**"On the general solution to Einstein's vacuum field and its implications for relativistic degeneracy"**
Stephen J. Crothers

This paper is well written, although unnecessarily polemical in places, e.g. "a bungled analysis of Hilbert's solution". The major question, however, is whether it is correct. It discusses the nature of the Schwarzschild metric in general relativity. I refer to the textbook by B.F. Schwartz, 'General Relativity', Chs. 9-11. Whether this metric is in Schwarzschild's original paper, or has merely been given his name, is beside the point.

The Schwarzschild metric gives the line element in a static, spherically symmetric configuration, corresponding to a spherically symmetric star of mass $M$, centred at the origin; and is valid outside the star, where the mass density is zero. There is apparently a theorem, Birkhoff's theorem, that this is the only such solution which is asymptotically flat. It can be written in the form given in equation (6) of the paper:

$$ds^2 = (\frac{r^* - \alpha}{r^*})dt^2 - (\frac{r^*}{r^* - \alpha})dr^{*2} - r^{*2}(d\theta^2 + \sin^2\theta d\theta^2) \quad (1)$$

where the space-time coordinates are $(t, r^*, \theta, \phi)$, $t$ = time, $\theta, \phi$ are the usual spherical angles from the origin, and $r^*$ is the radial coordinate called the "curvature coordinate". Crothers works in terms of a generalized radial coordinate

$$r^* = C(r) \quad (2)$$

I emphasize that this metric is valid for perfectly ordinary stars, and matches on to the usual Newtonian solution in the far field or weak field region.

I discuss section 2 of this paper. The mathematics seems generally correct: it is the interpretation I would question. After discussing the line interval in various coordinates (note a misprint in the second half of equation (1)), he discusses the proper distance between two points at radii $r$ and $r_0$, presumably located at the same angles, say $\theta = \phi = 0$. He calculates the proper distance between these points

$$R_\rho = \int_{r_0}^{r} \sqrt{g_{11}} dr = \int_{r_0^*}^{r^*} \sqrt{\frac{r^*}{r^* - \alpha}} dr^* \quad (3)$$

where $\alpha = 2M$, and gets what appears to be the right answer, which vanishes as it should do when $r = r_0$.

At the bottom of col. 1, p. 69, he says: "Let the test particle at $r_0$ acquire mass. This produces a gravitational field centred at the point $r_0 \geq 0$." If this is so, the original assumptions are violated. The gravitational field and metric will no longer be spherically symmetric about the origin, and we can proceed no further. He proceeds as if the test mass were zero: correspondingly, he finds later that $r_0$ is arbitrary and has no effect (bottom col. 2, p. 69), which of course must be true. In other words, he appears to be confusing the effects of the point mass at $r_0$, and the original star of mass $M$ at the origin. I'm afraid the analysis from here on makes very little sense to me.

His conditions on $C(r) = r^{*2}$ are:

1. $C'(r) > 0$: OK - $r^*$ is monotonically increasing;

2. $\lim_{r \to \infty} \frac{C(r)}{(r - r_0)^2} = 1$; OK, although the coordinate "$r$" has not really been defined yet;

3. $C(r_0) = \alpha^2$ - this makes no sense: $r_0$ was an arbitrary radius to begin with.

Paragraph middle col. 1, p. 70:

"The invalid conventional assumption that $0 < r < \alpha \cdots$" (presumably for $r$ we should read $r^*$): there is no such assumption, but in these coordinates one has to treat the regions $0 < r^* < \alpha$ and $\alpha < r^* < \infty$ separately; in Kruskal-Szekeres coordinates one can cross smoothly from one region to the other. " .. the incorrect conclusion that $r^* = \alpha$ is a 2-sphere.." : if one fixes $t = 0$ and $r^* = \alpha$, and allows $\theta$ and $\phi$ to vary freely, one will indeed trace out a 2-sphere. If you dispute this, you are negating the whole original geometrical framework, and you are not discussing the standard theory of general relativity. We can proceed no further.

I have not followed the argument any further.

General conclusions: I do not believe the theoretical analysis in this paper is correct. I do not recommend that Steve Crothers submit a thesis based upon this work, because it will be rejected by the referees. I recommend that if he wants to earn a Ph.D. in physics, that he start again with his supervisor John Webb on a project approved by John.