Dynamics of the solar wind: Eugene Parker's treatment and the laws of thermodynamics

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Abstract: In 1958, Eugene Parker advanced that the solar wind must be produced through the thermal expansion of coronal gas. At the time, he introduced a dimensionless parameter, $\lambda = GM_S M_H / 2k_B T_0 a$, where G corresponds to the universal constant of gravitation, M_S to the solar mass, M_H to the mass of the hydrogen atom, k_B to Boltzmann's constant, T_0 to the temperature at the location of interest, and a is the distance to the effective surface, or the radial distance, to the outer solar corona, the location of interest, relative to the center of the Sun. It is straightforward to demonstrate that this equation stands in violation of the 0th and 2nd laws of thermodynamics by simply rearranging the expression in terms of temperature: $T_0 = GM_S M_H / 2k_B \lambda a$. In that case, then temperature, an intensive property, is now being defined in terms of an extensive property, M_S, and the radial position, a, which is neither intensive nor extensive. All other terms in this expression are constants and unable to affect the character of a thermodynamic property. As a result, temperature in this expression is not intensive. Consequently, the expression advanced by Parker is not compatible with the laws of thermodynamics. This analysis demonstrates that solar winds cannot originate from the thermal expansion of coronal gas, as is currently accepted. © *2019 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-32.1.1]

Résumé: En 1958, Eugene Parker a avancé que le vent solaire devait être produit par la dilatation thermique du gaz coronal. A l'époque, il introduisit un paramètre sans dimension, $\lambda = GM_S M_H / 2k_B T_0 a$, où G correspond à la constante universelle de gravitation, M_S à la masse solaire, M_H à la masse de l'atome d'hydrogène, k_B à la constante de Boltzmann, T_0 à la température à l'emplacement d'intérêt, et a est la distance à la surface effective, ou la distance radiale à la couronne solaire extérieure, la position d'intérêt, par rapport au centre du soleil. Il est simple de démontrer que cette équation viole les $1^{\text{ière}}$ et $2^{\text{ième}}$ lois de la thermodynamique en réarrangeant simplement l'expression en termes de température: $T_0 = GM_S M_H / 2k_B \lambda a$. Dans ce cas, la température, une propriété intensive, est maintenant définie en termes d'une propriété extensive, M_S, et de la position radiale, a, qui n'est ni intensive ni éxtensive. Tous les autres termes de cette expression sont constants et incapables d'affecter le caractère d'une function d'état thermodynamique. En conséquence, la température dans cette expression n'est pas intensive. De même, l'expression avancée par Parker n'est pas compatible avec les lois de la thermodynamique. Cette analyse démontre que les vents solaires ne peuvent provenir de la dilatation thermique du gaz coronal, comme cela est actuellement accepté

Key words: Solar Wind; Thermodynamics; Parker Solar Probe.

Dedicated to Robert George Murdoch Crothers (16 October 1924–20 June 2018) and Marion Crothers.

I. INTRODUCTION

In thermodynamics, systems are described in terms of properties which are classified as either intensive or extensive.[1–5] Intensive properties can be determined at every spatial location and are independent of any changes in the mass of a system by definition. Temperature, pressure, velocity, thermal conductivity, and density are examples. It is well recognized that temperature maintains the same value at all spatial locations within a system in thermodynamic equilibrium. However, it might take on varying values in a system out of equilibrium. In either case, temperature always remains intensive, as it can be defined at every spatial location. Conversely, extensive properties are defined over a certain spatial extent. Typical examples are mass, volume, internal energy, and heat capacity. Extensive properties are additive. The thermodynamic coordinates necessary and sufficient to describe any thermodynamic system are determined by experiment.

Consider a homogeneous system in thermodynamic equilibrium. Divide it into two equal parts, each having equal mass. Those properties of the original system that remain unchanged in each half of the original system are intensive.

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Those properties that are halved are extensive. Those which change but not by half are neither intensive nor extensive.[4] It is also important to note that the quotient of two extensive properties is intensive. Density, $\rho = M/V$, is the best known example of such a quotient. Mass and volume are both extensive, but their quotient results in density, which is intensive. The quotient of two nonextensive properties, which behave identically with changes in spatial extent, is also intensive. However, the quotient (or the product) of an intensive property and an extensive property is always extensive.

Constants such as Boltzmann's constant, k_B, the Stefan-Boltzmann constant, σ, and Planck's constant, h, do not alter the intensive or extensive nature of a thermodynamic expression. For instance, the total energy of a simple monoatomic gas, E, can be given by the following simple expression, $E = 3Nk_BT/2$. In this expression, the extensive nature of E on the left-hand side is imparted by the extensive nature of the number of particles, N, on the right, given that temperature must remain intensive. Boltzmann's constant does not contribute toward establishing the thermodynamic nature of energy as extensive in this expression. At the same time, variables that are neither intensive nor extensive, but have units, affect the thermodynamic balance of expressions. Variables without units have no influence on thermodynamic character. In the end, it is important to remember the following rules: (1) extensive and intensive properties exist, (2) some properties are neither extensive nor intensive, and (3) physical constants (e.g., G, k_B, h) play no role in establishing whether a property is intensive, extensive, or neither. Landsberg[2] has argued that the nature of properties as intensive or extensive is so important to the study of thermodynamics that the concept should be adopted as the 4th law.

It is also true that any proposed thermodynamic equation must be thermodynamically balanced, as just demonstrated. If one side of the equation is intensive, or extensive, then the other side must also be intensive or extensive, respectively.[5] When a thermodynamic property in an expression is being defined in terms of other thermodynamic properties, the correct nature of the sought property must be obtained. Temperature cannot become nonintensive simply as a result of a mathematical expression. Temperature must always be intensive, in keeping with its role relative to defining the laws of thermodynamics.

The 0th law of thermodynamics requires thermal equilibrium between objects in defining temperature. Consider two isolated systems,[c)] each in thermodynamic equilibrium. Remove a section of the thermal insulating material from the surface of each system and place them in contact via the uncovered sections. When there are no observable changes in any thermodynamic properties of either system, they are each at the same temperature. The 0th law of thermodynamics not only makes a statement about thermal equilibrium of systems, it also includes the intensive character of temperature: "when two systems are at the same temperature as a third, they are at the same temperature as each other,"[6] "Two systems in thermal equilibrium with a third are in thermal equilibrium with each other."[7] Take two systems, A and B, at the same temperature in accordance with the foregoing method. The temperature of A is the same as that at every spatial location in B: as every part of B. Divide the system B into two parts, B_1 and B_2. Since A has the temperature of B, it has the temperature of B_1 and B_2: parts of B. Therefore, B_1 and B_2 are at the same temperature. Thus, the intensive nature of temperature is contained within the very definition of the 0th law of thermodynamics. Such equilibrium cannot exist if temperature is no longer intensive. Similarly, entropy must always remain extensive, in order to preserve the 2nd law.

If a system has spherical symmetry, its area can be expressed as $A = 4\pi r^2$. Clearly, r is neither intensive nor extensive, as it is not additive. This can also be established relative to a volumetric system with spherical symmetry. The radius is not extensive since volume, V, is given by $V = 4\pi r^3/3$. In this expression, it is the volume of a sphere which is an extensive property, along with r^3. It is clear that radius r is not additive. Hence, the radius of a sphere can never be considered as an extensive property.

Length is generally not extensive, as radius attests. However, length can become extensive in certain limited circumstances, as for example, in stretched wires,[7] having the thermodynamic coordinates of tension (intensive), length (extensive), and temperature (intensive). The spatial extent of this system is length. It is extensive, in this case, as it is directly related to the mass of the system. Any change in length of the wire is directly associated with a change in its mass.

It is also true that extensive properties in one system might not be extensive in another. A prime example is surface area. For a planar system composed of a single monolayer, area is extensive. Such systems arise when considering surface tension which, in turn, is an intensive property. However, the area of a sphere is never extensive. That is because such area is not additive. If one takes a sphere and divides it into two spheres of equal volume, the area of each sphere is not half of the initial.

As an additional example, consider the Stefan-Boltzmann law[8] describing a system in which area is a thermodynamic coordinate

$$L = \varepsilon\sigma AT^4. \quad (1)$$

In this expression, L is the luminosity of the object, ε is emissivity of the material (a unitless property), σ is the Stefan-Boltzmann constant, A is the area, and T is temperature. In this case, note that neither luminosity nor area are extensive, because these properties are not additive. However, both luminosity and area change in identical fashion relative to spatial extent. Temperature is defined at every spatial location in the system and remains intensive. For any given temperature, the luminosity is directly proportional to the area. Hence, the luminosity per unit area (L/A), also known as the emissive power, is intensive and so is the temperature, as required by the laws of thermodynamics. Equation (1) is therefore thermodynamically balanced.

The intensive nature of temperature is also necessary to the understanding of entropy as defined in the 2nd law of thermodynamics, stated mathematically as,

[c)]An *isolated* system does not exchange any energy, either by mechanical work or flow of heat, with its surroundings.

$$dS = \frac{\delta Q}{T}, \tag{2}$$

where S is entropy, Q is heat, and T is temperature. Entropy and heat are extensive. By the 2nd law, temperature is always intensive, for, if it is not, then entropy would not be extensive, contrary to the definition of entropy and the character of heat.

II. PARKER'S EQUATION FOR THE SOLAR WIND

In 1958, Eugene N. Parker attempted to account for the production of the solar wind by invoking thermal expansion of coronal gas.[9] It is simple to demonstrate that this work constitutes a violation of the laws of thermodynamics. Parker has proposed the following equation, with dimensionless variables, for the solar wind:[9]

$$\psi - \ln \psi = -3 - 4 \ln \frac{\lambda}{2} + 4 \ln \xi + \frac{2\lambda}{\xi}, \tag{3}$$

where the dimensionless variables are defined as

$$\psi = \frac{M_H v^2}{2k_B T_0}, \tag{4}$$

$$\xi = \frac{r}{a}, \tag{5}$$

$$\lambda = \frac{G M_H M_\Theta}{2a k_B T_0}, \tag{6}$$

wherein $r \geq a$ is the radial distance from the center of the Sun, a is the coronal radial position from which the solar wind emanates, or the effective surface of the Sun, T_0 is the temperature at $r = a$, M_H is the mass of a hydrogen atom, M_Θ is the mass of the Sun, k_B is Boltzmann's constant, v is the speed of the solar wind, and G is the universal constant of gravitation. Rearranging Eq. (4) for temperature gives

$$T_0 = \frac{M_H v^2}{2k_B \psi}. \tag{7}$$

The mass M_H of the hydrogen atom is a constant, ψ is a dimensionless variable, and as the velocity v and the temperature T_0 are both intensive coordinates, Eq. (7) is thermodynamically balanced. Thus Eq. (4) is admissible.

Rearranging Eq. (5) for the radius r yields

$$r = \xi a. \tag{8}$$

Since ξ is a dimensionless variable, it cannot influence the thermodynamic character of r or a, the latter the lower bound on radius r relative to the solar wind. For $\xi = 1$, $r = a$, giving a spherical surface of area $A = 4\pi a^2$ from which the solar wind emanates, enclosing the Sun of volume $V = 4\pi R_\Theta^3/3$, $r = R_\Theta < a$ the radius of the Sun. Hence, both r and a are neither intensive nor extensive, but have the same thermodynamic character. Thus, Eq. (5), relative to Eq. (3), does not violate the laws of thermodynamics and is admissible.

That Eq. (3) violates the laws of thermodynamics is made clear by solving Eq. (6) for T_0

$$T_0 = \frac{G M_H M_\Theta}{2a k_B \lambda}, \tag{9}$$

which reveals that temperature T_0, an intensive property, is being defined in terms of the extensive property M_Θ and the radius a which is neither intensive nor extensive. This is essentially the same as the problem previously highlighted relative to the equation defining the temperature within a gaseous star.[10–13] This violation of thermodynamics is amplified by substituting into Eq. (3) the explicit values of the dimensionless variables, by which one obtains

$$T_0 = \frac{\left(\dfrac{M_H v^2}{2} - \dfrac{G M_H M_\Theta}{r}\right)}{k_B \left[\ln\left(\dfrac{128 v^2 k_B^3 r^4 T_0^3}{G^4 M_H^3 M_\Theta^4}\right) - 3\right]}. \tag{10}$$

Prima facie, this expression seems correct. Dimensionally, both sides are expressed in terms of Kelvin. However, on closer examination, it becomes evident that this expression is thermodynamically unbalanced. Note that temperature on the left must be intensive, as required by the laws of thermodynamics. However, while the first term in the numerator on the right is intensive,[d)] the second term is not intensive, since M_Θ is extensive but r is neither intensive nor extensive. The term in the natural logarithm of the denominator, although variable, has no units (is a pure number). Hence, the right side of Eq. (10) is not intensive, even though the laws of thermodynamics require that temperature always remains intensive. Consequently, Eq. (3) is inadmissible.

III. CONCLUSIONS

From this simple analysis, it has been demonstrated that Eugene Parker's expression for the production of the solar wind, through the thermal expansion of coronal gas, violates the laws of thermodynamics. Temperature must always be an intensive property. When astronomers first advanced the theory of a gaseous star,[14–16] they proposed an equation similar to Eq. (9). Robitaille[10–13] has demonstrated that, in analogous fashion, that expression is also thermodynamically invalid. It is not appropriate to utilize the virial theorem and introduce temperature through kinetic theory, when balancing kinetic energy with potential energy. Such an approach results in direct violations of the laws of thermodynamics. Gravitational collapse (i.e., self-compression) of a gas cannot occur: gravitational collapse of an ideal gas produces a perpetual motion machine of the first kind.[10–13] Eugene Parker's treatment has fallen victim to the same type of error in failing to respect the laws of thermodynamics.

[d)] Because M_H is a constant and velocity is intensive.

NOTE ADDED IN PROOF

After reading this work, one might be left with a sense of "Why did such an error relative to intensive and extensive properties arise in astrophysics?" The answer lies in the inappropriate treatment of the gravitational fields and their effect on the temperature of a gas. External forces, including gravity, must never be permitted to alter this property. From a historical perspective, the introduction of this type of error is evident by contrasting the analysis provided by Boltzmann[17] versus Jeans[18] for a column of air within an adiabatic cylinder under the influence of a gravitational field. Boltzmann correctly argues[17] that the temperature distribution within this column remains uniform and indeed, that the entire column remains in thermal equilibrium. This remains the case even though the density and pressure of the gas assume gradients with respect to height.[17] However, the ratio of pressure and density remains constant throughout the column, and therefore, by the ideal gas law, so does the temperature: "…the temperature is also the same everywhere in spite of the action of external forces."[17] Boltzmann's temperature remains intensive,[17] as it never becomes dependent of the force of gravity. Conversely, Jeans argues[18] that the temperature varies with elevation by assuming that the gas never reaches equilibrium. This directly leads to a violation of the 0th law of thermodynamics, as manifested by the analysis of his expressions involving both the force of gravity and temperature.[18] The temperatures which Jeans obtains are not intensive as a direct result. This problem has drawn the attention of educational works[19,20] demonstrating that the correct answer does indeed rest with Boltzmann.[17]

Parker has allowed temperature to become affected by an external force, namely, gravity, and has committed the same error as Jeans.[18] The fact that his temperature is not intensive in Eq. (9) is a direct reflection of this misstep. Parker makes temperature dependent on gravity which is not allowed by Boltzmann.[17] In fact, it is interesting to note that while Parker's temperature in Eq. (9) is not intensive, his temperature in Eq. (7) is, in fact, intensive. As a result, Parker is simultaneously advancing that temperature can be both intensive and nonintensive simultaneously at the same location. This emphasizes, once again, that Parker's treatment of this problem cannot be correct.

[1] P. Atkins, *The Law of Thermodynamics: A Very Short Introduction* (Oxford University Press, Oxford, UK, 2010), p. 3.
[2] P. T. Landsberg, *Thermodynamics With Quantum Statistical Illustrations* (Interscience Publishers, New York, 1961), p. 142.
[3] Y. A. Cengel and M. A. Boles, *Thermodynamics—An Engineering Approach*, 8th ed. (McGraw-Hill, New York, 2015), p. 15.
[4] G. N. Hatsopoulos and J. H. Keenan, *Principles of General Thermodynamics* (John Wiley & Sons, Inc., Hoboken, NJ, 1965), p. 1.
[5] S. G. Canagaratna, J. Chem. Educ. **69**, 957 (1992).
[6] F. W. Sears, *An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics*, 2nd ed. (Addison-Wesley Publishing Company, Cambridge, MA, 1955).
[7] M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics*, 6th ed. (McGraw-Hill Book Company, New York, 1981).
[8] J. Stefan, "Sitzungsberichte der mathematischnatur wissenschaftlichen classe der kaiserlichen akademie der wissenschaften," Wien **79**, 391 (1879).
[9] E. N. Parker, Astrophysical J. **128**, 664 (1958).
[10] P.-M. Robitaille, Prog. Phys. **14**, 179 (2018).
[11] P.-M. Robitaille, *How are Stars Formed? The Standard Model: Gravitational Collapse, Black Holes, and the Big Bang!* (Sky Scholar, Columbus, OH, 2017), https://www.youtube.com/watch?v=LoqsG7V13G8
[12] P.-M. Robitaille, *Does Gravitational Collapse Occur? Insight from the Laws of Thermodynamics!* (Sky Scholar, Columbus, OH, 2018), https://www.youtube.com/watch?v=DtMee3rrHDY
[13] P.-M. Robitaille, *Gravitational Thermodynamics—Is it Science*? (Sky Scholar, Columbus, OH, 2018), https://www.youtube.com/watch?v=mZ2F2Kw5-nQ
[14] A. S. Eddington, *The Internal Constitution of the Stars* (Cambridge University Press, Cambridge, UK, 1926), pp. 79–96.
[15] J. Jeans, *Astronomy and Cosmogony* (Cambridge University Press, Cambridge, UK, 1928), pp. 65–69.
[16] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (Dover Publications, Mineola, NY, 1939), pp. 61–83.
[17] L. Boltzmann, *Lectures on Gas Theory*, translated by Stephen G. Brush (University of California Press, Berkeley, CA, 1964), pp. 141–143.
[18] J. H. Jeans, *The Dynamical Theory of Gases* (Cambridge University Press, Cambridge, UK, 1904), pp. 310–312.
[19] C. A. Coombes and H. Laue, Am. J. Phys. **53**, 272 (1985).
[20] S. Velasco, F. L. Román, and J. A. White, Eur. J. Phys. **17**, 43 (1996).