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The Schwarzschild Solution and its Implications for Gravitational Waves: Part II

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Abstract. According to Einstein his 'Principle of Equivalence' and his laws of Special Relativity must manifest in his gravitational field. It is shown herein that $Ric = R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence'. This has major implications for gravitational waves generally because it results in the total energy of Einstein's gravitational field always being zero so that gravitational energy cannot be localised. Attempts to rectify this problem by means of Einstein's pseudo-tensor fail because of mathematical non-existence. Linearisation of the field equations also fails because of mathematical non-existence.

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INTRODUCTION

Recall that 'Schwarzschild's solution' (using $c = 1$, $G = 1$) is,

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (1)$$

where it is asserted by inspection that r can go down to zero in some way, producing a black hole. It is reported in the companion paper **Part I** that this is not Schwarzschild's actual solution and that eq. (1) as usually interpreted is inconsistent with Schwarzschild's true solution (Schwarzschild 1916), which is

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (2)$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{1/3}, \quad 0 < r < \infty,$$

where α is an undetermined constant and $R(r)$ is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section .

Schwarzschild spacetime is a solution to the 'field equations' $R_{\mu\nu} = 0$. The latter has significant relevance to gravitational waves because it violates the physical foundations of Einstein's Theory.

RIC = 0 AND GRAVITATIONAL WAVES

The question of the localisation of gravitational energy is related to the validity of the field equations $R_{\mu\nu} = 0$, for according to Einstein, matter is the cause of the gravitational field and the causative matter is described in his theory by a mathematical object called the energy-momentum tensor, which is coupled to geometry (i.e.

spacetime) by his field equations, so that matter causes spacetime curvature (his gravitational field). Einstein's field equations,

> "*... couple the gravitational field (contained in the curvature of spacetime) with its sources.*"
> (Foster and Nightingale, 1995)

> "*Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter.*"
> (Pauli, 1981)

> "*Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that* ***the metrical field*** *(or, in mathematical language, the tensor with components g_{ik})* ***is related to the material filling the world***." (Weyl, 1952)

> "*... we have, in following the ideas set out just above, to discover the* ***invariant law of gravitation, according to which matter determines the components $\Gamma^{\alpha}_{\beta\iota}$ of the gravitational field****, and which replaces the Newtonian law of attraction in Einstein's Theory.*" (Weyl, 1952)

> "*Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field.*" (Landau and Lifshitz, 1951)

Qualitatively Einstein's field equations are,

spacetime geometry = $-\kappa$ x ***matter***

where ***matter*** is described by the energy-momentum tensor and κ is a constant. The spacetime geometry is described by a mathematical object called Einstein's tensor, $G_{\mu\nu}$ (μ, ν = 0, 1, 2, 3) and the energy-momentum tensor is $T_{\mu\nu}$. So Einstein's full field equations[a] are,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} \,. \tag{3}$$

Einstein asserted that his 'Principle of Equivalence' and his laws of Special Relativity must hold in a sufficiently small region of his gravitational field. Here is what Einstein (1967) said in 1954, the year before his death:

> "*Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K', uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the `cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is' at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For, according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K').*

> "*Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy.*"

[a] (The 'cosmological constant' is not included.)

In their textbook, J. Foster and J. D. Nightingale (1995) succinctly state the ‘Principle of Equivalence’ thus:

> “*We may incorporate these ideas into the principle of equivalence, which is this: In a freely falling (nonrotating) laboratory occupying a small region of spacetime, the laws of physics are the laws of special relativity.*”

Taylor and Wheeler (2000) state in their book:

> “*General Relativity requires more than one free-float* [i.e. inertial] *frame*.”

According to Pauli (1981):

> “*We can think of the physical realization of the local coordinate system K_o in terms of a freely floating, sufficiently small, box which is not subjected to any external forces apart from gravity, and which is falling under the influence of the latter. ... It is evidently natural to assume that the special theory of relativity should remain valid in K_o.*”

Note that the ‘Principle of Equivalence’ involves at least two masses. Similarly, the laws of Special Relativity involve the presence of at least two masses, for otherwise relative motion between two bodies cannot manifest. The two postulates of Special Relativity are couched in terms of inertial systems, which are in turn defined in terms of mass via Newton’s First Law of motion.

In the space of Newton’s theory of gravitation one can simply put in as many masses as one pleases. Although solving for the gravitational interaction of these masses rapidly becomes beyond our capacity, there is nothing to prevent us inserting masses conceptually. This is essentially the ‘Principle of Superposition’. However, one cannot do this in General Relativity, because Einstein’s field equations are non-linear. In General Relativity, each and every configuration of matter must be described by a corresponding energy-momentum tensor and the field equations solved separately for each and every such configuration, because matter and geometry are coupled, as eq. (3) describes. Not so in Newton’s theory where geometry is independent of matter. The ‘Principle of Superposition’ does not apply in General Relativity (Landau and Lifshitz, 1951).

> “*In a gravitational field, the distribution and motion of the matter producing it cannot at all be assigned arbitrarily -- on the contrary it must be determined (by solving the field equations for given initial conditions) simultaneously with the field produced by the same matter*.” (Landau and Lifshitz, 1951)

Now Einstein and the relevant physicists assert that the gravitational field “*outside*” a mass contains no matter, and so they assert that $T_{\mu\nu} = 0$, and that there is only one mass in the whole Universe with this particular problem statement. But setting the energy-momentum tensor to zero means that that there is no matter present by which the gravitational field can be caused! Nonetheless, it is so claimed, and it is also claimed that the field equations then reduce to the much simpler form,

$$R_{\mu\nu} = 0\,. \tag{4}$$

However, since this is a spacetime that **by construction** contains no matter, Einstein’s ‘Principle of Equivalence’ and his laws of Special Relativity cannot manifest, thus violating the physical requirements of his theory. It has never been “proven” that Einstein’s ‘Principle of Equivalence’ and laws of Special Relativity can manifest in a spacetime that by construction contains no matter. In fact, it is a contradiction. So $R_{\mu\nu} = 0$ fails. Now eq. (1) relates to eq. (4). However, there is allegedly mass present, denoted by m in eq. (1). This mass is not described by an energy-momentum tensor. That m is actually responsible for the alleged gravitational field associated with eq. (1) is confirmed by the fact that if $m = 0$, eq. (1) reduces to Minkowski spacetime, and hence no gravitational field. If not for the presence of m there is no curvature of spacetime. But this contradicts Einstein’s relation between geometry and matter, since m is introduced into eq. (1) *post hoc*, not via an energy-momentum tensor. So $R_{\mu\nu} = 0$ fails.

It is also claimed by the physicists that gravitational fields that can have no material cause. An example is de Sitter’s empty spherical Universe, based upon the following field equations (Tolman, 1987; Eddington, 1960):

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{5}$$

where λ is the so-called 'cosmological constant'. In the case of eq. (1) the field equations are given by eq. (4). On the one hand de Sitter's empty world is *devoid of matter* ($T_{\mu\nu} = 0$) and so has no material cause for the alleged gravitational field. On the other hand it is claimed that the spacetime described by eq. (4) has a material cause, *post hoc* as m in eq. (1), even though $T_{\mu\nu} = 0$ there as well: a contradiction. This is amplified by the co-called Schwarzschild-de Sitter line-element,

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{6}$$

which is a standard solution for eq. (5). Once again, m is identified *post hoc* as mass at the *centre of spherical symmetry* of the manifold, said to be at $r = 0$. The completely empty universe of de Sitter (Tolman, 1987; Eddington, 1960) can be obtained by setting $m = 0$ in eq. (6) to yield,

$$ds^2 = \left(1 - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{7}$$

Also, if $\lambda = 0$, eq. (5) reduces to eq. (4) and eq. (6) reduces to eq. (1). If both $\lambda = 0$ and $m = 0$, eqs. (6) and (7) reduce to Minkowski spacetime. Now in eq. (5) the term $\lambda g_{\mu\nu}$ is not an energy-momentum tensor. The universe described by eq. (7), which also satisfies eq. (5), is completely empty and so its curvature has no material cause; in eq. (5), just as in eq. (4), $T_{\mu\nu} = 0$. So eq. (7) is alleged to describe a gravitational field that has no material cause. Furthermore, although in eq. (4), $T_{\mu\nu} = 0$, its usual solution, eq. (1), is said to contain a (*post hoc*) material cause, denoted by m therein. Thus for eq. (1) it is claimed that $T_{\mu\nu} = 0$ supports a material cause of a gravitational field, but at the same time, for eq. (7), $T_{\mu\nu} = 0$ is also claimed to preclude material cause of a gravitational field. So $T_{\mu\nu} = 0$ is claimed to include and to exclude material cause. This is not possible. The contradiction is due to the *post hoc* introduction of matter, as m, in eq. (1). Furthermore, there is no experimental evidence to support the claim that a gravitational field can be generated without a material cause. Material cause is codified theoretically in eq. (3).

Since $R_{\mu\nu} = 0$ cannot describe Einstein's gravitational field, Einstein's field equations cannot reduce to $R_{\mu\nu} = 0$ when $T_{\mu\nu} = 0$. In other words, if $T_{\mu\nu} = 0$ (i.e. there is no matter present) then there is no gravitational field. Consequently Einstein's field equations must take the form (Lorentz, 1916; Levi-Civita, 1917),

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0. \tag{8}$$

The $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor; the total energy is *always zero*; the $G_{\mu\nu}/\kappa$ and the $T_{\mu\nu}$ *must vanish identically*; there is *no possibility* for Einstein gravitational waves. Furthermore, this means that Einstein's gravitational field violates the usual conservation of energy and momentum. There is no experimental evidence that the usual conservation of energy and momentum is invalid. It was early pointed out to Einstein by a number of his contemporaries that his General Theory violated the usual conservation of energy and momentum. To circumvent this problem Einstein invented his pseudo-tensor, having a two-fold purpose: (a) to bring his theory into line with the usual conservation of energy and momentum, (b) to enable him to get gravitational waves that propagate with speed c. First, it is not a tensor, and therefore not in keeping with his theory that all equations be tensorial. Second, he constructed his pseudo-tensor in such a way that it behaves like a tensor in one particular situation, that in which he could get gravitational waves with speed c. Now Einstein's pseudo-tensor is claimed to represent the energy and momentum of the gravitational field and it is routinely applied in relation to the localisation of gravitational energy, the conservation of energy and the flow of energy and momentum.

According to Dirac (1996),

> "*It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy*

within a definite (three dimensional) region at a certain time is independent of the coordinate system. Thus, in general, gravitational energy cannot be localized. The best we can do is to use the pseudo-tensor, which satisfies condition (i) but not condition (ii). It gives us approximate information about gravitational energy, which in some special cases can be accurate."

On gravitational waves Dirac (1996) remarks,

"*Let us consider the energy of these waves. Owing to the pseudo-tensor not being a real tensor, we do not get, in general, a clear result independent of the coordinate system. But there is one special case in which we do get a clear result; namely, when the waves are all moving in the same direction.*"

About the propagation of gravitational waves Eddington (1960) commented ($g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$),

$$\frac{\partial^2 h_{\mu\nu}}{\partial t^2} - \frac{\partial^2 h_{\mu\nu}}{\partial x^2} - \frac{\partial^2 h_{\mu\nu}}{\partial y^2} - \frac{\partial^2 h_{\eta\nu}}{\partial z^2} = 0 ,$$

"*… showing that the deviations of the gravitational potentials are propagated as waves with unit velocity, i.e. the velocity of light. But it must be remembered that this representation of the propagation, though always permissible, is not unique. … All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials $g_{\mu\nu}$ pertain not only to the gravitational influence which is objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with the **speed of thought**, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of the $g_{\mu\nu}$.*

"*The statement that in the relativity theory gravitational waves are propagated with the speed of light has, I believe, been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle."*

Einstein's pseudo-tensor, $\sqrt{-g}\, t^{\mu}{}_{\nu}$, is defined by (Misner, Thorne and Wheeler, 1970; Tolman, 1987; Landau and Lifshitz, 1951; Pauli, 1981; Dirac, 1996; Eddington, 1960),

$$\sqrt{-g}\, t^{\mu}_{\nu} = \frac{1}{2}\left(\delta^{\mu}_{\nu} L - \frac{\partial L}{\partial g^{\sigma\rho}_{\ ,\mu}} g^{\sigma\rho}_{\ ,\nu} \right), \tag{9}$$

where L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^{\gamma}_{\alpha\kappa}\Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta}\Gamma^{\kappa}_{\lambda\kappa}\right). \tag{10}$$

T. Levi-Civita (1917) gave a clear and rigorous proof that Einstein's pseudo-tensor is meaningless, and therefore any argument relying upon it is fallacious. Contracting eq. (9) produces a linear invariant (Levi-Civita, 1917; Eddington, 1960), thus

$$\sqrt{-g}\, t^{\mu}_{\mu} = \frac{1}{2}\left(4L - \frac{\partial L}{\partial g^{\sigma\rho}_{\ ,\mu}} g^{\sigma\rho}_{\ ,\mu} \right). \tag{11}$$

Since L is, according to (10), quadratic and homogeneous with respect to the Riemann-Christoffel symbols (and therefore also with respect to $g^{\sigma\rho}{}_{,\mu}$), one can apply Euler's theorem to obtain,

$$\frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\mu} = 2L. \tag{12}$$

Substituting (12) into (11) yields the linear invariant at *L*. This is a first-order intrinsic differential invariant that depends only upon the components of the metric tensor and their first derivatives. However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita (1900) proved that such invariants ***do not exist***! This renders Einstein's pseudo-tensor entirely meaningless, and all arguments relying on it false. Similarly, Einstein's field equations cannot be linearised because linearisation implies the existence of a tensor that, except for the trivial case of being precisely zero, ***does not otherwise exist***, as proven by Hermann Weyl (1944).

The LIGO project and its international counterparts have not detected gravitational waves (Sintes, 2008). The Lense-Thirring or 'frame dragging' effect was not detected by the Gravity Probe B and NASA has terminated further funding of that project (GPB, 2008).

3-D METRIC MANIFOLDS – FIRST PRINCIPLES

In the companion paper **Part I**, a proof was given that *r* in eq. (1) is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section and so of itself does not describe any distance in the manifold. This can be proven from first principles in the broader context of 3-dimensional metric manifolds, as follows.

Following the method suggested by Palatini, and developed by Levi-Civita (1977) denote ordinary Euclidean 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point O be in $\mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation **T** of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric dl'^2 unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of dl'^2 of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$ except possibly at an arbitrary point, the centre of spherical symmetry O' in $\mathbf{M}^3$.

Let a ray *i* emanate from an arbitrary point O in $\mathbf{E}^3$. There is then a corresponding geodesic *i*' in $\mathbf{M}^3$ issuing from the corresponding point O' in $\mathbf{M}^3$. Let P be any point on *i* other than O. There corresponds a point P' on *i*' in $\mathbf{M}^3$ different to O'. Let *g*' be a geodesic in $\mathbf{M}^3$ that is tangential to *i*' at P'.

Taking *i* as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on *i*' unchanged. If *g*' is distinct from *i*', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about *i* would cause *g*' to occupy an infinity of positions in $\mathbf{M}^3$ wherein *g*' has for each position the property of being tangential to *i*' at P' in the same direction, which is impossible. Hence, *g*' coincides with *i'*.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point O in $\mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' in $\mathbf{M}^3$ with centre of spherical symmetry at the corresponding point O' in $\mathbf{M}^3$.

Let Q be a point in Σ in $\mathbf{E}^3$ and Q' the corresponding point in Σ' in $\mathbf{M}^3$. Let $d\sigma^2$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma'^2$ in Σ' issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates *r*, *θ, φ*. Then

$$d\sigma^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{13}$$

$$r = \left|OQ\right|.$$

Clearly, if *r*, *θ, φ* are known, Q is determined and hence also Q' in Σ'. Therefore, *θ* and *φ* can be considered to be curvilinear coordinates for Q' in Σ' and the line element $d\sigma'^2$ in Σ' will also be represented by a quadratic form similar to eq. (13). To determine $d\sigma'^2$, consider two elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma'_1$ and $d\sigma'_2$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis |OQ| in $\mathbf{E}^3$, and so $d\sigma'_1$ and $d\sigma'_2$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $d\sigma'/d\sigma$ is the same for the two different directions irrespective of θ and φ, and so the foregoing ratio is a function of position, i.e. of r, θ, φ. But Q is an arbitrary point in Σ, and so $d\sigma'/d\sigma$ must have the same ratio for any corresponding points Q and Q'. Therefore, $d\sigma'/d\sigma$ is a function of r alone, thus

$$\frac{d\sigma'}{d\sigma} = H(r),$$

and so

$$d\sigma'^2 = H^2(r)\,d\sigma^2 = H^2(r)\,r^2\left(d\theta^2 + \sin^2\theta\, d\varphi\ \right), \tag{14}$$

where $H(r)$ is *a priori* unknown. For convenience set $R_c = R_c(r) = H(r)r$, so that eq. (14) becomes

$$d\sigma'^2 = R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi\ \right), \tag{15}$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing eq. (15) with eq. (13) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point Q', i.e. in terms of the relation $K = 1/R_c^2$ since the Gaussian curvature of eq. (13) is $K = 1/r^2$. This is an intrinsic property of all line elements of the form (15) (Levi-Civita, 1977). Accordingly, R_c, the inverse square root of the Gaussian curvature, can be regarded as the radius of Gaussian curvature. Therefore, in (13) the radius of Gaussian curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilic.

Let the element of radial distance from O in $\mathbf{E}^3$ be dr. Clearly, the radial lines issuing from O cut the surface Σ orthogonally. Combining this with (13) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$,

$$dl^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{16}$$

Let the corresponding radial geodesic from the point O' in $\mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from O' cut the geodesic surface Σ' orthogonally. Combining this with (15) by the theorem of Pythagoras gives the line element in $\mathbf{M}^3$ as,

$$dl^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{17}$$

where dR_p is, by spherical symmetry, also a function only of R_c. Set $dR_p = \sqrt{B(R_c)}\, dR_c$, so that (17) becomes

$$dl^2 = B(R_c)dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{18}$$

where $B(R_c)$ is an *a priori* unknown function.

Expression (18) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point O' in $\mathbf{M}^3$ (Levi-Civita, 1977).

Considering (16), the distance R_p = |OQ| from the point at the centre of spherical symmetry O to a point Q in Σ is given by

$$R_p = \int_0^r dr = r = R_c.$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature of a spherically symmetric surface in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point O in $\mathbf{E}^3$ and the point Q in Σ. Thus, in this case, $K = 1/R_c^2 = 1/R_p^2 = 1/r^2$. However, this is not a general relation, since according to (17) and (18), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point O' in $\mathbf{M}^3$ is not the same as the radius of Gaussian curvature of the spherically symmetric geodesic surface in $\mathbf{M}^3$, but is given by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c(r) = \int_0^r \sqrt{B(R_c(r))} \frac{dR_c(r)}{dr} dr$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \le r < \infty$ in (16) that it must follow in (17) and (18) that $0 \le R_c(r) < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from (17) and (18) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the centre of spherical symmetry to some other point in a metric manifold such as $\mathbf{M}^3$, or indeed of itself any distance at all in the associated manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the radius of Gaussian curvature of the spherically symmetric surface in $\mathbf{E}^3$ and the radial distance from the point at the centre of spherical symmetry of $\mathbf{E}^3$, owing to the fact that R_p and R_c are identical in that special case, as determined from the line-element.

It should also be noted that in writing expressions (16) and (17) it is implicit that O in $\mathbf{E}^3$ is defined as being located at the origin of the coordinate system of (16), i.e. O is located where R_p (= R_c = r) = 0, and by correspondence O' is defined as being located at the origin of the coordinate system of (17) and of (18), i.e. O' in $\mathbf{M}^3$ is located where R_p = 0. Furthermore, since it is well known that a geometry is completely determined by the form of the line-element describing it (Tolman, 1987), expressions (16), (17) and (18) share the very same fundamental geometry because they are line-elements of the same form.

CONCLUSION

$R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence'. The generalisation of Minkowski spacetime to Schwarzschild spacetime via $R_{\mu\nu} = 0$ is therefore not a generalisation of Special Relativity.

The black hole is fictitious and so there are no black hole generated gravitational waves. The international search for black holes and their gravitational waves is ill-fated, despite claims for discovery of black holes.

Curved spacetimes without material cause violate the physical principles of General Relativity. There is no experimental evidence supporting the notion of gravitational fields generated without material cause.

Einstein's gravitational waves are fictitious, so the international search for them is destined to detect nothing. Einstein's pseudo-tensor is meaningless and linearisation of Einstein's field equations inadmissible.

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