The Black Hole Catastrophe: A Reply to J. J. Sharples

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A recent Letter to the Editor (Sharples J. J., Coordinate transformations and metric extension: a rebuttal to the relativistic claims of Stephen J. Crothers, *Progress in Physics*, v.1, 2010) has analysed a number of my publications in *Progress in Physics*. There are serious problems with this treatment which should be brought to the attention of the Journal's readership. Dr. Sharples has committed errors in both mathematics and physics. For instance, his notion that $r = 0$ in the so-called "Schwarzschild solution" marks the point at the centre of the related manifold is false, as is his related claim that Schwarzschild's actual solution describes a manifold that is extendible. His *post hoc* introduction of Newtonian concepts and related mathematical expressions into Schwarzschild's actual solution are invalid; for instance, Newtonian two-body relations into what is alleged to be a one-body problem. Each of the objections are treated in turn and their invalidity fully demonstrated. Black hole theory is riddled with contradictions. This article provides definitive proof that black holes do not exist.

1 Introduction

A number of criticisms have been levelled in [1] against the arguments I have adduced to show that the black hole is not predicted by General Relativity. In reality, the black hole is a meaningless entity, without basis in any theory or in observation.

In the usual interpretation of Hilbert's [2–5] corrupted version of Schwarzschild's solution, the quantity r has *never* been properly identified by astrophysics. It has been variously and vaguely called a "*distance*" [6, 7], "*the radius*" [6, 8–22], the "*radius of a 2-sphere*" [1, 22, 23], the "*coordinate radius*" [24], the "*radial coordinate*" [1, 11, 16, 25–28], the "*Schwarzschild r-coordinate*" [26] , the "*radial space coordinate*" [29], the "*areal radius*" [24,25,27,30,31], the "*reduced circumference*" [28], and even "*a gauge choice: it defines the coordinate r*" [32]. In the particular case of $r = 2m = 2GM/c^2$ it is almost invariably referred to as the "*Schwarzschild radius*" or the "*gravitational radius*" [26]. However, none of these various and vague concepts of r are correct because the *irrefutable* geometrical fact is that r, in the spatial section of Hilbert's version of the Schwarzschild/Droste line-elements, is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section [33–35], and as such, it *does not* itself determine the geodesic radial distance from the centre of spherical symmetry located at an arbitrary point in the related pseudo-Riemannian metric manifold. It does not denote any distance in the spherically symmetric metric manifold for "Schwarzschild spacetime". It must also be emphasized that a geometry is completely determined by the *form* of its line-element [36, 37].

The correct geometric identification of the quantity r in Hilbert's solution *completely subverts* all claims for black holes, demonstrated herein, and hence proves the invalidity of the concerns advanced against my work [1]. In particular, proof is given that the said quantity r is not a distance in Schwarzschild spacetime but is related to the Gaussian curvature of the surface in the spatial section thereof. Similarly, it is proven that $r = 2m$ denotes the parametric *point* marking the centre of spherical symmetry of Hilbert's solution and so $r = 0$ does not signify an infinitely dense point-mass singularity. Thus, $0 \leq r < 2m$ is meaningless for Hilbert's solution. Moreover, I prove, by means of counter-example, that the Kruskal-Szekeres "coordinates" do not extend the so-called "Schwarzschild solution" because the latter is a maximal manifold; the Kruskal-Szekeres "coordinates" are therefore proven invalid.

It is demonstrated that the usual *post hoc* inclusion of mass, denoted by m in the "Schwarzschild solution", is invalid because it involves the arbitrary insertion of Newton's expression for escape velocity, which is a two-body relation (one body escapes from another), into what is alleged to be a solution for one body in an otherwise completely empty universe. It is also shown that this arbitrary inclusion of Newton's relation is effected in order to satisfy the claim that a massive source is nonetheless present in a spacetime that *by construction* contains no matter (i. e. $R_{\mu\nu} = 0$), and hence no sources.

According to Einstein, his 'Principle of Equivalence' and his laws of Special Relativity must manifest in sufficiently small regions of his gravitational field, regions which can be located anywhere in the gravitational field. Since Special Relativity forbids infinite density, General Relativity consequently forbids infinite density; demonstrated herein. Consequently, the infinitely dense point-mass singularity of the black hole is forbidden by General Relativity. Furthermore, since both the 'Principle of Equivalence' and the laws of Special Relativity are defined in terms of the *a priori* presence of multiple arbitrarily large finite masses, neither can manifest in a spacetime that *by construction* contains no matter: $R_{\mu\nu} = 0$ is a spacetime that *by construction* contains no matter.

It is shown that the notions of black holes existing in multitudes, interacting with one another and other matter, are invalid, because there are no known solutions to Einstein's field equations for two or more masses and no existence theorem by which it can even be asserted that his field equations contain latent solutions for such configurations of matter. All alleged black hole solutions pertain to a universe that contains only one mass. Since the 'Principle of Superposition' does not apply in General Relativity it cannot be asserted by an analogy with Newton's theory that black holes can be components of binary systems, collide, or merge. Thus, contrary to the usual claims, one cannot assert that two black holes can simultaneously persist in, and mutually interact in, a spacetime that *by construction* contains no matter.

In light of the foregoing, the astrophysics community must reconsider its theory of black holes.

2 Concerning spherically symmetric metric spaces

Consider the usual line-element used for Minkowski spacetime;

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{1}$$

$$0 \le r < \infty.$$

The spatial section of this line-element is ordinary Euclidean 3-Space, thus

$$d\sigma^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2}$$

and hence eq. (1) can be written as

$$ds^2 = c^2dt^2 - d\sigma^2 \tag{3}$$

where $d\sigma^2$ is a positive definite quadratic form. Thus, Minkowski spacetime is characterised by the *fixed* signature $(+,-,-,-)$. It cannot change signature to $(-,+,-,-)$, for instance.

Often, the speed of light in vacuum, c, is set to unity, so that eq. (1) is rendered as,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{4}$$

$$0 \le r < \infty.$$

Similarly, in Hilbert's "Schwarzschild solution", both c and Newton's gravitational constant G are set to unity, thus

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{5}$$

Schwarzschild's actual solution is [39]

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1}dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{5b}$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty, \quad \alpha = const.$$

Schwarzschild's solution is singular only at $r = 0$, and therefore does not "contain" a black hole. Despite the many claims to the contrary, Schwarzschild did not discuss black holes, because his solution does not predict them.

In section 3 of [1] it is claimed, without qualification, that I write the general static spherically symmetric line-element in [35] as

$$ds^2 = A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2. \tag{6}$$

(where $d\Omega^2 = (d\theta^2 + \sin^2\theta d\varphi^2)$). This is incorrect, as I clearly wrote the metric in all my relevant papers as

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{7}$$

$A, B, C > 0$, in order to maintain Minkowski spacetime signature $(+,-,-,-)$, and hence the time-like character of the quantity t and the space-like character of the quantities r, θ, φ. Maintenance of correct signature is an important issue [7,36]. The following metric is then adduced in [1],

$$ds^2 = A^*(\rho)dt^2 + B^*(\rho)d\rho^2 + \rho^2 d\Omega^2, \tag{8}$$

and it is asserted that I have denied that this corresponds to the most general metric. Interestingly, this expression is written in the dummy variable ρ "*to avoid confusion*" [1], despite the fact astrophysics always uses r, and thereby introduces confusion. Because r is the first letter of the word *radius* the astrophysics community erroneously takes r in the "Schwarzschild solution" to be the geodesic radial distance from the point at the centre of spherical symmetry in "Schwarzschild" spacetime, simply because r is the radial distance in the usual expression for Minkowski spacetime. The particular value $r = 2m$ is called the "Schwarzschild radius" which is alleged to be the radius of a sphere the effective "surface" of which is the "event horizon" of a "Schwarzschild" black hole. Now, the actual metric I wrote in relation to eq. (8) is

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{9}$$

where $A, B > 0$ once again to maintain signature. Here is what I wrote in the paper [35] cited in [1] (note that eq. (7) above corresponds to eq. (2a) in [35]):

"The standard analysis writes (2a) as,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2b}$$

and claims it the most general, which is incorrect. The form of $C(r)$ cannot be preempted, and must in fact be rigorously *determined from the general solution to (2a). ... Thus, the solution to (2b) can only produce a particular solution, not a general solution in terms of $C(r)$, for the gravitational field. ... The orthodox assumptions distort the fact that r is only a real parameter in the gravitational field and therefore that (2b) is not a general, but a particular expression, in which case the form of $C(r)$ has been fixed to $C(r) = r^2$. ... the form of $C(D(r))$ might be determined to obtain a means by which all particular solutions, in terms of an infinite sequence, may be constructed, according to the general prescription of Eddington. ... The Schwarzschild forms obtained from (24) satisfy Eddington's prescription for a general solution. Clearly, the correct form of $C(D(r))$ must naturally yield the Droste/Weyl/(Hilbert) solution, as well as the true Schwarzschild solution, and the Brillouin solution, amongst the infinitude of particular solutions that the field equations admit."*

Besides the important matter of the required fixed signature $(+,-,-,-)$, Dr. Sharples has evidently misunderstood my point: that the usual effective writing of $C(r) = r^2$ to get eq. (9) from eq. (7) *preempts* the form of the *a priori* unknown analytic function $C(r)$, and in that sense alone is eq. (9) not most general. It is obvious that simple relabelling of quantities does not alter the geometry of the manifold described by the metric. Thus, eqs. (7) and (9) are geometrically equivalent, since the geometry is fully determined by the *form* of the line-element [36, 37], not by the labelling of variables in the line-element. Moreover, in [38] I developed the relevant geometry from first principles, and explicitly stated that the most general 3-dimensional metric having spherical symmetry about an arbitrary point is [33],

$$ds^2 = A^2(R)dR^2 + R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

wherein R can be a function of some parameter. With R and $A(R)$ real-valued functions of a real variable r, this is a positive definite quadratic form, according to the Theorem of Pythagoras. The objective in my relevant papers is determination of the admissible form of the function $C(r)$, bearing in mind the solutions due to Schwarzschild [39], Droste [40], and Brillouin [41]. It is an irrefutable fact that r in the "Schwarzschild solution" (eq. (5) above) can be replaced by *any* analytic function of r without disturbing spherical symmetry and without violation of $R_{\mu\nu} = 0$ [4, 35, 42]. This being the case, $C(r)$ can be retained throughout the derivation of the solution, from which the form of $C(r)$ can be ascertained according to the conditions it must satisfy. Alternatively, r^2 in eq. (5) can be replaced by $C(r) > 0$ yet to be determined, making $A(r)$ and $B(r)$ functions of the parameter r by virtue of them being functions of $C(r)$, thus:

$$\begin{aligned} ds^2 = A\left(\sqrt{C(r)}\right)dt^2 - B\left(\sqrt{C(r)}\right)\, d\left(\sqrt{C(r)}\right)^2 - \\ -C(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \end{aligned} \qquad (10)$$

which is a simple restatement of eq. (9). The radial quantity r of Minkowski spacetime, appearing in the sought after solution for Schwarzschild spacetime, thus acts as a parameter for all the components of the metric tensor sought. But any analytic function will not do: for instance, $C(r) = \exp(2r)$ does not satisfy *all* the conditions required for the sought after solution. That there must be an infinite number of geometrically equivalent *particular* metrics is clear, as Eddington [42] has also noted. The solutions due to Schwarzschild, to Droste, and to Brillouin, for example, are geometrically equivalent metrics, differing only in the *particular* expression for $C(r)$. They all describe the same manifold. In the case of Schwarzschild, $C(r) = \left(r^3 + \alpha^3\right)^{2/3}$, $0 < r < \infty$, α a constant, the metric singular only at $r = 0$; for Droste, $C(r) = r^2$, $2m < r < \infty$, m a constant, and the metric singular only at $r = 2m$; and for Brillouin, $C(r) = (r + \alpha)^2$, $0 < r < \infty$, singular only at $r = 0$. Consequently, the claim in [1] that I maintain that "*solutions of the gravitational field equations that are derived from the metric ansatz (9) are particular solutions rather than general solutions*" is inaccurate. Such solutions differ only by the specific assignment of $C(r)$. There is no change in geometry by such admissible assignments of $C(r)$. My retention of $C(r)$ throughout, rather than using eq. (9) and replacing r^2 in the resultant by $C(r)$ is to monitor changes of variables and their boundary values. Eq. (7) is a generalisation of eq. (4). Carrying over the range $0 \le r < \infty$ from eq. (4) into eq. (7) requires that $C(0) \le C(r) < \infty$, and without knowing $C(r)$ it is invalid to conclude that $C(0) = 0$, and hence that in eq. (9) $0 \le r < \infty$. This was first pointed out by the late American physicist L. S. Abrams [2]. Dr. Sharples fails to see that $r = 0$ does not imply that $\sqrt{C(0)} = 0$, because $C(r)$ is *a priori* unknown. Clearly, the parameter r is not a distance in the solution space, and hence not the radial geodesic distance in that space. The question therefore arises as to the geometric identity of $C(r)$ and the relationship between $C(r)$ and r, which I have previously addressed.

The Standard Derivation

As explained in a number of my papers [35, 38, 43–46], the usual derivation of the "Schwarzschild solution" begins with eq. (4) for Minkowski spacetime and proposes a generalisation thereof as, or equivalent to,

$$ds^2 = F(r)dt^2 - G(r)dr^2 - H(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (11)$$

where $F, G, H > 0$ and r is the radial variable which appears in the usual metric for Minkowski spacetime, making r in eq. (4) a parameter for the components of the metric tensor of eq. (11). The functions $F(r), G(r), H(r)$ are to be determined such that the signature of metric (4) is maintained in metric (11), at $(+,-,-,-)$. The substitution $r^* = \sqrt{H(r)}$ is then usually made, to get,

$$ds^2 = W(r^*)dt^2 - M(r^*)dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (11b)$$

$W, M > 0$. Then, the * is simply dropped (i. e. $r^* = r$), and it is *assumed* that $0 \le r < \infty$ can be carried over from eq. (4) into eq. (11b), to get, by introduction of exponential functions [2, 7, 12, 16, 22, 24–26, 29, 33, 36, 42, 47–57],

$$ds^2 = e^{2\lambda} dt^2 - e^{2\beta} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (12)$$

$$0 \le r < \infty,$$

the real-valued exponential functions in r being introduced to emphasise the required fixed signature $(+, -, -, -)$. It is then required that $e^{2\lambda(r)}$ and $e^{2\beta(r)}$ be determined such as to satisfy $R_{\mu\nu} = 0$.

Note that in going from eq. (11) to eq. (12), it is *assumed* that $\sqrt{H(0)} = 0$ [2], making $0 \le r^* < \infty$ in eq. (11b) (and hence $0 \le r < \infty$ in eq. (12)), since $r^* = \sqrt{H(r)}$: but this cannot be known since $H(r)$ is *a priori* unknown [2–4]. One simply cannot treat r^* in eq. (11b), and hence r in eq. (12), as the r in eq. (4). Also note that eq. (12) not only retains the signature -2, but *also retains the signature* $(+, -, -, -)$, because $e^{2\lambda} > 0$ and $e^{2\beta} > 0$ *by construction*. Thus, neither $e^{2\lambda}$ nor $e^{2\beta}$ can change sign or become zero [7, 16, 36, 50, 57]. This is a requirement since there is no possibility for Minkowski spacetime (eq. (4)) to change signature from $(+, -, -, -)$ to, for example, $(-, +, -, -)$.

The astrophysics community then obtains the solution given by eq. (5), wherein the constant m is assigned to the mass causing the alleged associated gravitational field. By inspection of eq. (5), it is asserted that there are two singularities, one at $r = 2m$ and one at $r = 0$. It is claimed that $r = 2m$ is a removable coordinate singularity, and that $r = 0$ a physical singularity. It is also asserted that $r = 2m$ gives the event horizon (the "Schwarzschild radius") of a black hole, from which the "escape velocity" is that of light (in vacuo), and that $r = 0$ is the position of the infinitely dense point-mass singularity of a black hole, produced by irresistible gravitational collapse.

However, these claims cannot be true. First, the construction of eq. (12) to obtain eq. (5) in satisfaction of $R_{\mu\nu} = 0$ is such that neither $e^{2\lambda}$ nor $e^{2\beta}$ can change sign, because $e^{2\lambda} > 0$ and $e^{2\beta} > 0$. Therefore, the claim that r in metric (5) can take values less than $2m$ is false; a contradiction by the very construction of the metric (12) leading to metric (5). Furthermore, since neither $e^{2\lambda}$ nor $e^{2\beta}$ can ever be zero, the claim that $r = 2m$ is a removable coordinate singularity is also false. In addition, the true nature of r in both eqs. (12) and (5) is entirely overlooked, and the geometric relations between the components of the metric tensor, fixed by the *form* of the line-element, are not applied, in consequence of which astrophysics fatally falters.

In going from eq. (11) to eq. (12) the astrophysics community has failed to realise that in eq. (11) all the components of the metric tensor are functions of r by virtue of the fact that all the components of the metric tensor are functions of $C(r)$ (see eq. (10)). Indeed, to *illuminate* this, consider the metric,

$$ds^2 = B(R)\,dR^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

$$B(R) > 0.$$

This is the most general expression for the metric of a three-dimensional spherically symmetric metric-space [33, 38]. If R is a function of some parameter r, then the metric in terms of r is,

$$ds^2 = B(R(r))\left(\frac{dR}{dr}\right)^2 dr^2 + R^2(r)(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

$$= A^2(r)\, dr^2 + R^2(r)(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

$$A^2(r) = B(R(r))\left(\frac{dR}{dr}\right)^2 > 0.$$

Eq. (11) is given in terms of the parameter r of Minkowski spacetime; not explicitly in terms of the function $H(r)$. In eq. (11), set $G(r) = N\left(\sqrt{H(r)}\right)\left(d\sqrt{H}/dr\right)^2$, then eq. (11) becomes,

$$ds^2 = F\left(\sqrt{H(r)}\right)dt^2 - N\left(\sqrt{H(r)}\right)\left(\frac{d\sqrt{H}}{dr}\right)^2 dr^2 - H(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (11c)$$

or simply

$$ds^2 = F(\sqrt{H})dt^2 - N(\sqrt{H})d\sqrt{H}^{\,2} - H\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (11d)$$

wherein $H = H(r)$. Similarly, working backwards from eq. (11b), using $r^* = \sqrt{H(r)} = R(r)$, eq. (11b) becomes,

$$ds^2 = W(R(r))dt^2 - M(R(r))dR(r)^2 - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (11e)$$

or simply,

$$ds^2 = W(R)dt^2 - M(R)dR^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

wherein $R = R(r)$; and in terms of the parameter r of Minkowski spacetime, this becomes,

$$ds^2 = W(R(r))dt^2 - M(R(r))\left(\frac{dR}{dr}\right)^2 dr^2 - R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (11f)$$

Writing $W(R(r)) = F(r)$ and $G(r) = M(R(r))\,(dR/dr)^2$ gives,

$$ds^2 = F(r)dt^2 - G(r)dr^2 - H(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

which is eq. (11). Thus, eq. (11) is a disguised form of eq. (11d), and so there is no need for the implicit transformations applied by astrophysics to get their eq. (12), from which they get their eq. (5), the "Schwarzschild solution". In other words, what the astrophysics community calls r in their eq. (5) is actually $R(r)$, for which they have not given any definite admissible form in terms of the parameter r (except for $R(r) = r$), and they incorrectly treat their $R(r)$, labelled r in eqs. (12) and (5) as the r in eq. (4), manifest in the miscarry of the range $0 \leq r < \infty$ from eq. (4) into eqs. (11b) and (12) under the misconception that r is radial distance in the manifold described by eqs. (11b) and (12) (and hence in eq. (5) as well).

Notwithstanding the fixing of the spacetime signature to $(+, -, -, -)$ in the writing of eq. (12), the astrophysics community permits $0 < r < 2m$ in eq. (5), changing the signature to $(-, +, -, -)$, and then admits that the rôles of t and r are thereby exchanged i. e. t becomes spacelike and r becomes timelike. But this violates the construction at eq. (12), which has the fixed signature $(+, -, -, -)$, and is therefore inadmissible. To further illustrate this violation, when $2m < r < \infty$ the signature of eq. (5) is $(+, -, -, -)$; but if $0 < r < 2m$ in eq. (5), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) \text{ is negative, and}$$

$$g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1} \text{ is positive.}$$

Therefore, the signature of metric (5) changes to $(-, +, -, -)$. Thus, the rôles of t and r are exchanged. According to Misner, Thorne and Wheeler [26], who use the spacetime signature $(-, +, +, +)$ instead of $(+, -, -, -)$,

> *"The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial/\partial t$, is spacelike ($g_{tt} > 0$) and $\partial/\partial r$, is timelike ($g_{rr} < 0$).*
>
> *"What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at $r = 2M$ always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside $r = 2M$. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate $r = 2M$ to the later time coordinate $r = 0$. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty, and with never one halt along the way, r drops from $2M$ to 0.*
>
> *"At $r = 2M$, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite."*

Chandrasekhar [27] has expounded the same claim as follows,

> *'There is no alternative to the matter collapsing to an infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: "all the King's horses and all the King's men" cannot prevent it.'*

Carroll [22] also says,

> *"This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for $r < 2GM$, t becomes spacelike and r becomes timelike.) Thus you can no more stop moving toward the singularity than you can stop getting older."*

Vladimirov, Mitskiévich and Horský [58] assert,

> *"For $r < 2GM/c^2$, however, the component g_{oo} becomes negative, and g_{rr}, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t".*

To amplify this, set $t = r^*$ and $r = t^*$. Then, for $0 < r < 2m$, eq. (5) becomes,

$$ds^2 = \left(1 - \frac{2m}{t^*}\right) dr^{*2} - \left(1 - \frac{2m}{t^*}\right)^{-1} dt^{*2} - t^{*2} \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$0 < t^* < 2m.$$

It is now evident that this is a *time-dependent metric* since all the components of the metric tensor are functions of the timelike t^*, and thus this metric *bears no relationship to the original time-independent problem to be solved* [4, 40, 41]. In other words, this metric is a *non-static solution to a static problem*: contra hype! Thus, in eq. (5), $0 < r < 2m$ is meaningless.

Furthermore, if the signature of "Schwarzschild" spacetime is permitted to change from $(+, -, -, -)$ to $(-, +, -, -)$ in the fashion for the black hole, then there must be, for the latter signature, a corresponding generalisation of the Minkowski metric, taking the fundamental form

$$ds^2 = -e^{2\lambda}dt^2 + e^{2\beta}dr^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

where λ, β and R are all unknown real-valued functions of only the real variable r, and where $e^{2\lambda} > 0$ and $e^{2\beta} > 0$. But this is impossible because the Minkowski spacetime metric has the fixed signature $(+, -, -, -)$. The spatial section of Minkowski spacetime is a positive definite quadratic form. Therefore, the foregoing generalised metric is not a generalistion of Minkowski spacetime at all.

I developed the relevant geometry of 3-dimensional spherically symmetric metric spaces from first principles in [38].

Conclusions:

1. In [1] the play on the words "particular solutions" and "general solution" does not constitute a scientific argument.

2. Dr. Sharples has been selective in his references to my papers; outside the context of my analysis in terms of an *a priori* unknown analytic function $R_c^2(r) = C(r)$ by which all metrics obtained thereby are geometrically equivalent.

3. The change of signature from $(+, -, -, -)$ to $(-, +, -, -)$ in eq. (5) violates the required fixed Minkowski spacetime signature $(+, -, -, -)$ embodied in the generalisations from which eq. (5) is derived.

4. The range $0 < r < 2m$ on eq. (5) produces a non-static 'solution' to a static problem, and is therefore invalid.

3 The five additional criticisms

It is alleged in [1] that I have erred in holding the following five points true:

1. "*The coordinate 'ρ' appearing in (9)**, *is not a proper radius,*

2. *The "Schwarzschild" solution as espoused by Hilbert and others is different to the Schwarzschild solution obtained originally by Schwarzschild,*

3. *The original Schwarzschild solution is a complete (i.e. inextendible) metric,*

4. *There are an infinite number of solutions to the static, spherically symmetric solutions to the field equations corresponding to a point mass,*

5. *For line-elements of the Schwarzschild form, the scalar curvature f remains bounded everywhere, and hence there is no 'black hole'.*"

*See eq. (8) herein.

I shall now demonstrate that Sharples' attack on each of these points is invalid.

Claim 1. This 'criticism' involves a change of meaning. Nowhere in my writings have I ever asserted that this quantity ρ which appears in eq. (9) of [1] (i. e. eq. (8) above) cannot be "*a proper radius*" in some set of circumstances, such as by embedding into Euclidean 3-space the spherically symmetric geodesic surface of the "Schwarzschild solution". I have, in fact, repeatedly asserted and proven that this ρ (r in eq. (9) above) is not even a distance, let alone a radial one, in the "Schwarzschild solution". Thus, it is *not* the proper radius in Schwarzschild spacetime. Here again is the proof.

Recall that the squared differential element of arc-length of a curve in a surface is given by *the First Fundamental Quadratic Form* for a surface,

$$ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2,$$

wherein u and v are curvilinear coordinates. If either u or v is constant, the resulting line-elements are called *parametric curves* in the surface. The differential element of surface area is given by,

$$dA = \left|\sqrt{EG - F^2}\,du\,dv\right|.$$

An expression which depends only on E, F, G and their first and second derivatives is called a bending invariant. It is an intrinsic (or absolute) property of a surface. The Gaussian (or Total) curvature of a surface is an important intrinsic property of a surface.

The 'Theorema Egregium' of Gauss

> *The Gaussian curvature K at any point P of a surface depends only on the values at P of the coefficients in the First Fundamental Form and their first and second derivatives.* [59–61]

Thus,

> *"The Gaussian curvature of a surface is a bending invariant."* [60]

The plane has a constant Gaussian curvature of $K = 0$. *"A surface of positive constant Gaussian curvature is called a spherical surface."* [59]

Now a line-element, or squared differential element of arc-length, in spherical coordinates, for 3-dimensional Euclidean space is,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right), \qquad (13)$$

$$0 \le r < \infty.$$

The scalar r can be construed, verified by calculation, as the magnitude of the radius vector $\mathbf{r}$ from the origin of the coordinate system, the said origin coincident with the centre of the associated sphere. Relations between the components of the metric tensor are fixed by the form of the line-element. Indeed, the radius R_p of the associated sphere, for which $\theta = const., \varphi = const.$ is given by,

$$R_p = \int_0^r dr = r,$$

the length of the geodesic C_p (a parametric curve; $r = const.$, $\theta = \pi/2$) in an associated surface is given by,

$$C_p = r\int_0^{2\pi} d\varphi = 2\pi r,$$

the area A_p of the associated spherically symmetric surface ($r = const.$) is,

$$A_p = r^2\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi r^2,$$

and the volume V_p of the sphere is,

$$V_p = \int_0^r r^2 dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}\pi r^3.$$

Note that the point at the centre of spherical symmetry for any problem at hand need not be coincident with the origin of the coordinate system used to describe the problem. For example, the equation of a sphere of radius ρ centered at the point C located at the extremity of the fixed vector $\mathbf{r}_o$ in Euclidean 3-space, is given by

$$(\mathbf{r} - \mathbf{r}_o)\cdot(\mathbf{r} - \mathbf{r}_o) = \rho^2.$$

If $\mathbf{r}$ and $\mathbf{r}_o$ are collinear, the vector notation can be dropped, and this expression becomes,

$$\left|r - r_o\right| = \rho,$$

where $r = |\mathbf{r}|$ and $r_o = \left|\mathbf{r}_o\right|$, and the common direction of $\mathbf{r}$ and $\mathbf{r}_o$ becomes entirely immaterial. This scalar expression for a shift of the centre of spherical symmetry away from the origin of the coordinate system plays a significant rôle in the equivalent line-elements for Schwarzschild spacetime [2,35].

Consider next the generalisation of eq. (13) to a spherically symmetric metric manifold, by the line-element,

$$\begin{gathered} ds^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) = \\ = \Psi\left(R_c\right) dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (14) \\ R_c = R_c(r), \quad \Psi\left(R_c\right) > 0, \\ R_c(0) \le R_c(r) < \infty, \end{gathered}$$

where both $\Psi(R_c)$ and $R_c(r)$ are *a priori* unknown analytic functions. Since neither $\Psi(R_c)$ nor $R_c(r)$ are known, eq. (14) may or may not be well-defined at $R_c(0)$: one cannot know until $\Psi(R_c)$ and $R_c(r)$ are somehow specified. There is a one-to-one point-wise correspondence between the manifolds described by metrics (13) and (14), i.e. a mapping between the auxiliary Euclidean manifold described by metric (13) and the generalised non-Euclidean manifold described by metric (14), as those versed in differential geometry have explained [33]. If $R_c(r)$ is constant, metric (14) reduces to a 2-dimensional spherically symmetric geodesic surface described by the first fundamental quadratic form,

$$ds^2 = R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (15)$$

If r is constant, eq. (13) reduces to the 2-dimensional spherically symmetric surface described by the first fundamental quadratic form,

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (16)$$

Although R_c and r are constants in equations (15) and (16) respectively, they share a definite geometric identity in their respective surfaces by virtue of the *form* of the line-elements: but the identity is *not* that of being a radial quantity, or even of a distance. What then is this geometric identity?

A surface is a manifold in its own right. It need not be considered in relation to an embedding space. Therefore, quantities appearing in its line-element must be identified in relation to the surface:

> *"And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space."* [37]

Note that eqs. (13) and (14) have the same metric ground-form and that eqs. (15) and (16) have the same metric ground-form. Metrics of the same form share the same fundamental relations between the components of their respective metric tensors. For example, consider eq. (14) in relation to eq. (13). For eq. (14), the radial geodesic distance (i.e. the proper radius) from the point at the centre of spherical symmetry ($\theta = const., \varphi = const.$) is,

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi(R_c(r))} dR_c(r) = \\ = \int_0^r \sqrt{\Psi(R_c(r))}\frac{dR_c(r)}{dr}dr,$$

the length of the geodesic C_p (a parametric curve; $R_c(r) = const.$, $\theta = \pi/2$) in an associated surface is given by,

$$C_p = R_c(r)\int_0^{2\pi} d\varphi = 2\pi R_c(r),$$

the area A_p of an associated spherically symmetric geodesic surface ($R_c(r) = const.$) is,

$$A_p = R_c^2(r) \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi R_c^2(r),$$

and the volume V_p of the geodesic sphere is,

$$V_p = \int_0^{R_p} R_c^2(r)\, dR_p \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi =$$

$$= 4\pi \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi(R_c(r))} R_c^2(r) dR_c$$

$$= 4\pi \int_0^{r} \sqrt{\Psi(R_c(r))} R_c^2(r) \frac{dR_c(r)}{dr} dr.$$

Remarkably, in relation to metric (5), Celotti, Miller and Sciama [10] make the following false assertion:

> *"The 'mean density' $\overline{\rho}$ of a black hole (its mass M divided by $\frac{4}{3}\pi r_s^3$) is proportional to $1/M^2$"*

where r_s is the so-called "Schwarzschild radius". The volume they adduce for a black hole cannot be obtained from metric (5): it is a volume associated with the Euclidean 3-space described by metric (13); it is not at all the volume associated with the "Schwarzschild" manifold.

In the case of the 2-dimensional metric manifold given by eq. (15) the Riemannian (or Sectional) curvature associated with eq. (14) (which depends upon both position and direction) reduces to the Gaussian curvature K (which depends only upon position), and is given by [16,33,38,59,60,63–66],

$$K = \frac{R_{1212}}{g}, \tag{17}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$ (because the metric tensor of eq. (15) is diagonal). Gaussian curvature is an intrinsic geometric property of a surface (Theorema Egregium*); independent of any embedding space.

Recall from elementary differential geometry and tensor analysis that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\gamma}_{.\ \nu\rho\sigma}$$

$$R^1_{.\ 212} = \frac{\partial \Gamma^1_{22}}{\partial x^1} - \frac{\partial \Gamma^1_{21}}{\partial x^2} + \Gamma^k_{22}\Gamma^1_{k1} - \Gamma^k_{21}\Gamma^1_{k2}$$

$$\Gamma^i_{ij} = \Gamma^i_{ji} = \frac{\partial\left(\frac{1}{2}\ln|g_{ii}|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}} \frac{\partial g_{jj}}{\partial x^i}, \quad (i \neq j) \tag{18}$$

*i.e. Gauss' Most Excellent Theorem.

and all other Γ^i_{jk} vanish. In the above, $i, j, k = 1, 2$, $x^1 = \theta$, $x^2 = \varphi$. Applying expressions (17) and (18) to expression (15) gives,

$$K = \frac{1}{R_c^2} \tag{19}$$

by which $R_c(r)$ is the inverse square root of the Gaussian curvature. Hence, in eq. (16) the quantity r is the inverse square root of the Gaussian curvature. This Gaussian curvature is intrinsic to all geometric surfaces having the form of eq. (15) [33], and a geometry is completely determined by the *form* of its line-element [36]. Note that according to eqs. (13), (16) and (17), the radius calculated for (13) gives the same value as the associated inverse square root of the Gaussian curvature of a spherically symmetric surface embedded in the space described by eq. (13). Thus, the Gaussian curvature (and hence the inverse square root of the Gaussian curvature) of the spherically symmetric surface embedded in the space of (13) can be directly associated with the radius calculated from eq. (13). This is a consequence of the Euclidean nature of the space described by metric (13), which also describes the spatial section of Minkowski spacetime. However, this is *not* a general relationship. The inverse square root of the Gaussian curvature is not a distance of any sort in Schwarzschild spacetime but in fact determines the Gaussian curvature of the spherically symmetric geodesic surface containing any point in the spatial section of the manifold, as proven by expression (19). Thus, the quantity r in eq. (5) is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, not the radial geodesic distance from the centre of spherical symmetry of the spatial section, or any other distance.

The nature of the concepts "*reduced circumference*" (i. e. $r = C_p/2\pi$) and "*areal radius*" (i. e. $r = \sqrt{A_p/4\pi}$) is now plainly evident - neither concept correctly identifies the geometric nature of the quantity r in metric (5). The geodesic C_p in the spherically symmetric geodesic surface in the spatial section of eq. (5) is a function of the curvilinear coordinate φ and the surface area A_p is a function of the curvilinear coordinates θ and φ where, in both cases, r is a constant. However, r therein has a clear and definite geometrical meaning, as eq. (19) attests. The said Gaussian curvature K is a positive constant bending invariant of the surface, independent of the values of θ and φ. Thus, neither C_p nor A_p rightly identify what r is in line-element (5). To illustrate further, when $\theta = constant$, the arc-length in the spherically symmetric geodesic surface is given by:

$$s = s(\varphi) = r \int_0^{\varphi} \sin\theta\, d\varphi = r \sin\theta\, \varphi, \quad 0 \leq \varphi \leq 2\pi,$$

where $r = constant$. This is the equation of a straight line, of gradient $ds/d\varphi = r\sin\theta$. If $\theta = const. = \frac{1}{2}\pi$, then $s = s(\varphi) = r\varphi$, which is the equation of a straight line of gradient $ds/d\varphi = r$. The maximum arc-length of the geodesic $\theta =$

const. $= \frac{1}{2}\pi$ is therefore $s(2\pi) = 2\pi r = C_p$. Similarly, the surface area is:

$$A = A(\varphi, \theta) = r^2 \int_0^\theta \int_0^\varphi \sin\theta \, d\theta \, d\varphi = r^2 \varphi \left(1 - \cos\theta\right),$$

$$0 \le \varphi \le 2\pi, \quad 0 \le \theta \le \pi, \quad r = constant.$$

The maximum area (i.e. the area of the entire surface) is $A(2\pi, \pi) = 4\pi r^2 = A_p$. Clearly, neither s nor A are functions of r, because r is a constant here, not a variable. And since r appears in each expression (thereby having the same value in each expression), neither s nor A rightly identify the geometrical significance of r in the First Fundamental Form for the spherically symmetric geodesic surface described by $ds^2 = r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$, because r is *not* a distance in the spherical surface and is *not* the "radius" of the spherical surface (unless it is embedded into Euclidean 3-space). The geometrical significance of r is intrinsic to the surface and is determined from the components of the metric tensor for the surface, and their derivatives (Gauss' Theorema Egregium): it is the inverse square root of the Gaussian curvature K of the spherically symmetric surface described (the constant is $K = 1/r^2$). The "*reduced circumference*" $r = C_p/2\pi$ and the "*areal radius*" $r = \sqrt{A_p/4\pi}$ are expressions that do not identify the geometric nature of r in either metric (16) or metric (5), the former appearing in the latter. The claims made by the astrophysics community that the "*areal radius*" and the "*reduced circumference*" each define [24, 28, 50] (in two different ways) the constant r in eq. (5) are entirely false. The "*reduced circumference*" and the "*areal radius*" are in fact one and the same, namely the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of eq. (5), as proven above. This simple geometric fact completely subverts all claims that General Relativity predicts black holes.

The Proper Radius

Dr. Sharples [1] objects to my use of the indefinite integral

$$R_p = \int \sqrt{B(r)}dr$$

for the proper radius in Schwarzschild spacetime. It is asserted that I have erred because this "*... does not take into account the effect of coordinate transformations*" [1]. The objection is entirely groundless because there is a constant of integration associated with this integral, which I evaluate by means of the metric and the conditions it must satisfy. I obtained the line-element

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

where $R_c = R_c(r)$ is an as yet unknown analytic function. To ascertain the admissible form of $R_c(r)$ I take the integral for the proper radius, thus

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} =$$

$$= \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left[\sqrt{R_c} + \sqrt{R_c - \alpha}\right] + k, \qquad (20)$$

where k is a constant. Now for some r_o, $R_p(r_o) = 0$; bearing in mind that $0 \le R_p < \infty$ (see [38]). Then, by eq. (20), it is required that $R_c(r_o) = \alpha$ and $k = -\alpha \ln\sqrt{\alpha}$, by which

$$R_p(r) = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left[\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right], \qquad (21)$$

$$R_c = R_c(r).$$

It is thus also determined that the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section ranges not from ∞ to 0, as it does for Euclidean 3-space, but from α^{-2} to 0. This is an inevitable consequence of the peculiar non-Euclidean geometry described by eq. (5b). The indefinite metrics associated with Einstein's Theory of Relativity admit of other geometric oddities, such as null vectors, which are non-zero vectors that have zero magnitude, or equivalently, non-zero vectors that are orthogonal to themselves (to which the astrophysics community raises no objections).

Schwarzschild's true solution, eq. (5b), must be a particular case of the general expression sought for $R_c(r)$. Brillouin's solution [2, 41] must also be a particular case, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r+\alpha}\right)dt^2 - \left(1 - \frac{\alpha}{r+\alpha}\right)^{-1} dr^2 - \\ - (r+\alpha)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (22)$$

$$0 < r < \infty,$$

and Droste's solution [40] must as well be a particular solution, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

$$\alpha < r < \infty. \qquad (23)$$

All these solutions must be particular cases in an infinite set of equivalent metrics [42]. The admissible form for $R_c(r)$ is [35],

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \ne r_o, \qquad (24)$$

where r_o and n are entirely arbitrary constants. Hence the solution for $R_{\mu\nu} = 0$ is,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$
$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o. \tag{25}$$

Then, if $r_o = 0$, $r \rightarrow r_o^+$, $n = 1$, Brillouin's solution eq. (22) results. If $r_o = 0$, $r \rightarrow r_o^+$, $n = 3$, then Schwarzschild's actual solution eq. (5b) results. If $r_o = \alpha$, $r \rightarrow r_o^+$, $n = 1$, then Droste's solution eq. (23) results, which is the correct solution in terms of the line-element of eq. (5). In addition, the required infinite set of equivalent metrics is thereby obtained, all of which are asymptotically Minkowski spacetime. Furthermore, if the constant α is set to zero, eqs. (25) reduce to Minkowski spacetime, and if r_o is set to zero, then the usual Minkowski metric of eq. (4) is obtained. The significance of the term $\left|r - r_o\right|$ was given in Section 2: it is a shift of the location of the centre of spherical symmetry in the spatial section of the auxiliary manifold away from the origin of coordinates of the auxiliary manifold, along a radial line, to a point at distance r_o from the origin of coordinates, the direction of shift being immaterial. The point r_o in the auxiliary manifold is mapped into the point $R_p(r_o) = 0$ in Schwarzschild space, irrespective of the choice of the parametric point r_o in the auxiliary manifold. Minkowski spacetime is the auxiliary manifold for Schwarzschild spacetime. The arbitrary point r_o is mapped to $R_c(r_o) = \alpha \ \forall \ r_o \ \forall \ n$.

In [1] it is asserted, in relation to Schwarzschild's actual solution, that

> *...the manifold...is foliated by 2-spheres of radius greater than $\alpha = 2m$ – the spacetime has a hole in its centre!*

This is incorrect. The 2-spheres referred to in [1] are *not* in the Schwarzschild manifold because the 2-spheres referred to relate to a Euclidean 3-space in which the spherical surface described by $ds^2 = R^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$ is considered to be embedded; making this 3-space an auxiliary manifold. Radial distance in Schwarzschild spacetime is given by $R_p(r)$. Consequently, distances between two points (one fixed at $r = r_o$) in the spatial section of Minkowski spacetime (which is precisely Euclidean 3-space entire) are mapped into distances in the Euclidean 3-space involving Schwarzschild's R, where the point at the centre of spherical symmetry is at $R(0) = \alpha$, *not* at $R = 0$. In other words, the Euclidean embedding 3-space involving Schwarzschild's R is a copy of Euclidean 3-space entire. In general, according to eqs. (25) herein, $R_c(r)$ maps Euclidean 3-space into itself and thence maps distances therein into all the components of the metric tensor for Schwarzschild spacetime, as depicted in figure 1.

It is clear from expressions (25) that there is only one singularity, at the arbitrary constant r_o, where $R_c(r_o) = \alpha \ \forall \ r_o \ \forall \ n$ and $R_p(r_o) = 0 \ \forall \ r_o \ \forall \ n$, and that *all* components of the metric tensor are affected by the constant α. Hence, the "removal" of the singularity at $r = 2m$ in eq. (5) is fallacious because it is clear from expressions (25), in accordance with the intrinsic geometry of the line-element as given in Section 2 and [38], that there is no singularity at $r = 0$ in eq. (5) and so $0 \leq r < 2m$ therein is meaningless [2–4,7,35,38–40,62,66,67]. The usual claims for eq. (5) violate the geometry fixed by the *form* of its line-element and contradict the generalisations at eqs. (11) and (12) from which it has been obtained by the usual method. Therefore, there is no black hole associated with eq. (5) since there is no black hole associated with eq. (5b) and none with eq. (25), of which Schwarzschild's actual solution, eq. (5b), Brillouin's solution, eq. (22), and Droste's solution, eq. (23), are just particular equivalent cases.

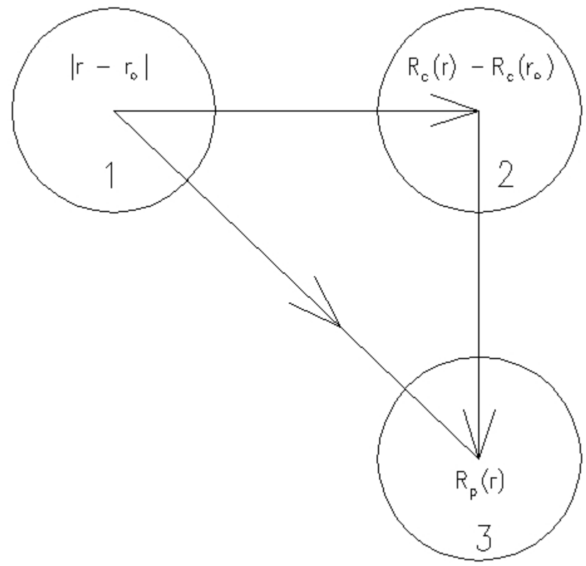

Fig. 1: Distances between two points (one fixed arbitrarily at the centre of spherical symmetry r_o) in the spatial section of Minkowski spacetime 1 (Euclidean 3-space) are mapped by $R_c(r)$ into Euclidean 3-space 2 (where the relevant centre of spherical symmetry is at the point $R_c(r_o) = \alpha \ \forall r_o \ \forall n$) and thence into all the components of the metric tensor for Schwarzschild space 3 where the point at the centre of spherical symmetry is located at $R_p(r_o) = 0 \ \forall r_o \ \forall n$. There are no holes in any of the manifolds.

All arguments for the black hole are based upon the same fundamental idea in that they conceive of a region that in actual fact does not exist. This fictitious region the astrophysics community calls the "interior", i. e. a region thought to be contained by a sphere the surface of which is called the "event horizon". But there is no such region. The notion comes from a failure to recognise that the centre of spherical symmetry of the problem at hand is *not* located at $r = 0$ in eq. (5).

The Centre of Spherical Symmetry

Consider the sphere in figure 2, radius ρ, centred at the origin of the system of coordinates O. The intrinsic geometry of this sphere is independent of its position in the space described by the system of coordinates. Shift the sphere to

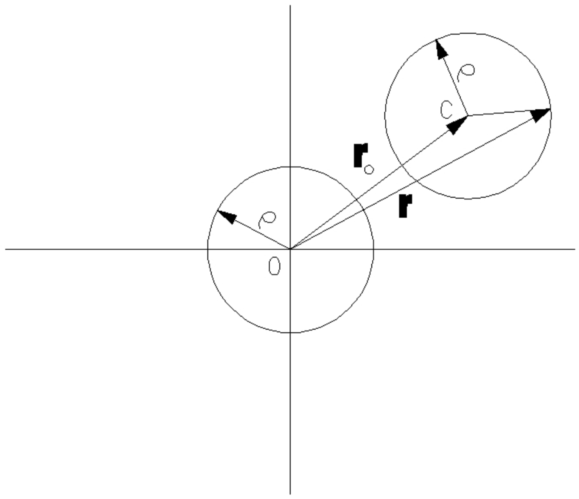

Fig. 2: The intrinsic geometry of the sphere is independent of its position in 3-space. The centre of the sphere is translated with the sphere - its centre point is not left at the origin of the coordinate system.

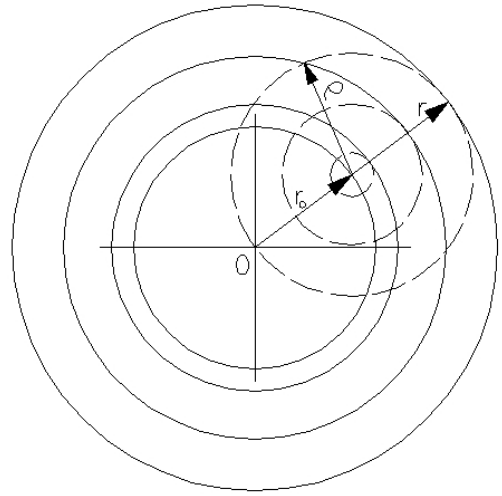

Fig. 3: The meaning of r_o – it is the relevant point at the centre of spherical symmetry in the spatial section of Minkowski spacetime. This point is arbitrary and need not be coincident with the origin of the coordinate system. The equation of a sphere of radius ρ centre C at the extremity of the fixed vector $\mathbf{r}_o$ is $(\mathbf{r} - \mathbf{r}_o) \cdot (\mathbf{r} - \mathbf{r}_o) = \rho^2$. If the vectors are collinear the equation is $|r - r_o| = \rho$; as depicted.

some arbitrary point C in the space, away from the origin of the coordinate system, as depicted. The centre of the shifted sphere is now located at the extremity of the fixed vector $\mathbf{r}_o$, relative to the origin of coordinates. The surface of the shifted sphere is the locus of points at the extremity of the variable vector $\mathbf{r}$, and the radius of the sphere, which is unaltered by this translation, is $\rho = \left|\mathbf{r} - \mathbf{r}_o\right|$. The centre of the translated sphere is no longer at the origin of coordinates. Consider a point on the surface of the translated sphere. It is at a distance ρ from the centre of the sphere. Let this point approach the centre of the sphere along any radius of the sphere. This can be described by the scalar $\rho \to 0$. The direction of approach is immaterial, since only radial approach is considered.

Now, the centre of the shifted sphere is shifted with the sphere. It would be quite absurd to suggest that although the sphere has been shifted away from the origin of the coordinate system, the centre of the sphere is still located at the origin of the coordinate system. If one shifted the sphere away from the origin of the coordinate system, without realising it, and treated the origin of the coordinate system as still the centre of the shifted sphere, then the resulting analysis would quickly lead to erroneous conclusions. The black hole is precisely a result of this misconception.

Consider again the situation in figure 2, except that the vectors $\mathbf{r}$ and $\mathbf{r}_o$ are always collinear. In this case, the vector notation can be dropped, so that $\rho = \left|r - r_o\right|$ where $r = |\mathbf{r}|$ and $r_o = \left|\mathbf{r}_o\right|$. Now consider figure 3. If one studies the intrinsic geometry of the shifted sphere without realising that it is no longer at the origin of the coordinate system, how would one interpret $r_o = \left|\mathbf{r}_o\right|$? In other words, as a point on the surface of the shifted sphere approaches the centre of the sphere along the collinear radial line, so that $r \to r_o^{\pm}$, how would this be interpreted when ignorant of the fact that the sphere has been shifted away from the origin of the coordinate system? In the case of the black hole, it has been misinterpreted as the said point approaching the surface of a sphere of radius r_o (circle through the tip of r_o in figure 3), and hence the spherical space contained by that radius is misinterpreted as the interior of a black hole, with the event horizon at radius r_o. Under this misconception, the usual method is to construct the Kruskal-Szekeres coordinates, and the Eddington-Finkelstein coordinates, thinking, erroneously, that there is an "interior" region. It is evident from figure 3 that as $r \to r_o^{\pm}$, $\rho \to 0^{+}$. It is also clear that when $r = 0$, $\rho = r_o$.

Minkowski spacetime actually plays the rôle of a parametric space for the generalisation to Schwarzschild spacetime. There is a mapping of distance between two arbitrary points, one fixed, in the spatial section of Minkowski space into all the components of the metric tensor for the "generalised" metric eq. (9). In other words, what the astrophysics community has unknowingly done by writing this metric, is to shift the parametric sphere in the spatial section of Minkowski spacetime away from the origin of coordinates of Minkowski spacetime (described by eq. (4) above), whilst thinking that the centre of the shifted parametric sphere is still located at $r = 0$ in Minkowski spacetime. This is compounded by misinterpretation of r in eq. (9) above as the radius in the spatial section thereof, simply because it is the radius in the spatial section of the usual Minkowski metric from which the typical analysis starts. With that, the astrophysics community thinks that $0 \le r < \infty$ in the general metric, which is determined finally as the "Schwarzschild" solution at eq. (5) above. The shift of the location of the centre of spherical symmetry was pointed out explicitly by Abrams [2] in 1989, and implicitly by Schwarzschild [39] in January 1916. Note in figure 3 that, as $\rho \to \infty$, the whole of the spatial section of Minkowski spacetime is accounted for – there is no hole in the manifold.

According to [1], the manifold associated with Schwarzschild's actual solution is extendible:

> *"Indeed, in deriving this form of the line-element, Schwarzschild imposed a very specific boundary condition, namely that the line-element is continuous everywhere except at $r = 0$, where $r \in (0, \infty)$ is the standard spherical radial coordinate. Imposition of this boundary condition has significant implications for the solution obtained. In particular, as a consequence of the boundary condition the coordinate R is shifted away from the origin. Indeed, if $r \in (0, \infty)$ then $R \in (\alpha, \infty)$. Hence ... the spacetime has a hole in its centre!*

However, the argument is specious. First and foremost, Schwarzschild's R is *not even a distance* let alone a radial one in his manifold. Second, the erroneous argument is similar to that adduced by G. Szekeres [68] in 1960 in that it is not recognised that the shifting of R away from the origin is a shifting of the relevant *point* at the centre of spherical symmetry away from the point at the origin in a corresponding Euclidean 3-space, as depicted in the preceeding figures. To amplify the error, consider the spatial section of Minkowski spacetime,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$
$$0 \le r < \infty.$$

Now, make the substitution $r = \overline{r} - 2m$, m a positive number. Then, the metric becomes

$$ds^2 = d\overline{r}^2 + (\overline{r} - 2m)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

According to Szekeres [68], there is now *"an apparent singularity on the sphere $\overline{r} = 2m$, due to a spreading out of the origin over a sphere of radius $2m$."* In [1] this is rendered as *"... the spacetime has a hole in its centre!"*. The claim made by Szekeres is easily proven false, as follows:

$$\begin{aligned} ds^2 &= d\overline{r}^2 + (\overline{r} - 2m)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= d\overline{r}^2 + |\overline{r} - 2m|^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= \frac{(\overline{r} - 2m)^2}{|\overline{r} - 2m|^2} d\overline{r}^2 + |\overline{r} - 2m|^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= (d\,|\overline{r} - 2m|)^2 + |\overline{r} - 2m|^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \\ \rho &= |\overline{r} - 2m| \ge 0, \end{aligned}$$

which describes the *whole* of Euclidean 3-space [38] and is therefore inextendible. There is no *"hole in its centre"* at all, and no separate manifold; the relevant centre has simply been shifted away from the origin of coordinates to some point at distance $2m$ from it, the direction of the translation being immaterial, as figures 2 and 3 illustrate.

Dr. Sharples has not understood Schwarzschild's argument for fixing his value of r_o to zero. In his paper, Schwarzschild [39] obtained a constant of integration ρ relating to his r_o and his function $R(r)$, thus

$$r_o^3 = \alpha^3 - \rho, \quad \text{and} \quad R(r) = \left(r^3 + \rho\right)^{\frac{1}{3}}.$$

He began his analysis with a generalisation that located the parametric point at the centre of spherical symmetry by construction at $r = r_o = 0$, thus [39]

$$ds^2 = F dt^2 - \left(G + Hr^2\right) dr^2 - Gr^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

where F, G, H are all functions of $r = \sqrt{x^2 + y^2 + z^2}$. Therefore, when x, y and z are all zero, r is zero. To make the origin $R_p = 0$ of his solution coincide with $r_o = 0$ of the auxiliary manifold, he chose his constant of integration at $\rho = \alpha^3$ to get $R_p(0) = 0$. However, Schwarzschild was at liberty to choose any real value for his constant. Indeed, if he set $\rho = 0$ he would have obtained $R = r$ and hence the line-element (5) above, with the range $\alpha < r < \infty$, with $R_p(\alpha) = 0$, the metric being singular only at $r = \alpha$, as Droste [40] determined independently in May 1916. If he did so choose his constant, Schwarzschild would have *moved* the centre of spherical symmetry from the point $r_o = 0$ in the auxiliary manifold to the point $r_o = \alpha$; and as demonstrated above, this does not make a *"hole"* appear in the auxiliary manifold or in the Schwarzschild manifold.

All points in the three manifolds depicted in figure 4 are in one-to-one correspondence with one another (see [38]). Consequently, the arbitrary point r_o of the (Euclidean) spatial section of Minkowski spacetime corresponds to the point $R_c(r_o) = \alpha\ \forall r_o\ \forall n$ in the Euclidean R_c space and thence to the point $R_p(r_o) = 0\ \forall r_o\ \forall n$ in the (non-Euclidean) spatial section of Schwarzschild spacetime. Then, as $r \to r_o^{\pm}$, $R_c(r) \to \alpha^+$ and $R_p(r) \to 0^+$, as depicted in figure 4. In Schwarzschild spacetime, the quantity R_c is not a distance of any sort therein: it strictly plays the geometric rôle of the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section.

It is evident from figure 4 that $r \to \pm\infty \Rightarrow \rho \to \infty$, then $R_c(r) \to \infty \Rightarrow \rho_c \to \infty$, then $R_p(r) \to \infty$. Similarly, $r \to r_o^{\pm} \Rightarrow \rho \to 0^+$, then $R_c(r) \to \alpha^+ \Rightarrow \rho_c \to 0^+$, then $R_p(r) \to 0^+$. It is noteworthy that $R_p = R_c$ when $R_c \approx 1.467\alpha$. Hence, $R_p > R_c$ when $R_c > 1.467\alpha$ and $R_p < R_c$ when $\alpha \le R_c < 1.467\alpha$. According to figure 4,

$$\rho_c = R_c(r) - R_c(r_o) = R_c(r) - \alpha$$

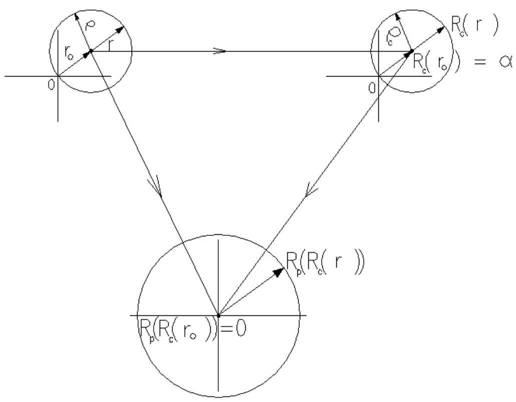

Fig. 4: This is a schematic representation of the relations between the three manifolds (Minkowski spacetime, the R_c intermediary manifold, and Schwarzschild spacetime). The following implications are apparent: $r \to \pm\infty \Rightarrow \rho \to \infty$, then $R_c(r) \to \infty \Rightarrow \rho_c \to \infty$, and then $R_p(r) \to \infty$. Similarly, $r \to r_o^{\pm} \Rightarrow \rho \to 0^+$, and so $R_c(r) \to \alpha^+ \Rightarrow \rho_c \to 0^+$, then $R_p(r) \to 0^+$. $R_c(r_o) = \alpha \;\forall\; r_o \;\forall\; n$ and $R_p(r_o) = 0 \;\forall\; r_o \;\forall\; n$. Each manifold is inextendible.

since by eqs. (25) $R_c(r_o) = \alpha \;\forall\; r_o \;\forall\; n$. Also by eqs. (25),

$$\rho_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}} - \alpha = (\rho^n + \alpha^n)^{\frac{1}{n}} - \alpha,$$

since $\rho = \left|r - r_o\right|$. Then, the proper radius for Schwarzschild spacetime can be written as

$$R_p\left(\rho_c\right) = \sqrt{\rho_c\left(\rho_c + \alpha\right)} + \alpha \ln\left(\frac{\sqrt{\rho_c + \alpha} + \sqrt{\rho_c}}{\sqrt{\alpha}}\right).$$

Therefore, $\rho \to 0^+ \Rightarrow \rho_c \to 0^+ \Rightarrow R_p \to 0^+$ and $\rho \to \infty \Rightarrow \rho_c \to \infty \Rightarrow R_p \to \infty$. Hence, all three manifolds are inextendible, as Abrams [2] proved by a different method. Also of importance, is the fact that Hagihara [69] proved, in 1931, that all geodesics that do not run into the boundary of the "Schwarzschild" metric at $r = 2m$ (i. e. at $R_p(r_o = 2m) = 0$) are complete, which therefore holds for eqs. (25) as well.

Kruskal-Szekeres: A Counter-Example

The following remark [1],

> *"In fact it is well-known that there exist coordinates in which the difficulty at $R = 2m$ can be removed, resulting in a single manifold that satisfies the field equations."*

is apparently an allusion to Eddington-Finkelstein [42,70] coordinates and Kruskal-Szekeres [68, 71] coordinates. I have shown elsewhere [72] that the Eddington-Finkelstein coordinates are without scientific merit, although the foregoing also implicitly demonstrates their invalidity, and the invalidity of the Kruskal-Szekeres method as well.

According to the astrophysics community the solution for Einstein's so-called static vacuum gravitational field must satisfy the following conditions [26,36,39,42,48,50,52–55,70]:

(a) It must be static; i.e. all the components of the metric tensor must be independent of time and the geometry must be unchanged under time reversal;

(b) It must be spherically symmetric;

(c) It must satisfy the equations $R_{\mu\nu} = 0$; no matter present;

(d) It must be asymptotically Minkowski spacetime.

Consider the metric

$$ds^2 = \left(1 - \frac{2m}{2m - r}\right) dt^2 - \left(1 - \frac{2m}{2m - r}\right)^{-1} dr^2 - \\ -(r - 2m)^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right). \tag{26}$$

This metric satisfies all the conditions (a) – (d); therefore metric (26) *is as good as* metric (5). I now apply to eq. (26) the very same methods that the astrophysics community applies to eq. (5) and begin by assuming that $0 \le r < \infty$ on eq. (26), and that "the origin" $r = 0$ marks the point at the centre of spherical symmetry of the manifold [73]. There are two "singularities"; at $r = 2m$ and at $r = 0$, just as in the case of eq. (5). When $r > 2m$, the signature of (26) is $(+, -, -, -)$, just as in eq. (5). When $0 < r < 2m$, the signature is $(-, +, -, -)$, again just as in eq. (5). Now when $r = 2m$, the coefficient of dt^2 in eq. (5) is zero, but in eq. (26) it is undefined. Similarly, when $r = 0$, the coefficient of dt^2 in eq. (5) is undefined, but in eq. (26) it is zero. Furthermore, when $r = 2m$, the Kretschmann scalar is $f = 3/4m^2$ in eq. (5), but is undefined, in eq. (26), and when $r = 0$, the Kretschmann scalar is $f = 3/4m^4$ in eq. (26), but is undefined in eq. (5). Therefore, according to the usual methods, there is an infinitely dense point-mass singularity at $r = 2m$ and an event horizon at $r = 0$ in eq. (26). Thus, the 'point-mass' singularity is encountered *before* the event horizon (a naked singularity!) and has the "radius" $r = 2m$, and the "Schwarzschild radius" of the "event horizon" is $r = 0$. The "event horizon" is "inside" the singularity! Again, following the same methods that the astrophysics community applies to eq. (5), apply the Kruskal-Szekeres method to remove the "coordinate singularity" at $r = 0$ in eq. (26) by setting

$$u = \left(1 - \frac{2m - r}{2m}\right)^{\frac{1}{2}} e^{\frac{2m-r}{4m}} \sinh\frac{t}{4m}$$

$$v = \left(1 - \frac{2m - r}{2m}\right)^{\frac{1}{2}} e^{\frac{2m-r}{4m}} \cosh\frac{t}{4m}.$$

Then, metric (26) becomes,

$$ds^2 = \frac{32m^3}{r - 2m} e^{\frac{r-2m}{2m}} \left(du^2 - dv^2\right) + (r - 2m)^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \tag{27}$$

where r is a function of u and v, by means of

$$\left(\frac{r}{2m}\right)e^{\frac{2m-r}{2m}} = v^2 - u^2.$$

It is apparent that eq. (27) is not singular at $r = 0$. The singularity at the event horizon with its "Schwarzschild radius" $r = 0$ has been removed. The metric is singular only at $r = 2m$, where according to the proponents of the black hole there must be an infinitely dense 'point-mass' singularity (but now with a "radius" of $r = 2m$, and therefore of finite density).

In obtaining eq. (27), nothing more was done than that which is usually done to eq. (5), and since (5) and (26) satisfy conditions (a) – (d), the one is as good as the other. Consequently, eq. (26) is as valid as eq. (5), insofar as the methods of the astrophysics community apply. The foregoing analysis of eq. (26) shares the same faults as those usual for eq. (5). Thus, the methods employed by the proponents of the black hole are flawed; they try to extend a manifold that is already maximal.

The usual form of eq. (5) in isotropic coordinates is,

$$ds^2 = \frac{\left(1 - \frac{m}{2r}\right)^2}{\left(1 + \frac{m}{2r}\right)^2}dt^2 - \left(1 + \frac{m}{2r}\right)^4 \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right],$$

wherein it is again alleged that r can go to zero. This expression has the same metrical groundform as eq. (5) and therefore, shares the same geometric character. The coefficient of dt^2 is zero when $r = m/2$, which, according to the astrophysics community, marks the "radius" or "event horizon" of a black hole, and where m is the alleged infinitely dense point-mass of the black hole singularity located at $r = 0$, just as in eq. (5). This further amplifies the fact that the quantity r appearing in both eq. (5) and its isotropic coordinate form is not a distance in the manifold described by these line-elements. Applying the intrinsic geometric relations detailed in Section 2 above, it is clear that the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of the isotropic coordinate line-element is given by,

$$R_c(r) = r\left(1 + \frac{m}{2r}\right)^2$$

and the proper radius is given by,

$$R_p(r) = r + m\ln\left(\frac{2r}{m}\right) - \frac{m^2}{4r}.$$

Hence, $R_c(m/2) = 2m$, and $R_p(m/2) = 0$, which are scalar invariants necessarily consistent with eq. (25). Furthermore, applying the same geometric analysis leading to eq. (25), the generalised solution in isotropic coordinates is [62],

$$ds^2 = \frac{\left(1 - \frac{\alpha}{4h}\right)^2}{\left(1 + \frac{\alpha}{4h}\right)^2}dt^2 - \left(1 + \frac{\alpha}{4h}\right)^4 \left[dh^2 + h^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right],$$

$$h = h(r) = \left[\left|r - r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{\frac{1}{n}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o,$$

wherein r_o and n are entirely arbitrary constants. Then,

$$R_c(r) = h(r)\left(1 + \frac{\alpha}{4h(r)}\right)^2 = \frac{1}{\sqrt{K(r)}},$$

$$R_p(r) = h(r) + \frac{\alpha}{2}\ln\left(\frac{4h(r)}{\alpha}\right) - \frac{\alpha^2}{16h(r)},$$

and

$$R_c(r_o) = \alpha, \quad R_p(r_o) = 0, \quad \forall\, r_o\ \forall\, n,$$

which are scalar invariants, in accordance with eq. (25). Clearly in these isotropic coordinate expressions, r does not denote any distance in the manifold, just as it does not denote any distance in eq. (25), of which eqs. (5) and (5b) are particular cases. It is a parameter for all the components of the metric tensor and hence for the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section and for the proper radius (i.e. the radial geodesic distance from the point at the centre of spherical symmetry of the spatial section). The so-called "interior" of the alleged "Schwarzschild" black hole does not form part of the solution space of the "Schwarzschild" manifold [2, 4, 5, 7, 35, 38, 43, 62, 66, 67] because there is *no* interior.

In the same fashion, it is easily proven [35, 43] that the general expression for the Kerr-Newman geometry is given by,

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi^2\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(R^2 + a^2\right)d\varphi - a dt\right]^2 -$$

$$-\frac{\rho^2}{\Delta}dR^2 - \rho^2 d\theta^2,$$

$$R = R(r) = \left(\left|r - r_o\right|^n + \beta^n\right)^{\frac{1}{n}},$$

$$\beta = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - (q^2 + a^2\cos^2\theta)}, \quad a^2 + q^2 < \frac{\alpha^2}{4},$$

$$a = \frac{2L}{\alpha}, \quad \rho^2 = R^2 + a^2\cos^2\theta, \quad \Delta = R^2 - \alpha R + q^2 + a^2,$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o.$$

Metrical Groundform

Dr. Sharples' claim that

> "*... the proper radius does not depend on the form of the line-element*" [1]

is patently false. Indeed, one can relabel the line-element for Schwarzschild spacetime with Φ, thus

$$ds^2 = \left(1 - \frac{\alpha}{\Phi}\right) dt^2 - \left(1 - \frac{\alpha}{\Phi}\right)^{-1} d\Phi^2 - \Phi^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

and then integrate over the relevant function and still get the same groundform for the proper radius, because the *form* of the line-element fully determines the geometry [36, 37]. The proper radius does not depend upon labels. One must not confuse labels with the *form* of the metric, as Dr. Sharples' has done. Hence, the claim that I have asserted that the contested quantity denoted by ρ in [1] is "*not a proper radius*" is false - it can be a proper radius if related to a 3-D Euclidean embedding space, but it is certainly *not* the proper radius in the "Schwarzschild" manifold, which is what I have actually argued and proven.

Concerning the "Schwarzschild" solution, Carroll and Ostlie remark [25]:

> *"The 'curvature of space' resides in the radial term. The radial distance measured simultaneously* ($dt = 0$) *between two nearby points on the same radial line* $d\theta = d\varphi = 0$ *is just the proper distance*
>
> $$dL = \sqrt{-(ds)^2} = \frac{dr}{\sqrt{1 - 2GM/rc^2}}.$$
>
> ... *"The factor of* $1/\sqrt{1 - 2GM/rc^2}$ *must be included in any calculation of spatial distances."**

Conclusions:

1. Dr. Sharples' play on the words "*not a proper radius*", by substituting the article "a" for the article "the" in relation to the "Schwarzschild solution", is not a scientific argument.
2. The quantity r in the "Schwarzschild" solution is not a distance in "Schwarzschild" spacetime - this irrefutable geometric fact completely subverts all claims that General Relativity predicts black holes.
3. My use of the proper radius is valid.

Claims 2 and 3. Although it is acknowledged in [1] that the "Schwarzschild solution" is not Schwarzschild's solution, I am nonetheless criticised for making the same point. One can plainly see that Schwarzschild's solution is different to that of Hilbert. Here again is Schwarzschild's actual solution [39],

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

$$R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty \quad \alpha = const.$$

In relation to this metric it is asserted in [1] that

> "... *by imposing the additional boundary condition at infinity, that the solution be consistent with the predictions of Newtonian gravitational theory, it is found that the constant* $\alpha = 2m$*, where* m *is the mass at the origin*".

This reveals once again Dr. Sharples' erroneous notion that $r = 0$ marks the point at the centre of spherical symmetry of Schwarzschild spacetime (where in fact $\rho = r_o$ as in figure 3). Furthermore, his introduction of Newtonian relations and concepts into Schwarzschild's solution is inadmissible. The "Schwarzschild" solution is for a problem that is alleged by the astrophysics community to pertain to one mass in an otherwise completely empty universe, *by construction*. But the Newtonian gravitational potential is a *two-body* concept; it is defined as the work done per unit mass against Newton's gravitational field. There is no meaning to a Newtonian potential for a single mass in an otherwise empty Universe. Newton's theory of gravitation is defined in terms of the *a priori* interaction of *two* masses in a space for which the 'Principle of Superposition' applies. In Newton's theory, there is no limit set to the number of masses that can be piled up in space, although the analytical relations for the gravitational interactions of many bodies upon one another quickly become intractable. In Einstein's theory, matter *cannot* be piled up in a given spacetime because the matter itself determines the structure of the spacetime containing the matter. In General Relativity, spacetime and matter are causally linked. It is clearly impossible for Schwarzschild spacetime, which is alleged by the astrophysics community to contain *by construction* only one mass in an otherwise totally empty Universe, to reduce to, or otherwise contain, an expression that is defined in terms of the *a priori* interaction of *two* masses. This is illustrated even further by writing eq. (5) in terms of c and G explicitly,

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dt^2 - c^2 \left(c^2 - \frac{2Gm}{r}\right)^{-1} dr^2 - \\ -r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

The term $2Gm/r$ is immediately recognised as the square of the Newtonian escape velocity of a body from a mass m. From this arbitrary insertion of Newton's expression for escape velocity, the astrophysics community asserts that when the "escape velocity" is that of light in vacuum, there is an event horizon ("Schwarzschild radius") and hence a black hole. Yet escape velocity is a concept that involves *two bodies* - one body escapes from another body. Even though one mass appears in Newton's expression for escape velocity, it cannot be determined without recourse to a fundamental two-body

*Nonetheless, Carroll and Ostlie fail to apply this mathematical fact.

gravitational interaction. Recall that Newton's Universal Law of Gravitation is,

$$F_g = -G\frac{mM}{r^2},$$

where G is the gravitational constant and r is the distance between the centres of mass for m and M. A centre of mass is an infinitely dense point-mass, but it is not a physical object; merely a mathematical artifice. Newton's gravitation is clearly defined in terms of the interaction of two bodies. Newton's gravitational potential Φ is defined as,

$$\Phi = \lim_{\sigma\to\infty}\int_{\sigma}^{r} -\frac{F_g}{m}dr = -G\frac{M}{r},$$

which is the work done per unit mass against the gravitational field due to masses m and M. This is a two-body concept. The potential energy P of a mass m in the gravitational field due to masses m and M is therefore given by,

$$P = m\Phi = -G\frac{mM}{r},$$

which is a two-body concept.

Similarly, the velocity required by a mass m to escape from the gravitational field due to masses m and M is determined by,

$$F_g = -G\frac{mM}{r^2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dr}.$$

Separating variables and integrating gives,

$$\int_{v}^{0} mv\,dv = \lim_{r_f\to\infty}\int_{R}^{r_f} -GmM\,\frac{dr}{r^2},$$

whence

$$v = \sqrt{\frac{2GM}{R}},$$

where R is the radius of the mass M. Thus, escape velocity necessarily involves two bodies: m escapes from M. In terms of the conservation of kinetic and potential energies,

$$K_i + P_i = K_f + P_f$$

whence,

$$\frac{1}{2}mv^2 - G\frac{mM}{R} = \frac{1}{2}mv_f^2 - G\frac{mM}{r_f}.$$

Then, as $r_f \to \infty$, $v_f \to 0$, and the escape velocity of m from M is,

$$v = \sqrt{\frac{2GM}{R}}.$$

Once again, the relation is derived from a two-body gravitational interaction.

It is also noteworthy that the denominators in Newton's expressions for escape velocity, gravitational potential, gravitational potential energy, gravitational force and such, is the relevant radial distance. It is not even a distance in the "Schwarzschild" solution let alone the radial distance therein, and so it cannot be treated as such, despite claims to the contrary by the astrophysics community.

Physical Principles of General Relativity

According to Einstein, matter is the cause of the gravitational field and the causative matter is described in his theory by a mathematical object called the energy-momentum tensor, which is coupled to geometry (i.e. spacetime) by his field equations, so that matter causes spacetime curvature (his gravitational field) and spacetime constrains motion of matter when gravity alone acts. According to the astrophysics community, Einstein's field equations,

> *"... couple the gravitational field (contained in the curvature of spacetime) with its sources."* [48]

> *"Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter."* [54]

> *"Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that* the metrical field *(or, in mathematical language, the tensor with components g_{ik})* is related to the material filling the world." [7]

> *"... we have, in following the ideas set out just above, to discover the* invariant law of gravitation, according to which matter determines the components $\Gamma^{\alpha}_{\beta\iota}$ of the gravitational field, *and which replaces the Newtonian law of attraction in Einstein's Theory."* [7]

> *"Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field."* [52]

> *"Clearly, the mass density, or equivalently, energy density $\varrho(\vec{x}, t)$ must play the role as a source. However, it is the* 00 *component of a tensor $T_{\mu\nu}(x)$, the mass-energy-momentum distribution of matter. So, this tensor must act as the source of the gravitational field."* [21]

> *"In general relativity, the stress-energy or energy-momentum tensor T^{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation"*. [16]

"Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move." [25]

Qualitatively Einstein's field equations are:

Spacetime geometry = -κ × *causative matter*

where *causative matter* is described by the energy-momentum tensor and κ is a constant. The spacetime geometry is described by a mathematical object called Einstein's tensor, $G_{\mu\nu}$, $(\mu, \nu = 0, 1, 2, 3)$ and the energy-momentum tensor is $T_{\mu\nu}$. Einstein's full field equations are*:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa T_{\mu\nu}. \qquad (28)$$

In the transition from Minkowski spacetime to Schwarzschild spacetime matter is not involved. The speed of light c that appears in the Minkowski spacetime line-element is a speed, not a photon. For this speed to be assigned to a photon, the photon must be present *a priori*. Similarly, for the relations of Special Relativity to hold, multiple arbitrarily large finite masses must also be present *a priori*. Minkowski spacetime is not Special Relativity because the latter requires the presence of matter, whereas the former does not. Similarly, the presence of the constant c in the line-element for Schwarzschild spacetime does not mean that a photon is present. The transition from Minkowski spacetime to Schwarzschild spacetime is thus not a generalisation of Special Relativity at all, merely a generalisation of the geometry of Minkowski spacetime. In the usual derivation of Schwarzschild spacetime, mass is included by means of a sophistic argument, viz. $R_{\mu\nu} = 0$ describes the gravitational field "outside a body". When one inquires of the astrophysics community as to what is the source of this alleged gravitational field "outside a body", one is told that it is *the body*, in which case the body must be described by a non-zero energy-momentum tensor since Einstein's field equations ... *couple the gravitational field... with its sources"* [48]. Dirac [55] tells us that

> *"... the constant of integration m that has appeared... is just the mass of the central body that is producing the gravitational field."*

We are told by Einstein [53] that,

> *"... M denotes the sun's mass centrally symmetrically placed about the origin of coordinates."*

According to Weyl [7],

> *"... the quantity m_o introduced by the equation $m = km_o$ occurs as the* field-producing mass *in it; we call m the* gravitational radius *of the matter causing the disturbances of the field."*

Foster and Nightingale [48] assert that

> *"... the corresponding Newtonian potential is $V = -GM/r$, where M is the mass of the body producing the field, and G is the gravitational constant ... we conclude that $k = -2GM/c^2$ and Schwarzschild's solution for the empty space outside a spherical body of mass M is ... "*

After the "Schwarzschild" solution is obtained there is no matter present. This is because the energy-momentum tensor is set to zero and Minkowski spacetime is not Special Relativity. The astrophysics community merely inserts (Weyl says *"introduced"*) mass and photons by erroneously appealing to Newton's theory through which they also get any number of masses and any amount of radiation by applying the 'Principle of Superposition'. This is done despite the fact that the 'Principle of Superposition' does not apply in General Relativity. However, Newton's relations, as explained above, involve two bodies and the 'Principle of Superposition'. Conversely, $R_{\mu\nu} = 0$ contains *no bodies* and cannot accommodate the 'Principle of Superposition'. The astrophysics community removes all matter on the one hand by setting $R_{\mu\nu} = 0$ and then puts it back in at the end with the other hand by means of Newton's theory. The whole procedure constitutes a violation of elementary logic.

Einstein asserted that his 'Principle of Equivalence' and his laws of Special Relativity must hold in sufficiently small regions of his gravitational field, and that these regions can be located anywhere in his gravitational field. Here is what Einstein [53] himself said in 1954, the year before his death:

> *"Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K', uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and*

*The so-called "cosmological constant" is not included.

signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For, according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K').

"Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy."

In their textbook, Foster and Nightingale [48] succinctly state the 'Principle of Equivalence' thus:

"We may incorporate these ideas into the principle of equivalence, which is this: In a freely falling (nonrotating) laboratory occupying a small region of spacetime, the laws of physics are the laws of special relativity."

According to Pauli [54],

"We can think of the physical realization of the local coordinate system K_o in terms of a freely floating, sufficiently small, box which is not subjected to any external forces apart from gravity, and which is falling under the influence of the latter. ... "It is evidently natural to assume that the special theory of relativity should remain valid in K_o."

Taylor and Wheeler state in their book [28],

"General Relativity requires more than one free-float frame."

Carroll and Ostlie write [25],

"**The Principle of Equivalence:** *All local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments. . . . Note that special relativity is incorporated into the principle of equivalence. . . . Thus general relativity is in fact an extension of the theory of special relativity."*

In the Dictionary of Geophysics, Astrophysics and Astronomy [11],

"Near every event in spacetime, in a sufficiently small neighborhood, in every freely falling reference frame all phenomena (including gravitational ones) are exactly as they are in the absence of external gravitational sources."

Note that the 'Principle of Equivalence' involves the *a priori* presence of multiple arbitrarily large finite masses. Similarly, the laws of Special Relativity involve the *a priori* presence of at least two arbitrarily large finite masses (and at least one photon); for otherwise relative motion between two bodies cannot manifest. The postulates of Special Relativity are themselves couched in terms of inertial systems, which are in turn defined in terms of mass via Newton's First Law of motion. "Schwarzschild's solution", and indeed all black hole "solutions", pertain to one mass in a universe that contains no other masses. According to the astrophysics community, "Schwarzschild" spacetime consists of one mass in an otherwise *totally empty universe*, and so its alleged black hole is the only matter present - it has nothing to interact with, including "observers" (on the assumption that any observer is material).

In the space of Newton's theory of gravitation, one can insert as many masses as desired. Although solving for the gravitational interaction of these masses rapidly becomes beyond our capacity, there is nothing to prevent us inserting masses conceptually. This is essentially the 'Principle of Superposition'. However, one cannot do this in General Relativity, because Einstein's field equations are non-linear. In General Relativity, each and every configuration of matter must be described by a corresponding energy-momentum tensor and the field equations solved separately for each and every configuration, because matter and geometry are coupled, as eq. (28) describes. This is not the case in Newton's theory, where geometry is independent of matter. The 'Principle of Superposition' does not apply in General Relativity:

"In a gravitational field, the distribution and motion of the matter producing it cannot at all be assigned arbitrarily — on the contrary it must be determined (by solving the field equations for given initial conditions) simultaneously with the field produced by the same matter." [52]

"An important characteristic of gravity within the framework of general relativity is that the theory is nonlinear. Mathematically, this means that if g_{ab} and γ_{ab} are two solutions of the field equations, then $ag_{ab} + b\gamma_{ab}$ (where a, b are scalars) may not be a solution. This fact manifests itself physically in two ways. First, since a linear combination may not be a solution, we cannot take the overall gravitational field of the two bodies to be the summation of the individual gravitational fields of each body." [16]

The astrophysics community claims that the gravitational field "outside" a mass contains no matter, and thereby asserts that the energy-momentum tensor $T_{\mu\nu} = 0$. Despite this, it is routinely alleged that there is only one mass in the whole Universe with this particular problem statement. But setting the energy-momentum tensor to zero means that there is no matter present by which the gravitational field can be caused! It is also claimed that the field equations then reduce to the much simpler form,

$$Ric = R_{\mu\nu} = 0. \tag{29}$$

This is a statement that spacetime is devoid of matter.

> *"Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat spacetime."* [11]

However, since this is a spacetime that *by construction* contains no matter, Einstein's 'Principle of Equivalence' and his laws of Special Relativity cannot manifest, thus violating the physical requirements of the gravitational field. It has never been proven that Einstein's 'Principle of Equivalence' and his laws of Special Relativity, both of which are defined in terms of the *a priori* presence of multiple arbitrary large finite masses, can manifest in a spacetime that *by construction* contains no matter. Now eq. (5) relates to eq. (29). However, there is allegedly mass present, denoted by m in eq. (5). This mass is not described by an energy-momentum tensor. The reality that the constant m is actually responsible for the alleged gravitational field due to a black hole associated with eq. (5) is confirmed by the fact that if $m = 0$, eq. (5) reduces to Minkowski spacetime, and hence no gravitational field according to the astrophysics community. If not for the presence of the alleged mass m in eq. (5) there would be no cause of the gravitational field. But this contradicts Einstein's relation between geometry and matter, since m is introduced into eq. (5) *post hoc*, not via an energy-momentum tensor describing the matter causing the associated gravitational field. In Schwarzschild spacetime, the components of the metric tensor are only functions of one another, and reduce to functions of one component of the metric tensor. None of the components of the metric tensor contain matter, because the energy-momentum tensor is zero. There is no transformation of matter in Minkowski spacetime into Schwarzschild spacetime, and so the laws of Special Relativity are not transformed into a gravitational field by $Ric = 0$. The transformation is merely from a pseudo-Euclidean geometry containing no matter into a pseudo-Riemannian (non-Euclidean) geometry containing no matter. Matter is introduced into the spacetime of $Ric = 0$ by means of a vicious circle, as follows. It is stated at the outset that $Ric = 0$ describes spacetime "outside a body". The words "outside a body" introduce matter, contrary to the energy-momentum tensor, $T_{\mu\nu} = 0$, that describes the causative matter as being absent. There is no matter involved in the transformation of Minkowski spacetime into Schwarzschild spacetime via $Ric = 0$, since the energy-momentum tensor is zero, making the components of the resulting metric tensor functions solely of one another, and reducible to functions of just one component of the metric tensor. To satisfy the initial claim that $Ric = 0$ describes spacetime "outside a body", so that the resulting spacetime curvature is caused by the alleged mass present, the alleged causative mass is *inserted* into the resulting metric *ad hoc*. This is achieved by means of a contrived analogy with Newton's theory and his expression for escape velocity (a *two-body* relation), thus closing the vicious circle. Here is how Chandrasekhar [27] presents the vicious circle:

> *"That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that* $\frac{1}{2}v^2 > GM/R$; *and it cannot escape if* $v^2 < 2GM/R$. *On the basis of this last inequality, Laplace concluded that if* $R < 2GM/c^2 = R_s$ *(say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*
>
> *"By a curious coincidence, the limit* R_s *discovered by Laplace is* exactly *the same that general relativity gives for the occurrence of the* trapped surface *around a spherical mass."*

But it is not surprising that general relativity gives the same R_s "*discovered by Laplace*" because the Newtonian expression for escape velocity is deliberately inserted *post hoc* by the astrophysicists and astronomers, into the "Schwarzschild solution". Newton's escape velocity does not drop out of any of the calculations to Schwarzschild spacetime. Furthermore, although $R_{\mu\nu} = 0$ is said to describe spacetime "outside a body", the resulting metric (5) is nonetheless used to describe the *interior* of a black hole, since the black hole begins at the alleged "event horizon", not at its infinitely dense point-mass singularity (allegedly at $r = 0$ in eq. (5)).

In the case of a totally empty Universe, what would be the relevant energy-momentum tensor? It must be $T_{\mu\nu} = 0$. Indeed, it is also maintained by the astrophysics community that spacetimes can be intrinsically curved, i. e. that there are gravitational fields that have no material cause. An example is de Sitter's empty spherical Universe, based upon the following "field" equations [36, 42]:

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{30}$$

where λ is the so-called "cosmological constant". In the case of metric (5) the field equations are given by expression (29). On the one hand, de Sitter's empty world is devoid of matter ($T_{\mu\nu} = 0$) and therefore has no material cause for the alleged associated gravitational field. On the other hand, it is stated that the spacetime described by eq. (29) has a material cause, *post hoc* as m in metric (5), even though $T_{\mu\nu} = 0$ there as well: a contradiction. This is amplified by the so-called "Schwarzschild-de Sitter" line-element,

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - \\ -r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{31}$$

which is the standard solution for eq. (30). Once again, m is identified *post hoc* as mass at the centre of spherical symmetry of the manifold, said to be at $r = 0$. The completely empty universe of de Sitter [36, 42] can be obtained by setting $m = 0$ in eq. (31) to yield,

$$ds^2 = \left(1 - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{32}$$

Also, if $\lambda = 0$, eq. (30) reduces to eq. (29) and eq. (31) reduces to eq. (5). If both $\lambda = 0$ and $m = 0$, eqs. (31) and (32) reduce to Minkowski spacetime. Now in eq. (30), the term $\lambda g_{\mu\nu}$ is not an energy-momentum tensor, since according to the astrophysics community, expression (32) describes a world devoid of matter [36, 42]. The universe described by eq. (32), which also satisfies eq. (30), is completely empty and so its curvature has no material cause; in eq. (30), just as in eq. (29), $T_{\mu\nu} = 0$. Thus, eq. (32) is alleged to describe a gravitational field that has no material cause. Furthermore, although in eq. (29), $T_{\mu\nu} = 0$, its usual solution, eq. (5), is said to contain a (*post hoc*) material cause, denoted by m therein. Thus, for eq. (5), it is postulated that $T_{\mu\nu} = 0$ supports a material cause of a gravitational field. At the same time, for eq. (32), $T_{\mu\nu} = 0$ precludes material cause of a gravitational field. $T_{\mu\nu} = 0$ therefore includes and excludes material cause. This is not possible. The contradiction is due to the *post hoc* introduction of mass, as m, in eq. (5), by means of the Newtonian expression for escape velocity. Furthermore, there is no experimental evidence to support the claim that a gravitational field can be generated without a material cause. Material cause is codified theoretically in eq. (28).

Black Hole Interactions

The literature abounds with claims that black holes can interact in such situations as binary systems, mergers, collisions, and with surrounding matter generally. Bearing in mind that *all* black holes "solutions" pertain to a universe that contains only one mass (the black hole itself), concepts involving multiple black holes tacitly assumes application of the 'Principle of Superposition', which however *does not apply* in General Relativity. According to Chandrasekhar [27],

> *"From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.*
>
> *"In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease; it can at best remain unchanged (if the conditions are stationary).*
>
> *"Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass $\frac{1}{2}M$, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass $\overline{M}$. Then Hawking's theorem requires that*
>
> $$16\pi\overline{M}^2 \geq 16\pi\left[2\left(\frac{1}{2}M\right)^2\right] = 8\pi M^2$$
>
> *or*
>
> $$\overline{M} \geq M/\sqrt{2}.$$
>
> *Hence the maximum amount of energy that can be released in such a coalescence is*
>
> $$M\left(1 - 1/\sqrt{2}\right)c^2 = 0.293Mc^2."$$

Hawking [80] says,

> *"Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes."*

According to Schutz [50],

> *"... Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can*

never bifurcate spontaneously into two smaller ones.

"Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved."

Townsend [56] also arbitrarily applies the 'Principle of Superposition' to obtain charged black hole (Reissner-Nordström) interactions as follows:

"The extreme RN in isotropic coordinates is

$$ds^2 = V^{-2}dt^2 + V^2\left(d\rho^2 + \rho^2 d\Omega^2\right)$$

where

$$V = 1 + \frac{M}{\rho}$$

This is a special case of the multi black hole solution

$$ds^2 = V^{-2}dt^2 + V^2 d\vec{x}\cdot d\vec{x}$$

where $V^2 d\vec{x}\cdot d\vec{x}$ *is the Euclidean 3-metric and* V *is any solution of* $\nabla^2 V = 0$. *In particular*

$$V = 1 + \sum_{i=1}^{N} \frac{M_i}{\left|\vec{x} - \vec{x}_i\right|}$$

yields the metric for N *extreme black holes of masses* M_i *at positions* $\vec{x}_i$.

Carroll and Ostlie remark [25],

"The best hope of astronomers has been to find a black hole in a close binary system. ... If a black hole coalesces with any other object, the result is an even larger black hole. ... If one of the stars in a close binary system explodes as a supernova, the result may be either a neutron star or a black hole orbiting the companion star. ... the procedure for detecting a black hole in a binary x-ray system is similar to that used to measure the masses of neutron stars in these systems. ... What is the fate of a binary x-ray system? As it reaches the endpoint of its evolution, the secondary star will end up as a white dwarf, neutron star, or black hole."

But Einstein's field equations are non-linear. Thus, the 'Principle of Superposition' *does not apply* [16, 52, 81]. Therefore, before one can talk of black hole binary systems and the like it must first be proven that the two-body system is theoretically well-defined by General Relativity. This can be accomplished in only two ways:

(a) Derivation of an exact solution to Einstein's field equations for the two-body configuration of matter; or

(b) Proof of an existence theorem.

However, there are no known solutions to Einstein's field equations for the interaction of two (or more) masses (charged or not). Furthermore, no existence theorem has ever been proven, by which Einstein's field equations can even be said to admit of latent solutions for such configurations of matter. The "Schwarzschild" black hole is allegedly obtained from a line-element satisfying $Ric = 0$. For the sake of argument, assume that black holes are predicted by General Relativity as alleged in relation to metric (5). Since $Ric = 0$ is a statement that there is no matter in the Universe, one cannot simply insert a second black hole into the spacetime of $Ric = 0$ of a given black hole, so that the resulting two black holes (each obtained separately from $Ric = 0$) simultaneously persist in, and mutually interact in, a spacetime that *by construction contains no matter*! One cannot simply assert by an analogy with Newton's theory that two black holes can be components of binary systems, collide, or merge [52, 81, 82], because the 'Principle of Superposition' does not apply in Einstein's theory. Moreover, General Relativity has to date been unable to account for the simple experimental fact that two fixed bodies will approach one another upon release. Thus, black hole binaries, collisions, mergers, black holes from supernovae, and other black hole interactions are all invalid concepts.

Gravitational Collapse

Much of the justification for the notion of irresistible gravitational collapse into an infinitely dense point-mass singularity, and hence the formation of a black hole, is given to the analysis due to Oppenheimer and Snyder [83]. Hughes [31] relates it as follows;

"In an idealized but illustrative calculation, Oppenheimer and Snyder ... showed in 1939 that a black hole in fact does form in the collapse of ordinary matter. They considered a 'star' constructed out of a pressureless 'dustball'. By Birkhof's Theorem, the entire exterior of this dustball is given by the Schwarzschild metric ... Due to the self-gravity of this 'star', it immediately begins to collapse. Each mass element of the pressureless star follows a geodesic trajectory toward the star's center; as the collapse proceeds, the star's density increases and more of the spacetime is described by the Schwarzschild metric. Eventually, the surface passes through $r = 2M$. *At this point, the Schwarzschild exterior includes an event horizon: A black hole has formed. Meanwhile, the matter which formerly constituted the star continues collapsing to ever smaller radii. In short order, all of the original*

> *matter reaches $r = 0$ and is compressed (classically!) into a singularity*[4].
>
> [4] *"Since all of the matter is squashed into a point of zero size, this classical singularity must be modified in a complete, quantum description. However, since all the singular nastiness is hidden behind an event horizon where it is causally disconnected from us, we need not worry about it (at least for astrophysical black holes)."*

Note that the 'Principle of Superposition' has again been arbitrarily applied by Oppenheimer and Snyder, from the outset. They first *assume* a relativistic universe in which there are multiple mass elements present *a priori*, where the 'Principle of Superposition' however, does not apply, and despite there being no solution or existence theorem for such configurations of matter in General Relativity. Then, all these mass elements "collapse" into a central point (zero volume; infinite density). However, such a collapse has not been given any specific general relativistic mechanism in this argument; it is simply asserted that the "collapse" is due to self-gravity. But the "collapse" cannot be due to Newtonian gravitation, given the resulting black hole, which does not occur in Newton's theory of gravitation. A Newtonian universe cannot "collapse" into a non-Newtonian universe. Moreover, the black hole so formed is in an empty universe, since the "Schwarzschild black hole" relates to $Ric = 0$, a spacetime that by construction contains no matter. Nonetheless, Oppenheimer and Snyder permit, within the context of General Relativity, the presence of and the gravitational interaction of many mass elements, which coalesce and collapse into a point of zero volume to form an infinitely dense point-mass singularity, when there is no demonstrated general relativistic mechanism by which any of this can occur.

Furthermore, nobody has ever observed a celestial body undergo irresistible gravitational collapse and there is no laboratory evidence whatsoever for such a phenomenon.

It is quite clear that the introduction of Newtonian relations into Schwarzschild's solution, and the corruption thereof by Hilbert, as given in [1], is invalid.

Escape Velocity

It is widely held by astrophysicists and astronomers that a black hole has an escape velocity c (or $\geq c$, the speed of light in vacuo) [6, 9, 11, 13–15, 19, 20, 27, 31, 80, 84–86]. Chandrasekhar [27] remarked,

> *"Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it.*
>
> *... A trapped surface is one from which light cannot escape to infinity."*

According to Hawking [80],

> *"Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. Its boundary is called the event horizon. It coincides with the paths of the light rays that just fail to escape from the black hole.*
>
> *"A neutron star has a radius of about ten miles, only a few times the critical radius at which a star becomes a black hole.*
>
> *"I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole."*

However, according to the alleged properties of a black hole, nothing at all can even *leave* the black hole. In the very same paper Chandrasekhar made the following quite typical contradictory assertion:

> *"The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge."*

Hughes [31] reiterates,

> *"Things can go into the horizon (from $r > 2M$ to $r < 2M$), but they cannot get out; once inside, all causal trajectories (timelike or null) take us inexorably into the classical singularity at $r = 0$.*
>
> *"The defining property of black holes is their event horizon. Rather than a true surface, black holes have a 'one-way membrane' through which stuff can go in but cannot come out."*

Taylor and Wheeler [28] assert,

"... Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon ... and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter."

In the Dictionary of Geophysics, Astrophysics and Astronomy [11], one finds the following assertions:

*"***black hole** *A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*

$$v_{esc} = \sqrt{\frac{2GM}{R}},$$

where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole. *The required radius R_{BH} follows from setting v_{esc} equal to c:*

$$R_{BH} = \frac{2GM}{c^2}.$$

... "In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression R_{BH} holds for the surface of a black hole. The surface of a black hole at R_{BH} is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon."

A. Guth [87] tells us,

". . . classically the gravitational field of a black hole is so strong that not even light can escape from its interior . . . "

And according to the Collins Encyclopedia of the Universe [85],

*"***black hole** *A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light."*

Now, if its escape velocity is really that of light in vacuo, then, by definition of escape velocity, light would escape from a black hole, and therefore be seen by all observers. If the escape velocity of the black hole is greater than that of light in vacuo, then light could emerge but not escape; there would therefore always be a class of observers that could see it. Not only that, if the black hole had an escape velocity, then material objects with an initial velocity less than the alleged escape velocity, could leave the black hole, and therefore be seen by a class of observers, but not escape (just go out, come to a stop and then fall back), even if the escape velocity is $\geq c$. Escape velocity does not mean that objects cannot leave; it only means they cannot escape if they have an initial velocity less than the escape velocity. Hence, on the one hand it is alleged that black holes have an escape velocity $\geq c$, but on the other hand that nothing, including light, can even leave the black hole. The claims are contradictory - nothing but a meaningless play on the words "escape velocity" [81, 82]. Furthermore, as demonstrated above, escape velocity is a *two-body* concept, whereas the black hole is derived not from a two-body gravitational interaction, but from an alleged one-body concept (but which is in fact a no-body situation). The black hole has no escape velocity.

The Michell-Laplace Dark Body

It is also routinely asserted that the theoretical Michell-Laplace (M-L) dark body of Newton's theory, which has an escape velocity $\geq c$, is a kind of black hole [6, 10, 11, 13, 25, 27, 30, 80] or that Newton's theory somehow predicts *"the radius of a black hole"* [28]. Hawking remarks [80],

"On this assumption a Cambridge don, John Michell, wrote a paper in 1783 in the Philosophical Transactions of the Royal Society of London. *In it, he pointed out that a star that was sufficiently massive and compact would have such a strong gravitational field that light could not escape. Any light emitted from the surface of the star would be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be a large number of stars like this. Although we would not be able to see them because light from them would not reach us, we could still feel their gravitational attraction. Such objects are what we now call black holes, because that is what they are – black voids in space."*

In the Cambridge Illustrated History of Astronomy [88] it is asserted that,

"Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work."

But the M-L dark body is not a black hole. The M-L dark body possesses an escape velocity, whereas the black hole has no escape velocity. Objects can leave the M-L dark body, but nothing can leave the black hole. There is no upper limit of the speed of a body in Newton's theory, so masses can always escape from the M-L dark body, provided they leave at or greater than the escape velocity. The M-L dark body does not require irresistible gravitational collapse, whereas the black hole does. It has no infinitely dense point-mass singularity, whereas the black hole does. It has no event horizon, whereas the black hole does. There is always a class of observers that can see the M-L dark body [81, 82], but there is no class of observers that can see the black hole. The M-L dark body can persist in a space which contains other matter and interact with that matter, but the spacetime of the "Schwarzschild" black hole (and variants thereof) is devoid of matter by construction and consequently cannot interact with anything. Thus, the M-L dark body does not possess the characteristics of the alleged black hole and so it is not a black hole.

Foliations

In [1], in relation to Schwarzschild's actual solution, Dr. Sharples remarks,

> "*... that if R and t are held constant (say $R = a$ and $t = t_0$ the line-element reduces to that of a 2-sphere with radius $a > 2m$. The line-element therefore defines a manifold that is foliated by 2-spheres with radii greater than $2m$.*"

This claim is spurious for the following reason: the setting of $C(r) = R_c^2(r) = r^2$ in the general expression for Schwarzschild spacetime [eq. (25)] introduces a shift of the corresponding centre of spherical symmetry in the auxiliary manifold described by the Minkowski line-element, away from the origin $r_o = 0$ to a point at distance $r_o = 2m$ [2, 38]. Moreover, the centre of spherical symmetry of the 3-D Euclidean space in which the said 2-spheres are embedded (thereby making them 2-spheres) is not at $R = 0$ for the problem at hand (namely, Schwarzschild's actual solution), but at the scalar invariant of Schwarzschild's spacetime, given by $R(0) = \alpha$. The manifold referred to in [1] is not foliated by 2-spheres of radii $R > 2m$, because this R is not even a distance, let alone a radial one, in Schwarzschild's actual solution. In the auxiliary Euclidean R-manifold the 2-spheres rightly relate to the point at the centre of spherical symmetry for the problem at hand (at $R(0) = \alpha$), not to the origin $R = 0$ of the auxiliary embedding space for the said 2-spheres. In other words, the centre of the 2-spheres referred to in [1] is *not* at $R = 0$ but at the *point* $R = \alpha$, as figure 4 illustrates (where the auxiliary 2-spheres have radii ρ_c with $0 \leq \rho_c < \infty$).

Infinite Density Forbidden

The black hole is alleged to contain an infinitely dense point-mass singularity, produced by irresistible gravitational collapse (see for example [12, 17, 25, 27, 80, 89]). According to Hawking [80]:

> *"The work that Roger Penrose and I did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole."*

Dodson and Poston [12] assert:

> *"Once a body of matter, of any mass m, lies inside its Schwarzschild radius $2m$ it undergoes gravitational collapse ... and the singularity becomes physical, not a limiting fiction."*

According to Carroll and Ostlie [25],

> *"A nonrotating black hole has a particularly simple structure. At the center is the* **singularity***, a point of zero volume and infinite density where all of the black hole's mass is located. Spacetime is infinitely curved at the singularity. ... The black hole's singularity is a real physical entity. It is not a mathematical artifact ... "*

The singularity of the alleged Big Bang cosmology is, according to many proponents of the Big Bang, also infinitely dense [87]. Yet, by Special Relativity, infinite densities are forbidden, because their existence implies that a material object can acquire the speed of light c in vacuo (or equivalently, the existence of infinite energies), thereby violating the very basis of Special Relativity. Since General Relativity cannot violate Special Relativity, General Relativity must therefore also forbid infinite densities. Point-mass singularities are alleged to be infinitely dense objects. Therefore, point-mass singularities are forbidden by the Theory of Relativity.

Let a cuboid rest-mass m_0 have sides of length L_0. Let m_0 have a relative speed $v < c$ in the direction of one of three mutually orthogonal Cartesian axes attached to an observer of rest-mass M_0. According to the observer M_0, the moving mass m is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and the volume V thereof is

$$V = L_0^3 \sqrt{1 - \frac{v^2}{c^2}}.$$

Thus, the density D is

$$D = \frac{m}{V} = \frac{m_0}{L_0^3 \left(1 - \frac{v^2}{c^2}\right)},$$

and thus $v \to c \;\Rightarrow\; D \to \infty$. Since, according to Special Relativity, no material object can acquire the speed c (this would require an infinite energy), infinite densities are forbidden by Special Relativity, and so point-mass singularities are forbidden. Since Special Relativity must manifest in sufficiently small regions of Einstein's gravitational field, and these regions can be located anywhere in the gravitational field, General Relativity too must thereby forbid infinite densities and hence forbid point-mass singularities. It does not matter how it is alleged that a point-mass singularity is generated by General Relativity because the infinitely dense point-mass cannot be reconciled with Special Relativity. Point-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

Observational Evidence

It is nowadays routinely reported that many black holes have been found. Yet the signatures of the black hole are (a) an infinitely dense 'point-mass' singularity and (b) an 'event horizon'. Nobody has ever found an infinitely dense 'point-mass' singularity and nobody has ever found an 'event horizon', so nobody has ever assuredly found a black hole. It takes an infinite amount of observer time to verify a black hole event horizon [27,31,48,50,55,56,58]. Nobody has been around and nobody will be around for an infinite amount of time and so no observer can ever verify the presence of an event horizon, and hence a black hole, in principle; the notion is irrelevant to physics. Moreover, an 'observer' cannot be present in a spacetime that by construction contains no matter (i. e. $R_{\mu\nu} = 0$), or in a universe that contains only one mass, by construction. All reports of black holes being found are patently false; the product of wishful thinking.

Conclusions:

1. The claim that matter is present in a spacetime that by construction contains no matter is false.
2. Schwarzschild spacetime cannot be extended because it is maximal.
3. The introduction of Newtonian two-body relations and concepts into Schwarzschild's solution is inadmissible.
4. The theoretical Michell-Laplace dark body is not a black hole.
5. Dr. Sharples' objection to the reality that the so-called "Schwarzschild's solution" is not Schwarzschild's solution is contrary to established fact.
6. The Theory of Relativity forbids infinitely dense point-masses.

Claim 4. It is alleged that I have maintained that there are an infinite number of solutions to Einstein's so-called static vacuum field. As noted already in Section 2, this claim is inaccurate. I have remarked in a number of my papers that the infinite set of particular solutions are *geometrically equivalent*, in accordance with Eddington's [42] observation. For instance, in the abstract of [62] I wrote:

> *"It is proved herein that the metric in the so-called 'isotropic coordinates' for Einstein's gravitational field is a particular case of an infinite class of equivalent metrics."*

In the abstracts of my conference papers [45, 46] I wrote:

> *"With the correct identification of the associated Gaussian curvature it is also easily proven that there is only one singularity associated with all Schwarzschild metrics, of which there is an infinite number that are equivalent."*

Recall that the only difference between the elements of this infinite set is the particular expression given to the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. Irrespective of the particular admissible expression for the said Gaussian curvature, all the relevant invariants are satisfied, as they must, being intrinsic properties of the geometry which is determined by the *form* of the line element [36, 37].

In [1] there is an appeal to the so-called Birkoff's Theorem:

> *"This theorem establishes, with mathematical certainty, that the Schwarzschild solution (exterior, interior or both) is the only solution of the spherically symmetric vacuum field equations."*

However, as Abrams [2] again pointed out, Birkoff's Theorem only establishes the *form* of the line-element, not the range on the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section - that has to be determined from the intrinsic geometry of the line-element. The *form* of the line-element associated with Birkoff's "Theorem" is

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right),$$

nothing more. Incidently, Nikias Stavroulakis [91] has argued that Birkoff's Theorem is not even a theorem.

In addition, [1] essentially reproduces variations in the notation of the line-element that already occur in my papers in order to claim that,

> *"... what appears to be an infinitude of particular solutions are actually just different coordinate expressions of the same solution ... "*

Yet, in my papers, it is repeatedly remarked that all the line-elements I adduce via the admissible form for $R_c(r)$ are equivalent, because they describe the same geometry.

Conclusions:

1. Dr. Sharples' claim that I have asserted that there is an infinite number of geometrically inequivalent solutions for $R_{\mu\nu} = 0$ is false.

2. Birkoff's "theorem" establishes only the *form* of the line-element. It says nothing about the range of values for the Gaussian curvature of the geodesic surface in the spatial section of the "Schwarzschild solution".

3. The range of values for the associated Gaussian curvature is determined from the line-element, by calculation.

Claim 5. Dr. Sharples [1] appeals next to the Riemann tensor scalar curvature invariant (the Kretschmann scalar) $f = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$, reiterating the usual argument that a singularity must occur where this invariant is unbounded (undefined) [22, 24, 26, 29, 51]. However curvature invariants are intrinsic properties of a geometry and are calculated from the line-element describing the geometry, not by *ad hoc* assumption of their values. Moreover, the Kretschmann scalar for Schwarzschild spacetime is not an independent curvature invariant because it is a function of the Gaussian curvature K of the spherically symmetric geodesic surface in the spatial section, and the said Gaussian curvature is itself constrained by the line-element to the range $0 < K < \alpha^{-2}$, not $0 < K < \infty$. The Kretschmann scalar is given incorrectly by Dr. Sharples [1] as $f = 12\alpha^2/R^3$. For Schwarzschild spacetime the Kretschmann scalar is actually given by

$$f = 12\alpha^2 K^3 = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{\left(|r - r_o|^n + \alpha^n\right)^{\frac{6}{n}}}.$$

Then,

$$f\left(r_o\right) = \frac{12}{\alpha^4} \quad \forall\ r_o\ \forall\ n,$$

which is a scalar invariant that corresponds to the scalar invariants $R_p\left(r_o\right) = 0$, $R_c\left(r_o\right) = \alpha$, $K\left(r_o\right) = \alpha^{-2}$. The usual assumption that the Kretschmann scalar must be unbounded (undefined) at a singularity in Schwarzschild spacetime is just that, and has no valid physical or mathematical basis. It is evident from the line-element that the Kretschmann scalar is finite everywhere. This is reaffirmed by the Riemannian (or Sectional) curvature K_s of the spatial section of Schwarzschild spacetime, given by

$$K_s = \frac{-\frac{\alpha}{2}W_{1212} - \frac{\alpha}{2}W_{1313}\sin^2\theta + \alpha R_c\left(R_c - \alpha\right)W_{2323}}{R_c^3 W_{1212} + R_c^3 W_{1313}\sin^2\theta + R_c^4\sin^2\theta\left(R_c - \alpha\right)W_{2323}}$$

$$R_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}}, \quad r \in \Re, \quad n \in \Re^+$$

where

$$W_{ijkl} = \begin{vmatrix} U^i & U^j \\ V^i & V^j \end{vmatrix} \begin{vmatrix} U^k & U^l \\ V^k & V^l \end{vmatrix}$$

and $< U^i >$ and $< V^i >$ are two arbitrary non-zero contravariant vectors at any point in the space. Thus, in general, the Riemannian curvature is dependent upon both position and direction (i.e. the directions of the contravariant vectors). Now,

$$K_s\left(r_o\right) = -\frac{1}{2\alpha^2} = -\frac{1}{2}K\left(r_o\right)$$

which is entirely *independent* of the contravariant vectors (and hence independent of direction) and is half the negative of the associated Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. This is a scalar invariant that corresponds to $R_c(r_o) = \alpha\ \forall r_o\ \forall n$ and $R_p(r_o) = 0\ \forall r_o\ \forall n$.

That the Kretschmann scalar is finite everywhere is reaffirmed in other ways, as follows. Doughty [92] has shown that the radial geodesic acceleration a of a point in a manifold described by a line-element with the form of eq. (25) is given by,

$$a = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)|g_{00,1}|}{2g_{00}}.$$

This gives,

$$a = \frac{\alpha}{R_c^{\frac{3}{2}}\left(r\right)\sqrt{R_c\left(r\right) - \alpha}}.$$

Now,

$$\lim_{r \to r_o^{\pm}} R_c\left(r\right) = \alpha,$$

and so

$$r \to r_o^{\pm} \Rightarrow a \to \infty \quad \forall\ r_o\ \forall\ n.$$

Accordingly, there is no possibility for $R_c(r) < \alpha$.

In the case of eq. (5), for which $r_o = \alpha = 2m$, $n = 1$, $r \to 2m^+$, the acceleration is,

$$a = \frac{2m}{r^{\frac{3}{2}}\sqrt{r - 2m}},$$

and hence $a \to \infty$ as $r \to 2m^+$. But the usual unproven (and invalid) assumption that r in eq. (5) can go down to zero means that there is an infinite acceleration at $r = 2m$ where, according to the community of astrophysics, *there is no matter*! However, r can't take the values $0 \le r < r_o = 2m$ in eq. (5), by virtue of the nature of the Gaussian curvature of spherically symmetric geodesic surfaces in the spatial section of the manifold and the intrinsic geometry of the line-element, as the acceleration reaffirms.

The change of signature described in Section 2 above also attests to the inextendibility of the Schwarzschild manifold and the finite nature of the Kretschmann scalar.

Conclusions:

1. The Kretschmann scalar is everywhere finite in Schwarzschild spacetime.

2. Schwarzschild spacetime is inextendible.

4 Recapitulation

Contrary to Dr. Sharples' claim [1], $R_c(r)$ is *not* the proper radius in the Schwarzschild manifold. At the same time, I have never said that it cannot be a proper radius in other circumstances, such as when the associated surface is embedded in Euclidean 3-space.

The solution by Hilbert is *not* the same as that obtained by Schwarzschild because there is only one singularity in Schwarzschild spacetime, whereas there are allegedly two singularities in Hilbert's corruption of the Schwarzschild and the Droste solutions. Schwarzschild's actual solution does not contain a black hole - it forbids black holes, as does the solution by Droste.

The Schwarzschild manifold is maximal and cannot therefore be extended. The Kruskal-Szekeres and the Eddington-Finkelstein coordinates are invalid because they attempt to extend a manifold that is already maximal.

Contrary to the charge in [1], I have remarked in several of my papers that the infinite set of particular solutions I obtain are all *geometrically equivalent*.

The Kretschmann scalar is finite everywhere in Schwarzschild spacetime; and it is not an independent curvature invariant.

The introduction of Newtonian two-body concepts and associated mathematical expressions into Schwarzschild's solution is inadmissible because there is no matter present, *by construction*, in Schwarzschild spacetime. All alleged black hole "solutions" pertain to a universe that contains only one alleged mass (as a source) and so cannot contain Newtonian relations. $R_{\mu\nu} = 0$ is *not* a two or more body problem - it is a statement that there is no matter present and hence no sources, and therefore constitutes a no-body problem, which cannot describe a gravitational field. The 'Principle of Superposition' *does not apply* in General Relativity and so one cannot, by an analogy with Newton's theory, assert that black holes exist in multitudes, merge, collide, be components of binary systems, or otherwise interact with one another and other matter. There are no known solutions to Einstein's field equations for two or more bodies and no existence theorem for latent solutions for such configurations of matter.

Notwithstanding Dr. Sharples' arguments, the black hole is in fact inconsistent with General Relativity, because all black hole "solutions" violate the intrinsic geometry of their metrical groundforms and the physical principles of Einstein's gravitational field.

The Theory of Relativity forbids infinitely dense point-mass singularities.

Despite the many claims made by the astrophysics community, nobody has ever found a black hole. The international search for black holes is destined to detect none.

The theoretician D. Rabounski [93] has reaffirmed my arguments that a black hole cannot form in Schwarzschild space.

Emulating Dr. Sharples, I close with an ageless and poignant adage:

> *"It was once told as a good joke upon a mathematician that the poor man went mad and mistook his symbols for realities; as M for the moon and S for the sun."*
>
> O. Heaviside (1893)

Dedication

I dedicate this article to my late brother,

Paul Raymond Crothers

12th May 1968 – 25th December 2008.

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