **My second point is: The undoubted facts of SgrA* are:**
>
> **- It is a radio source with a measured (!) size comparable to the Jupiter orbit.**
>
> **-The radio source has to have 4×10^5 solar masses at least, since it moves so straight. To within 2mas of the radio source, the orbits of stars show that a mass of 4×10^6 solar masses resides.**
>
> **-The radio/submm emission (including polarimetry) of SgrA* are well-describes by accretion models, so-called RIAFs. The existence of a hard surface in such models would lead to a detectable emission, which has not been seen however.**
>
> **-Please name any object that could fulfill these requirements; at least it is clear something is there, so what is it? (Assuming GR is correct, and assuming that the object be stable for some time further constrains it, this together will exclude any M-L dark body.)**

Other than the as yet undiscovered theoretical M-L dark body I can't suggest to you what object (hypothetical or otherwise) might be the source of your observations. However, one cannot attribute the observations to the theoretical black hole, since there is no evidence for the existence of its alleged signatures, and there are mathematical facts that nullify their alleged theoretical validity, as explained in my previous correspondence. You also raise another important point: "**Assuming GR is correct**". I will elaborate on this matter in the sequel.

> **Now let me try to answer some of your points: 1) - For the word escape velocity: We only disagree on whether the use of a semi-classical picture is fair or not. In public talks I actually give a 3-fold explanation to shed some light on what a BH is: a) escape velocity, b) closed light trajectories and c) infinite gravitational redshift.**
>
> **- For the M-L dark body: I think if you assume Newtonian gravity and special relativity then you can anticipate black holes. Newton's laws alone of course only can predict a dark body, but not an event horizon.**

I don't understand your remarks. In your previous correspondence you conceded that it is meaningless to talk of an escape velocity for a black hole. I therefore thought that you agreed that black holes have no escape velocity.

I do not see how augmenting Newton with Special Relativity "**can anticipate**" black holes. Even with SR there is no event horizon, because there is an escape velocity, there is no infinitely dense point-mass singularity and there is always a class of observers that can see the M-L dark body (even with SR).

> **2)- In principle I agree, a Schwarzschild BH is highly artificial since it really is a whole universe with just one BH and nothing else. But also note, that for weak gravitational fields, the superposition principle holds. (In the linear order of the equations.) Otherwise the fly-by of spacecraft on some planet would not work (3-body interaction of planet, Sun, spacecraft). So, the concept might be questionable, but it is correct to the observable order. That's at least something!**
>
> **- For what we see in SgrA*-observations nowadays, we only need a Newtonian 1/r potential. So we are not yet at the stage that we could tell one GR solution from another. (But we would like to do that, of course.)Again: We observe a mass enclosed in a small radius, the most conservative explanation for that is a BH in the eyes of the astrophysical community.**
>
> **- I strongly disagree with your rejection of numerical solutions. Of course, all solutions that are constructed in that way are asymptotically flat. If you a) believe GR, b) find a solution that fulfills the equations and c) also satisfies the boundary conditions, you have truly found a description of the Einstein gravitational field. Note that these solutions can be calculated to arbitrary precision, and that they can be verified by plugging them into the field equations.– Also note that a proof of existence can be given by giving an example. (If I show you a black swan, I have proven the sentence 'Black swans do exist'.)– Being able to write something down as a formula is not a necessary condition for being a solution to some equation.**

The alleged "Schwarzschild" BH is not a solution to a linear form of the field equations. The said black hole is obtained from a violation of the intrinsic geometry of the line-element, and therefore invalid.

Boundary conditions must be correctly identified and applied. The BH results from an incorrect near field boundary condition, explained in my previous correspondence; to wit, that $0 \leq r < \infty$ on the "Schwarzschild solution". This alleged range is an assertion that has never been proven. All attempts by argument on the Kretschmann scalar are not proofs, just unsubstantiated claims, as explained in my previous correspondence. Satisfaction of an incorrect, ad hoc boundary condition is not proof by example.

I reiterate that neither the problem nor the boundary conditions have been correctly defined by the relativists, and they have not demonstrated that their problem statement is well-defined in GR. The existence of exact solutions corresponding to a solution to the linearised equations must be investigated before perturbation analysis can be applied with any reliability. The relativists have not properly investigated. Indeed, linearisation of the field equations is inadmissible, even though the relativists write down linearised equations and proceed as though they are valid, because linearisation of the field equations implies the existence of a tensor which, except for the trivial case of being precisely zero, does not exist, as proven by Hermann Weyl in 1944. Here is Weyl's proof:

Thus, everything based upon linearisation of Einstein's field equations is flawed.

This is not a matter of belief in GR but one of what is and what is not consistent with its mathematical form and its physical principles. Belief does indeed motivate most astrophysical scientists; but it is unscientific.

3)- One cannot believe in singularities, since it would require knowing the physics at the Planck mass scale. I doubt that astrophysicists in general believe in the reality of the singularity.- But I think an event horizon can exist and might really be an option in nature. (It is actually not so difficult to form a BH, the density need not be high, just the mass has to be big. Since Rs is prop to M, the density is prop $1/M^2$.)

I note that you concede the invalidity of the notion of the BH singularity. Infinitely dense point-mass singularities are forbidden by SR. GR cannot violate SR. So GR forbids the said singularities too. One does not even need the hypothetical Planck mass scale to prove the singularity invalid. Notwithstanding your views, it is evident from even a cursory reading of the relevant literature that most astrophysical scientists do believe that infinitely dense point-mass singularities are real. This idea even appears in textbooks for students.

4) Equations (1) to (11):- You (correctly) state: s = 2 pi r, A = 4 pi^2 r^2, K = 1/r^2. Each is a relation between some quantity and r. But only the latter you accept to be a valid definition of r. Why? The structure of all 3 relations is the same.- The first (s=2 pi r) in my eyes is the best, since it gives a clear prescription how to measure r: You count how many rulers you need along the coordinate line and divide by 2 pi. The second is more difficult to execute. But for a spherical symmetric situation it is equivalent to the first. (since you are free to orient the angles as you wish). The third also gives a measurement prescription, one needs to measure curvature, i.e. by looking at triangles and checking the angle sum.

Now s is a function of the curvilinear coordinate φ and A is a function of the curvilinear coordinates θ and φ, and r is a constant. Thus K is a constant, irrespective of the values of θ and φ, and so the three equations are not of the same structure. If $y = ax$ and $p = ax^2$, both y and p are functions of x, not of the constant a. So $k = a$ is not on the same footing as y and p. The same applies to s, A and K in relation to the constant r. In the case of the spherically symmetric geodesic surface in the spatial section of the so-called "Schwarzschild solution", r is a constant, independent of the curvilinear coordinates θ and φ. However, it has a clear and definite geometrical meaning: it is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, as I proved in my previous correspondence. That proof is irrefutable. Thus, neither s nor A, or the infinite variations of them by the integrated values of θ and φ, adequately identify what r is in the said line-element. To amplify further: when $\theta = constant$, the arc-length is given by:

$$s = s(\varphi) = r\int_0^{\varphi} \sin\theta \, d\varphi = r\sin\theta\,\varphi, \quad 0 \leq \varphi \leq 2\pi,$$

where $r = constant$ and $\theta = constant$. This is the equation of a straight line, of gradient $ds/d\varphi = r\sin\theta$. If $\theta = \frac{1}{2}\pi$ then $s = s(\varphi) = r\varphi$, which is the equation of a straight line with gradient $ds/d\varphi = r$. The maximum arc-length of the geodesic $\theta = \frac{1}{2}\pi$ is therefore $s(2\pi) = 2\pi r$. Similarly the area is:

$$A = A(\varphi,\theta) = r^2\int_0^{\theta}\int_0^{\varphi} \sin\theta \, d\theta \, d\varphi = r^2\varphi\left(1-\cos\theta\right),$$

$$0 \leq \varphi \leq 2\pi, \quad 0 \leq \theta \leq \pi, \quad r = constant.$$

The maximum area is $A\left(2\pi,\pi\right) = 4\pi r^2$. Clearly, neither s nor A are functions of r, because r is a constant, not a variable. And since r appears in each expression, neither s nor A adequately identify the geometrical significance of r in the 1st fundamental form for the spherically symmetric geodesic surface: $ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$. The geometrical significance of r is determined from the components of the metric tensor and their derivatives (Gauss' Theorema Egregium): it is the inverse square root of the Gaussian curvature K of the spherically symmetric surface so described (the constant is $K = 1/r^2$). Thus, my identification of the quantity r is geometrically accurate and complete, whereas s and A are not, being merely platitudinous expressions containing the constant r.

5) Eq. (16): Now I looked it up in a text book: Your argument is correct, but it is not complete. Initially r is confined to $2m <= r <$ inf. But then by going to ingoing Eddington-Finkelstein coordinates one realizes that one can analytically continue the range of r to the whole positive axis. Why are you not familiar with the argument? (Of course, r ¡ 2m exchanges the role of r and t, you notice that later in the text)- Also, it is wrong that the solution is not static for $r < 2m$. Just the light cones are tilted by more than 45deg and no test particle can move at constant r. But the metric itself remains static (your $t* = -r$ is not the time coordinate)

On what basis is my calculation incomplete? One can easily see from my calculation that $0 \leq r < 2m$ produces complex values for the geodesic radius, which is meaningless. That this is so is easily verified by careful study of the description I gave of spherically symmetric metric manifolds from 1st principles in my previous correspondence. I am quite aware of the Eddington-Finkelstein construction. It is meaningless, because it is based upon the same unproven (indeed, demonstrably false) assertion that $0 \leq r < \infty$ in the so-called "Schwarzschild solution". Again clearly from my demonstration from 1st principles. If my calculation is incomplete, then please identify where and why it is incomplete, in relation to both my simple calculation and my development by 1st principles.

I do not think you have understood my argument to the non-static solution to a static problem. My substitution is a valid emphasis of the fact that for $0 \leq r < 2m$, the so-called "Schwarzschild solution" produces a metric tensor whose components are functions of time. Thus, it is non-static. Recall that you cited Misner, Thorne and Wheeler. Here is what they say in their book 'Gravitation' (Section 31.3 Behavior of Schwarzschild coordinates at $r = 2M$):

> "The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial/\partial t$ is spacelike ($g_{tt} > 0$) and $\partial/\partial r$, is timelike ($g_{rr} < 0$).
>
> "What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at $r = 2M$ always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside $r = 2M$. then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate $r = 2M$ to the later time coordinate $r = 0$. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away "the unforgiving minutes," with equal certainty, and with never one halt along the way, r drops from $2M$ to 0.
>
> "At $r = 2M$, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite."

Clearly, the argument adduced by Misner, Thorne and Wheeler is invalid. Yet it is an argument of the relativists in general.

6) For Schwarzschild's solution: Indeed I was not aware that he found a slightly different solution. Thanks for letting me know. (But so what?)

That "Schwarzschild's solution" is not Schwarzschild's solution is of utmost importance. As remarked in my previous correspondence his solution is no less valid for its age just as Gauss' Theorema Egregium is no less valid for its age. Schwarzschild's solution is in one-to-one correspondence with the fundamental manifold of Minkowski spacetime. However, "Schwarzschild's solution" is not. Schwarzschild's solution satisfies the intrinsic geometry of the line-element. "Schwarzschild's solution" does not. Schwarzschild's solution forbids the notion of black hole. "Schwarzschild's solution" conjures up black holes by faulty geometry. Here is Schwarzschild's original paper in English translation:

www.sjcrothers.plasmaresources.com/schwarzschild.pdf

More details are here:

www.sjcrothers.plasmaresources.com/Waves-1.pdf

and here:

www.sjcrothers.plasmaresources.com/MSAST-Crothers.pdf

Belief in GR?

Since $R_{\mu\nu}=0$ violates Einstein's Principle of Equivalence and forbids the manifestation of SR, Einstein's field equations cannot reduce to $R_{\mu\nu}=0$ when $T_{\mu\nu}=0$. In other words, if $T_{\mu\nu}=0$ (i.e. there is no matter present) then there is no gravitational field. Consequently Einstein's field equations must take the form,

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0. \tag{1}$$

This is an identity. The $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Thus the total energy of Einstein's gravitational field is *always* zero; the $G_{\mu\nu}/\kappa$ and the $T_{\mu\nu}$ *must vanish identically* (i.e. when $T_{\mu\nu}=0$ then $G_{\mu\nu}=0$ and vice-versa); there is *no possibility* for the localization of gravitational energy (i.e. there are no Einstein gravitational waves). This also means that Einstein's gravitational field violates the usual conservation of energy and momentum. Since there is no experimental evidence that the usual conservation of energy and momentum is invalid, Einstein's General Theory violates the experimental evidence.

It was early pointed out to Einstein by a number of his contemporaries that his General Theory violated the usual conservation of energy and momentum. To circumvent this problem Einstein invented his pseudo-tensor. His invention had a two-fold purpose (a) to bring his theory into line with the usual conservation of energy and momentum, (b) to enable him to get gravitational waves that propagate with speed *c*. First, it is not a tensor, and therefore not in keeping with his theory that all equations be tensorial. Second, he constructed his pseudo-tensor in such a way that it behaves like a tensor in one particular situation, that in which he could get gravitational waves with speed *c*. Now Einstein's pseudo-tensor is claimed to represent the energy and momentum of the gravitational field and it is routinely applied in relation to the localisation of gravitational energy, the conservation of energy and the flow of energy and momentum. Einstein's pseudo-tensor, $\sqrt{-g}\; t^{\mu}_{\nu}$, is defined by,

$$\sqrt{-g}\; t^{\mu}_{\nu} = \frac{1}{2}\left(\delta^{\mu}_{\nu} L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}\; g^{\sigma\rho}_{,\nu}\right), \tag{2}$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^{\gamma}_{\alpha\kappa}\Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta}\Gamma^{\kappa}_{\gamma\kappa}\right). \tag{3}$$

In a remarkable paper published in 1917, T. Levi-Civita provided a clear and rigorous proof that Einstein's pseudo-tensor is meaningless, and therefore any argument relying upon it is fallacious. I repeat Levi-Civita's proof. Contracting eq. (2) produces a linear invariant, thus

$$\sqrt{-g}\; t^{\mu}_{\mu} = \frac{1}{2}\left(4L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}\; g^{\sigma\rho}_{,\mu}\right). \tag{4}$$

Since L is, according to (3), quadratic and homogeneous with respect to the Riemann-Christoffel symbols, and therefore also with respect to $g^{\sigma\rho}_{,\mu}$, one can apply Euler's theorem to obtain,

$$\frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}\; g^{\sigma\rho}_{,\mu} = 2L. \tag{5}$$

Substituting (5) into (4) yields the linear invariant at L. This is a first-order, intrinsic differential invariant that depends only upon the components of the metric tensor and their first derivatives. However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita proved, in 1900, that such invariants ***do not exist***. This is sufficient to render Einstein's pseudo-tensor entirely meaningless, and hence all arguments relying on it false. Einstein's conception of the conservation of energy and momentum in the gravitational field is erroneous.

Levi-Civita's paper is here:

www.sjcrothers.plasmaresources.com/Levi-Civita.pdf

Further details are here:

www.sjcrothers.plasmaresources.com/Waves-2.pdf

and here:

www.sjcrothers.plasmaresources.com/Jangjeon-2008.pdf

Stephen J. Crothers

30 December 2008

In memory of my brother, Paul: 12 May 1968 — 25 December 2008.

from Stephen Crothers <thenarmis@gmail.com> hide details Jan 21
to Stefan Gillessen <ste@mpe.mpg.de>,
genzel@mpe.mpg.de,
eisenhau@mpe.mpg.de
date Wed, Jan 21, 2009 at 11:37 AM
subject Re: supermassive black hole at Sagittarius A*
mailed-by gmail.com

Dear Drs.

Do you intend to reply to my last email with attachment in response to that below dated 25th December 2008 from Dr. Gillessen? It has been a number of weeks since I sent it.

Yours faithfully,
Stephen J. Crothers.

from Stefan Gillessen <ste@mpe.mpg.de> hide details Jan 21
to Stephen Crothers <thenarmis@gmail.com>
date Wed, Jan 21, 2009 at 7:51 PM
subject Re: supermassive black hole at Sagittarius A*
mailed-by mpe.mpg.de

Dear Mr Crothers,

in principle yes, except I am busy with my work.

Stefan Gillessen

However, Gillessen, Genzel and Eisenhaur have not replied.