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Geometric and Physical Defects in the Theory of Black Holes

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The so-called 'Schwarzschild solution' is not Schwarzschild's solution, but a corruption of the Schwarzschild/Droste solution due to David Hilbert (December 1916), wherein m is allegedly the mass of the source of the alleged associated gravitational field and the quantity r is alleged to be able to go down to zero (although no proof of this claim has ever been advanced), so that there are two alleged 'singularities', one at $r = 2m$ and another at $r = 0$. It is routinely alleged that $r = 2m$ is a 'coordinate' or 'removable' singularity which denotes the so-called 'Schwarzschild radius' (event horizon) and that the 'physical' singularity is at $r = 0$. The quantity r in the usual metric has never been rightly identified by the physicists, who effectively treat it as a radial distance from the alleged source of the gravitational field at the origin of coordinates. The consequence of this is that the intrinsic geometry of the metric manifold has been violated in the procedures applied to the associated metric by which the black hole has been generated. It is easily proven that the said quantity r is in fact the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of Schwarzschild spacetime and so does not denote radial distance in the Schwarzschild manifold. With the correct identification of the associated Gaussian curvature it is also easily proven that there is only one singularity associated with all Schwarzschild metrics, of which there is an infinite number that are equivalent. Thus, the standard removal of the singularity at $r = 2m$ is actually a removal of the *wrong* singularity, very simply demonstrated herein.

I. Schwarzschild Spacetime

It is reported almost invariably in the literature that Schwarzschild's solution for $R_{\mu\nu} = 0$ is (using $c = G = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (1)$$

Simply by inspection of this line-element the physicists have asserted that r can go down to zero in some way, producing an infinitely dense point-mass singularity, with an event horizon at the 'Schwarzschild radius' at $r = 2m$. Contrast this metric with that actually obtained by K. Schwarzschild in 1915 (published January 1916),

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (2)$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty,$$

wherein α is an undetermined constant. There is only *one* singularity in Schwarzschild's solution, at $r = 0$. Contrary to the usual claims Schwarzschild did not set $\alpha = 2m$; he did not breathe a single word about the bizarre object that has come to be called a black hole; he did not derive the so-called 'Schwarzschild radius'; he did not claim that there is an 'event horizon' (by any other name); and his solution clearly forbids the black hole because when Schwarzschild's $r = 0$, his $R = \alpha$, and so there is no possibility for his R to be less than α, let alone take the value $R = 0$. All this can be easily verified by simply reading Schwarzschild's original paper [1]. Thus, eq. (1) for $0 < r < 2m$ is *inconsistent* with Schwarzschild's true solution, eq. (2).

II. 3-D Spherically Symmetric Metric Manifolds

A line-element, in spherical coordinates, for 3-dimensional Euclidean space is,

$$ds^2 = dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (3)$$

$$0 \le r < \infty.$$

The scalar r is the magnitude of the radius vector $\vec{r}$ from the origin of the coordinate system, the said origin coincident with the centre of the associated sphere. All the components of the metric tensor are well-defined and related geometrical quantities are fixed by the line-element: a geometry is completely determined by the *form* of its line-element [2]. Indeed, the radius R_p of the associated sphere is given by,

$$R_p = \int_0^r dr = r,$$

the circumference C_p of a great circle ($\theta = \pi/2$) is,

$$C_p = r \int_0^{2\pi} d\varphi = 2\pi r,$$

the surface area A_p of the sphere is,

$$A_p = r^2 \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi r^2,$$

and the volume V_p of the sphere is,

$$V_p = \int_0^r r^2 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}\pi r^3.$$

Consider the generalisation of eq. (3) to a non-Euclidean 3-dimensional spherically symmetric metric manifold by the line-element,

$$\begin{aligned} ds^2 &= dR_p^2 + R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \\ &= \Psi\left(R_c\right) dR_c^2 + R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \\ R_c &= R_c(r) \\ R_c(0) &\leq R_c(r) < \infty, \end{aligned} \tag{4}$$

where both $\Psi(R_c)$ and $R_c(r)$ are *a priori* unknown analytic functions. Since neither $\Psi(R_c)$ nor $R_c(r)$ are known, eq. (4) may or may not be well-defined at $R_c(0)$: one cannot know until $\Psi(R_c)$ and $R_c(r)$ are somehow specified. With this proviso, there is a one-to-one point-wise correspondence between the manifolds described by eqs. (3) and (4), i.e. a mapping, as the differential geometers have explained [3]. If R_c is constant, metric (4) reduces to a 2-dimensional spherically symmetric geodesic surface described by,

$$ds^2 = R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{5}$$

If in eq. (3) r is constant, then it reduces to the 2-dimensional spherically symmetric surface described by,

$$ds^2 = r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{6}$$

A surface is a manifold in its own right. It need not be considered in relation to an embedding space. Therefore, quantities appearing in its line-element must be identified in relation to the surface, not to any embedding space it might be in:

> *"And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space"* [4].

Note that eqs. (3) and (4) have the same metrical form and that eqs. (5) and (6) have the same metrical form. Metrics of the same form share the same fundamental relations between the components of their respective metric tensors. For example, consider eq. (4) in relation to eq. (3). For eq. (4), the radial geodesic distance (i.e. the proper radius) from the point at the centre of spherical symmetry is

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi(R_c(r))} dR_c(r) = \int_0^r \sqrt{\Psi(R_c(r))} \frac{dR_c(r)}{dr} dr,$$

the circumference C_p of a great circle ($\theta = \pi/2$) is,

$$C_p = R_c(r) \int_0^{2\pi} d\varphi = 2\pi R_c(r),$$

the surface area A_p of the geodesic sphere is,

$$A_p = R_c^2(r) \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi R_c^2(r),$$

and the volume V_p of the geodesic sphere is,

$$\begin{aligned} V_p &= \int_0^{R_p} R_c^2\,(r)\,dR_p \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi \\ &= \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi\left(R_c\,(r)\right)} R_c^2(r) dR_c \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi \\ &= \int_0^{r} \sqrt{\Psi\left(R_c\,(r)\right)} R_c^2(r) \frac{dR_c(r)}{dr} dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi. \end{aligned}$$

In the case of the 2-dimensional metric manifold given by eq. (5) the Riemannian curvature associated with eq. (4) (which depends upon both position and direction) reduces to the Gaussian curvature K (which depends only upon position), and is given by

$$K = \frac{R_{1212}}{g}, \tag{7}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$ (because the metric tensor of eq. (5) is diagonal). Now recall from elementary differential geometry and tensor analysis that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\gamma}_{.\ \nu\rho\sigma}$$

$$R^{1}_{.\ 212} = \frac{\partial \Gamma^1_{22}}{\partial x^1} - \frac{\partial \Gamma^1_{21}}{\partial x^2} + \Gamma^k_{22}\Gamma^1_{k1} - \Gamma^k_{21}\Gamma^1_{k2}$$

$$\Gamma^i_{ij} = \Gamma^i_{ji} = \frac{\partial\left(\frac{1}{2}\ln|g_{ii}|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i}, \quad (i \neq j) \tag{8}$$

and all other Γ^i_{jk} vanish. In the above, $i, j, k = 1, 2,\ x^1 = \theta,\ x^2 = \varphi$. Applying expressions (7) and (8) to expression (5) gives,

$$K = \frac{1}{R_c^2} \tag{9}$$

so that $R_c(r)$ is the inverse square root of the Gaussian curvature, i.e. the radius of Gaussian curvature, and hence, in eq. (6) the quantity r therein is the radius of Gaussian curvature because Gaussian curvature is intrinsic to all geometric surfaces having the form of eq. (5) (a geometry is completely determined by the *form* of its line-element). Indeed, any 2-D surface has an intrinsic Gaussian curvature. Note that according to eqs. (3), (6) and (7), the radius calculated for (3) gives the same value as the associated radius of Gaussian curvature of a spherically symmetric surface in the space of eq. (3). Thus, the Gaussian curvature (and hence the radius of Gaussian curvature) of the spherically symmetric surface in the space of (3) can be associated with the calculated radius, from eq. (3). This is a consequence of the Euclidean nature of the space of eq. (3). However, this is *not* a general relationship. The radius of Gaussian curvature does not directly determine any distance at all in Einstein's gravitational manifold but in fact determines the Gaussian curvature of the spherically symmetric geodesic surface through any point in the spatial section of the gravitational manifold, as proven by expression (9). Thus, the quantity r in eq. (1) gives the inverse square root of the Gaussian curvature (i.e. the radius of Gaussian curvature) of a spherically symmetric geodesic surface in the spatial section, not the radial geodesic distance from the centre of spherical symmetry of the spatial section. It necessarily follows from this simple geometric fact that all the claims made for black holes are entirely false.

III. The Standard Derivation

The usual derivation [5–13] begins with the following metric for Minkowski spacetime (using $c = 1$),

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{10}$$

and proposes a generalisation thereof as

$$ds^2 = e^\lambda dt^2 - e^\beta dr^2 - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{11}$$

where λ, β and R are all unknown functions of only r, to be determined, and so that the signature of (10) is maintained. The form of $R(r)$ is then *assumed* so that $R(r) = r$, to get,

$$ds^2 = e^\lambda dt^2 - e^\beta dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{12}$$

It is then required that e^λ and e^β be determined so as to satisfy $R_{\mu\nu} = 0$. Now note that eq. (12) **not only retains the signature -2, but also retains the signature $(+,-,-,-)$, because $e^\lambda > 0$ and $e^\beta > 0$.** Thus, neither e^λ nor e^β can change sign.

The Standard analysis then obtains the solution given by eq. (1), wherein the constant m is claimed to be the mass generating the alleged gravitational field. By inspection of (1) the Standard analysis asserts that there are two singularities, one at $r = 2m$ and one at $r = 0$. It is claimed that $r = 2m$ is a removable coordinate singularity, and that $r = 0$ a physical singularity. It is also asserted that $r = 2m$ gives the event horizon (the 'Schwarzschild radius') of a black hole and that $r = 0$ is the position of the infinitely dense point-mass singularity of the black hole.

However, these claims cannot be true. First, the construction of eq. (12) to obtain eq. (1) in satisfaction of $R_{\mu\nu} = 0$ is such that neither e^{λ} nor e^{β} can change sign, because $e^{\lambda} > 0$ and $e^{\beta} > 0$. Therefore the claim that r can take values less than $2m$ is false; a contradiction by the very construction of the metric (12) leading to metric eq. (1). Furthermore, since neither e^{λ} nor e^{β} can ever be zero, the claim that $r = 2m$ is a removable coordinate singularity is also false. In addition, the true nature of r in both eqs. (12) and (1) is entirely overlooked, and the geometric relations between the components of the metric tensor, fixed by the *form* of the line-element, are not applied, in consequence of which the Standard analysis fatally falters.

To highlight further the geometrical errors that produce the black hole, rewrite eq. (11) as,

$$ds^2 = A\left(R_c\right)dt^2 - B\left(R_c\right)dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{13}$$

where $A\left(R_c\right), B\left(R_c\right), R_c\left(r\right) > 0$. The solution for $R_{\mu\nu} = 0$ then takes the form,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1}dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{14}$$

$$R_c = R_c(r),$$

where α is a constant. It remains to determine the admissible form of $R_c\left(r\right)$, which, from **Section II**, is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the manifold associated with eq. (14), owing to the metrical *form* of eq. (14). From **Section II** herein the proper radius for a metric of the *form* eq. (14) is,

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha\ln\left[\sqrt{R_c} + \sqrt{R_c - \alpha}\right] + k, \tag{15}$$

where k is a constant. Now for some r_o, $R_p\left(r_o\right) = 0$. Then by (15) it is required that $R_c\left(r_o\right) = \alpha$ and $k = -\alpha\ln\sqrt{\alpha}$, so

$$R_p\left(r\right) = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha\ln\left[\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right]. \tag{16}$$

It is thus also determined that the Gaussian curvature of the spherically symmetric geodesic surface of the spatial section ranges not from ∞ to 0, as it does for Euclidean 3-space, but from α^{-2} to 0. This is an inevitable consequence of the non-Euclidean geometry described by eq. (14).

Schwarzschild's true solution, eq. (2), must be a particular case of the general expression we seek for $R_c(r)$. Brillouin's solution [14, 15] must also be a particular case, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r+\alpha}\right) dt^2 - \left(1 - \frac{\alpha}{r+\alpha}\right)^{-1} dr^2 - (r+\alpha)^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
$$0 < r < \infty, \tag{17}$$

and Droste's solution [16] must as well be a particular solution, viz.,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$
$$\alpha < r < \infty. \tag{18}$$

These particular solutions must all be particular solutions in an infinite set of equivalent metrics [17]. The only admissible form for $R_c(r)$ is [18],

$$R_c(r) = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$
$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o, \tag{19}$$

where r_o and n are entirely arbitrary constants. So the solution for $R_{\mu\nu} = 0$ is,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
$$R_c(r) = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$
$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o. \tag{20}$$

Then if $r_o = 0$, $r > r_o$, $n = 1$, Brillouin's solution eq. (17) results. If $r_o = 0$, $r > r_o$, $n = 3$, then Schwarzschild's actual solution eq. (2) results. If $r_o = \alpha$, $r > r_o$, $n = 1$, then Droste's solution eq. (18) results, which is the correct solution in the particular metric of eq. (1). In addition the required infinite set of equivalent metrics is thereby obtained, all of which are asymptotically Minkowski spacetime. Furthermore, if the constant α is set to zero, eq. (20) reduces to Minkowski spacetime, and if in addition r_o is set to zero, that the usual Minkowski metric of eq. (10) is obtained.

It is clear from expression (20) that there is only ever one singularity, at the arbitrary constant r_o, where $R_c(r_o) = \alpha \ \forall \ r_o \ \forall \ n$ and $R_p(r_o) = 0 \ \forall \ r_o \ \forall \ n$. Hence, the "removal" of the singularity at $r = 2m$ in eq. (1) is fallacious, and in a very real sense, is a removal of the *wrong* singularity, because it is clear from expression (19) and the *form* of the line-element at eq. (13), in accordance with the intrinsic geometry of the line-element as given in **Section II** and the generalisation at eq. (12), that there is no singularity at $r = 0$ in eq. (1) and that

$0 \leq r \leq 2m$ therein is meaningless. The Standard claims for eq. (1) violate the geometry fixed by the *form* of its line-element and contradict the generalisation at eq. (13) from which it has been obtained by the Standard method. There is no black hole associated with eq. (1).

IV. Other Violations

Special Relativity forbids infinite density because infinite density implies that a material body can acquire the speed of light in vacuo (or equivalently there is infinite energy) [19]. General Relativity cannot, by its very definition, violate Special Relativity. Therefore General Relativity also forbids infinite density. But the point-mass singularity of the alleged black hole is infinitely dense. Thus, General Relativity forbids black holes.

In writing eq. (12) the Standard Model incorrectly asserts that only the components g_{00} and g_{11} are modified by $R_{\mu\nu} = 0$, allegedly manifest in eq. (1), as it is usually interpreted. However, it is plain by expression (20) that this is false. All components of the metric tensor are modified by the constant α appearing in eq. (20), of which eq. (1) is but a particular case.

The Standard Model asserts in relation to eq. (1) that a 'true' singularity must occur where the Riemann tensor scalar curvature invariant (i.e. the Kretschmann scalar) is unbounded. However, it has never been proven that Einstein's field equations require such a curvature condition to be fulfilled. Since the Kretschmann scalar is finite at $r = 2m$ in eq. (1), it is then claimed that $r = 2m$ marks a "coordinate singularity" or "removable singularity". However, these assertions violate the intrinsic geometry of the manifold described by eq. (1). The Kretschmann scalar depends upon all the components of the metric tensor and all the components of the metric tensor are functions of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, owing to the form of the line-element. Einstein's gravitational field is manifest in the curvature of spacetime, a curvature induced by the presence of matter. It is therefore to be expected that the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the gravitational manifold is also modified from that of ordinary Euclidean space, and this is indeed the case. Eq. (20) gives the modification of the Gaussian curvature fixed by the intrinsic geometry of the line-element and the required boundary conditions specified by Einstein, in consequence of which the Kretschmann scalar is constrained by the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. Recall that the Kretschmann scalar f is,

$$f = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}.$$

Using eq. (20) gives,

$$f = 12\alpha^2 K^3 = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{\left(|r - r_o|^n + \alpha^n\right)^{\frac{6}{n}}},$$

then

$$f(r_o) = \frac{12}{\alpha^4} \quad \forall\ r_o\ \forall\ n,$$

which corresponds to $R_p(r_o) = 0$, $R_c(r_o) = \alpha$, $K(r_o) = \alpha^{-2}$.

Doughty [20] has shown that the radial geodesic acceleration a of a point in a manifold described by a line-element with the form of eq. (13) is given by,

$$a = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)|g_{00,1}|}{2g_{00}}.$$

Using eq. (20) once again gives,

$$a = \frac{\alpha}{R_c^{\frac{3}{2}}(r)\sqrt{R_c(r) - \alpha}}.$$

Now,

$$\lim_{r \to r_o^{\pm}} R_p(r) = 0,$$

$$\lim_{r \to r_o^{\pm}} R_c(r) = \alpha,$$

and so

$$r \to r_o^{\pm} \Rightarrow a \to \infty \quad \forall\ r_o\ \forall\ n.$$

According to eq. (20) there is no possibility for $R_c(r) < \alpha$.

Now according to eq. (1), for which $r_o = \alpha = 2m$, $n = 1$, $r > \alpha$, the acceleration is,

$$a = \frac{\alpha}{r^{\frac{3}{2}}\sqrt{r - \alpha}}.$$

which is infinite at $r = 2m$. But the usual unproven (and invalid) assumption that r in eq. (1) can go down to zero means that there is an infinite acceleration at $r = 2m$ where, according to the Standard Model, there is no matter! However, r can't take values less than $r = r_o = 2m$ in eq. (1), as eq. (20) shows, by virtue of the nature of the Gaussian curvature of spherically symmetric geodesic surfaces in the spatial section associated with the gravitational manifold and the intrinsic geometry of the line-element.

The proponents of the Standard Model admit that if $0 < r < 2m$ in eq. (1) above, the rôles of t and r are interchanged. But this violates their construction at eq. (12), which has the fixed signature (+,—,—,—), and is therefore inadmissible. To further illustrate this violation, when $2m < r < \infty$ the signature of eq. (1) is (+,–,–,–); but if $0 < r < 2m$ in eq. (1), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) \text{ is negative, and}$$

$$g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1} \text{ is positive.}$$

So the signature of eq. (1) changes to (–,+,–,–). Thus the rôles of t and r are interchanged. To amplify this, set $t = -r^*$ and $r = -t^*$, and so for $0 < r < 2m$, eq. (1) becomes,

$$ds^2 = -\left(1 + \frac{2m}{t^*}\right)^{-1} dt^{*2} + \left(1 + \frac{2m}{t^*}\right) dr^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$-2m < t^* < 0.$$

which has the signature (+,–,–,–). But this is now a *time-dependent metric* since all the components of the metric tensor are functions of the time t^*, and so this metric *bears no relationship to the original time-independent problem to be solved.* In other words, this metric is a non-static solution to a static problem: contra-hype! Thus, in eq. (1), $0 < r < 2m$ is meaningless, as eqs. (12) and (20) show.

The Gravity Probe B did not detect the alleged Lense-Thirring effect and so NASA has cancelled the project [21].

Nobody has ever found a black hole anywhere because nobody has found an infinitely dense point-mass singularity and nobody has found an event horizon. All claims for detection of black holes are patently false. And it is clear from the foregoing analysis that General Relativity does not predict the black hole and does not predict the big bang.

According to Einstein [22], his 'Principle of Equivalence' and his laws of Special Relativity must manifest in his gravitational field. Now $R_{\mu\nu} = 0$ is a construction by which there is no matter present in the Universe. Therefore, Einstein's 'Principle of Equivalence' and his laws of Special Relativity cannot manifest in the manifold for $R_{\mu\nu} = 0$, and so $R_{\mu\nu} = 0$ does not describe Einstein's gravitational field. Furthermore, there can be no freely falling inertial systems, no observers, and no energy in a spacetime that *by definition* contains no matter. Consequently, there is no black hole associated with eq. (1). The introduction of mass into eq. (1) is *post hoc* and therefore inadmissible.

Since $R_{\mu\nu} = 0$ does not describe Einstein's gravitational field, his field equations must take the form [23, 24],

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0,$$

where the $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. This is an identity. The total gravitational energy is always zero; the $G_{\mu\nu}$ and the $T_{\mu\nu}$ must vanish identically; there is no possibility for the localisation of gravitational energy (i.e. no Einstein gravitational waves). Furthermore, Einstein's General Theory of Relativity violates the usual conservation of energy and momentum so well established by experiment. It is therefore invalid. The current international efforts to detect gravitational waves are destined to detect nothing. They have detected nothing [25].

The Big Bang cosmology, spawned by General Relativity, not observation, is theoretically meaningless, owing to the failure of General Relativity.

It is also claimed by the physicists that spacetimes can be intrinsically curved, i.e. that there are gravitational fields that have no material cause. An example is de Sitter's empty spherical Universe, based upon the following field equations [2, 17]:

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{21}$$

wherein λ is the so-called 'cosmological constant'. Now in the case of line-element (1) the field equations are:

$$R_{\mu\nu} = 0. \tag{22}$$

Curiously, the physicists claim on the one hand that (21) is devoid of matter and so has no material cause for the associated alleged gravitational field (i.e. the curvature of spacetime), because the energy-momentum tensor is zero there, yet on the other hand they also claim that (22) has a material cause, which they insert *post hoc*, even though the energy-momentum tensor is zero there as well. The interpretations by the physicists of the alleged gravitational fields associated with (21) and (22) are therefore contradictory. Furthermore, despite the assertions of the physicists, there is no experimental evidence whatsoever to support the claim that a gravitational field can exist without a material cause.

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