CORRESPONDENCE

To the Editors of 'The Observatory'

Laplace's Alleged "Black Hole"

GENTLEMEN,—

In recent years it has become fashionable to claim that Laplace in the eighteenth century anticipated the "black hole" of general relativity by the use of Newtonian gravitational theory[1]. This assertion is to be found in expositions meant for non-specialist audiences[2] and versions of it are beginning to appear in textbooks for students[3,4]. The following analysis is therefore intended to show that the claim has little validity and is largely due to a play on words in expressions such as "the velocity of escape" or "the escape from a body".

The Newtonian velocity of escape. Consider a spherical body of finite mass M and finite non-zero outer radius R. A particle subject to the Newtonian inverse-square law of gravitation is projected radially outwards from the body's surface with speed V. As is well known, the particle's speed, v, when it has reached distance r from the centre of the body is given by

$$v^2 = V^2 + 2GM(1/r - 1/R)\,, \tag{1}$$

and the acceleration of the particle will then be

$$g(r) = -GM/r^2\,. \tag{2}$$

It is therefore possible for v to vanish at the finite value r_0 of r where

$$r_0 = R\{1 - V^2R/2(GM)\}^{-1} > R \tag{3}$$

and, since $g(r_0)$ is negative, the particle then begins to fall back towards $r = R$. Clearly a permissible r_0, which must be positive, exists only for

$$V^2 < 2GM/R\,. \tag{4}$$

An infinite value of r_0 is also possible and occurs when V is equal to the "velocity of escape", V_e, given by

$$V_e^2 = 2GM/R\,. \tag{5}$$

This is the minimum velocity of projection *that will carry the particle to infinity and bring it to rest there* and, since $g(\infty) = 0$, the particle will not thereupon start to fall back towards the central body.

The Laplacean "black hole". Laplace employs the corpuscular theory of light, each corpuscle of light, like a material particle, being subject to Newton's law of gravitation. I am not concerned with the validity or otherwise of this theory of light, but I may mention that the corpuscles cannot be photons because these do not possess the same kind of mass as material particles have. Laplace[1] assumes that when the escape velocity V_e—which he calls a—equals c, light cannot escape from the body of mass M and radius R. Chandrasekhar's succinct statement[2] of Laplace's result is that if $R < 2GM/c^2$ then "light will not be able to escape from such a body and we should not be able to see it". Suppose that Chandrasekhar's inequality is slightly modified to read

$$R = (2GM/c^2)n\,, \qquad 0 < n \leq 1\,. \tag{6}$$

A corpuscle projected radially with speed V from the body's surface will move outwards, according to equation (3), to a distance

$$r_0 = R(1 - nV^2/c^2)^{-1} \tag{7}$$

before falling back again. By equation (5) the velocity of escape that will carry the corpuscle to infinity is

$$V_e = c/\sqrt{n}, \geq c\,. \tag{8}$$

The last equation need cause no surprise, since velocities greater than c are perfectly compatible with Newtonian mechanics and gravitational theory. But, of course, the condition that the maximum velocity of projection is to be c may be imposed if desired. The result is, by (7), that the maximum value of r_0 is

$$r_0 = R/(1 - n)\,. \tag{9}$$

These results mean that an observer located at a value of r in $R < r \leq r_0$ will not only be able to see the body by means of the corpuscles reaching him directly but also, if he turns through 180°, by those which are falling back from $r = r_0$. It is only observers in $r_0 < r \leq \infty$ who fail to see the emitting body. However, when $n = 1$, *every observer*, wherever he may be, can see the emitter directly because in this case

$$r_0 \to \infty, \qquad V_e = c, \qquad R = 2GM/c^2\,. \tag{10}$$

Admittedly very remote observers would have to wait a long time for corpuscles to reach them!

Equally elementary arguments apply when corpuscles are projected at an angle to a radius. Hence the statement that "light cannot escape from the body" when equation (6) applies does not mean that light cannot leave the body's surface. Laplace's light corpuscles can always leave the surface of his "black hole", though they may subsequently fail to travel to arbitrarily great distances. They do so travel when the conditions (10) are satisfied. Hence there is always a class of external observers who can see the emitting body.

The general relativity (GR) black hole. The simplest type is the Schwarzschild spherically symmetric black hole whose basic property is that neither material particles nor photons can leave its surface and move outwards. A GR black hole is therefore invisible to an observer however close he may be to its surface. None of the phenomena experienced by observers in the region around the Laplacean "black hole" is present in the neighbourhood of a GR black hole. The supposed connection with a Newtonian velocity of escape of magnitude c arises thus: when curvature coördinates[5] are employed in the Schwarzschild space–time the coördinate-radius of the GR black hole is indeed

$$r_s = 2GM/c^2\,. \tag{11}$$

This has a superficial resemblance to the Newtonian formula for R in (10), which corresponds to a velocity of escape equal to c but implies that all observers, even those at infinity, can in principle see the emitting body. Moreover, the Schwarzschild space–time admits other radial coördinates: in the isotropic[5] system the black hole radius is $\frac{1}{2}GM/c^2$, while in Fock's harmonic[6] system it is GM/c^2.

I suggest that the important conclusion to be drawn is that there is no analogue of the GR black hole in Newtonian gravitational theory even when this is bolstered up with the long-discarded corpuscular theory of light. GR black holes occur in certain highly-specialized exact solutions of Einstein's field equations and so far only solutions containing a single centrally-placed black hole are known. There is, therefore, no way of asserting through some analogy with Newtonian gravitational theory that a black hole could be a component of a close binary system or that two black holes could collide. An existence theorem would first be needed to show that Einstein's field equations contained solutions which described such configurations.

I am, Gentlemen,

Yours faithfully,

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References

(1) S. W. Hawking & G. F. R. Ellis, *The large scale structure of space–time* (University Press, Cambridge), 1973, pp. 365–8 contain a translation of Laplace's essay.

(2) S. Chandrasekhar, *The Observatory*, **92**, 168, 1972.

(3) M. Berry, *Principles of cosmology and gravitation* (University Press, Cambridge), 1976, p. 101.

(4) S. P. Wyatt, *Principles of astronomy*, third edition (Allyn & Bacon, Boston, Mass.), 1977, p. 496.

(5) J. L. Synge, *Relativity: the general theory* (North-Holland, Amsterdam), 1971, pp. 268–270.

(6) V. A. Fock, *The theory of space, time and gravitation* (Pergamon Press, Oxford), 1959, pp. 193–4.

Radio Emission from X-ray Pulsars

Gentlemen,—

A considerable body of evidence exists that the X-ray emitting binary pulsars are rotating magnetic neutron stars. It is also believed that rotating magnetic neutron stars are the basis of radio pulsars. The radio emission according to one school of thought is supposed to arise from high-energy particles emanating from the poles of the neutron star[1]. This 'pulsar process'[1] is expected to be suppressed in the case of X-ray binaries because of the incoming gas at the poles. However, there are a few objects which 'turn off' in X-rays. The prime example is Her X–1, which has a periodicity of 35 days; it is 'on' for about 10 days and 'off' for about 25 days[2]. The models for this behaviour are of two categories: in one the accretion is completely shut off, and in the other accretion continues, but the observer moves out of the 'beam' owing to precession or other geometrical effects[2]. In the case of shut-off of accretion, the normal 'pulsar process' should occur at the poles, and Her X–1 may be seen as a 1·2-s radio pulsar. An upper limit of 100 mJy was given by Hartmann and Lapedes[3] for any pulsar radio emission from Her X–1. Using the observed radio strengths of pulsars in the 1–2-s range and a distance of 3 kpc for Her X–1 we anticipate a flux of 10–100 mJy for Her X–1. A radio observation of a pulsar in Her X–1 during the X-ray 'off'-period would be interesting. It would lend support to the ideas that rotating neutron stars are responsible for both X-ray and radio pulsars and that the pulsar radiation takes place at the poles of the neutron star. If a radio pulsar is not observed in Her X–1 during its 'off'-period, it would indicate that the 35-day cycle is probably due to some geometrical effect