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Concerning Fundamental Mathematical and Physical Defects in the General Theory of Relativity

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The physicists have misinterpreted the quantity 'r' appearing in the so-called "Schwarzschild solution" as it is neither a distance nor a geodesic radius but is in fact the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section of the Schwarzschild manifold, and so it does not directly determine any distance at all in the Schwarzschild manifold - in other words, it determines the Gaussian curvature at any point in a spherically symmetric geodesic surface in the spatial section of the manifold. The concept of the black hole is consequently invalid. It is also shown herein that the Theory of Relativity forbids the existence of point-mass singularities because they imply infinite energies (or equivalently, that a material body can acquire the speed of light in vacuo), and so the black hole is forbidden by the Theory of Relativity. That $Ric = R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence' and so does not describe Einstein's gravitational field, is demonstrated. It immediately follows that Einstein's conceptions of the conservation and localisation of gravitational energy are invalid - the General Theory of Relativity violates the usual conservation of energy and momentum.

I. Mathematical Preamble: Spherical Symmetry of Three-Dimensional Metrics

Following the method suggested by Palatini, and developed by Levi-Civita [1], denote ordinary Euclidean 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point $O \in \mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation T of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric $d\ell'^2$ unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of $d\ell'^2$ of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$, except possibly at an arbitrary point, the centre of spherical symmetry $O' \in \mathbf{M}^3$. Let a ray i emanate from an arbitrary point $O \in \mathbf{E}^3$. There is then a corresponding geodesic

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$i' \in \mathbf{M}^3$ issuing from the corresponding point $O' \in \mathbf{M}^3$. Let P be any point on i other than O. There corresponds a point P' on $i' \in \mathbf{M}^3$ different to O'. Let g' be a geodesic in $\mathbf{M}^3$ that is tangential to i' at P'.

Taking i as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on i' unchanged. If g' is distinct from i', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about i would cause g' to occupy an infinity of positions in $\mathbf{M}^3$ wherein g' has for each position the property of being tangential to i' at P' in the same direction, which is impossible. Hence, g' coincides with i'.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point $O \in \mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' in $\mathbf{M}^3$ with centre of symmetry at the corresponding point $O' \in \mathbf{M}^3$. Let Q be a point in $\Sigma \in \mathbf{E}^3$ and Q' the corresponding point in $\Sigma' \in \mathbf{M}^3$. Let $d\sigma$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma' \in \Sigma'$ issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates r, θ, φ. Then

$$d\sigma^2 = r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.1}$$

$$r = |\overline{OQ}|.$$

Clearly, if r, θ, φ are known, Q is determined and hence also Q' in Σ'. Therefore, θ and φ can be considered to be curvilinear coordinates for Q' in Σ' and the line element $d\sigma' \in \Sigma'$ will also be represented by a quadratic form similar to (1.1). To determine $d\sigma'$, consider two elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma'_1$ and $d\sigma'_2$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis $\overline{OQ}$ in $\mathbf{E}^3$, and so $d\sigma'_1$ and $d\sigma'_2$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $\frac{d\sigma'}{d\sigma}$ is the same for the two different directions irrespective of $d\theta$ and $d\varphi$, and so the foregoing ratio is a function of position, i.e. of r, θ, φ. But Q is an arbitrary point in Σ, and so $\frac{d\sigma'}{d\sigma}$ must have the same ratio for any corresponding points Q and Q'. Therefore, $\frac{d\sigma'}{d\sigma}$ is a function of r alone, thus

$$\frac{d\sigma'}{d\sigma} = H(r),$$

and so

$$d\sigma'^2 = H^2(r)d\sigma^2 = H^2(r)r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.2}$$

where $H(r)$ is *a priori* unknown. For convenience set $R_c = R_c(r) = H(r)r$, so that (2) becomes

$$d\sigma'^2 = R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.3}$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing (1.3) with (1.1) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point Q', i.e. in terms of the relation $K = \frac{1}{R_c^2}$ since the Gaussian curvature of (1.1) is $K = \frac{1}{r^2}$. This is an intrinsic property of all line elements of the form (1.3) [1]. Accordingly, R_c, the inverse square root of the Gaussian curvature, can be regarded as the radius of Gaussian curvature. Therefore, in (1.1) the radius of Gaussian curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilics.

Let the element of radial distance from $O \in \mathbf{E}^3$ be dr. Clearly, the radial lines issuing from O cut the surface Σ orthogonally. Combining this with (1.1) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$

$$d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1.4}$$

Let the corresponding radial geodesic from the point $O' \in \mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from O' cut the geodesic surface Σ' orthogonally. Combining this with (1.3) by the theorem of Pythagoras gives the line element in $\mathbf{M}^3$ as,

$$d\ell'^2 = dR_p^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.5}$$

where dR_p is, by spherical symmetry, also a function only of R_c. Set $dR_p = \sqrt{B(R_c)}dR_c$, so that (1.5) becomes

$$d\ell'^2 = B(R_c)dR_c^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.6}$$

where $B(R_c)$ is an *a priori* unknown function. Expression (1.6) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point $O' \in \mathbf{M}^3$.

Considering (1.4), the distance $R_p = |\overline{OQ}|$ from the point at the centre of spherical symmetry O to a point $Q \in \Sigma$, is given by

$$R_p = \int_0^r dr = r = R_c.$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature of the spherically symmetric geodesic surface containing any point in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point $O \in \mathbf{E}^3$ and the point $Q \in \Sigma$. Thus, in this case, $K = \frac{1}{R_c^2} = \frac{1}{R_p^2} = \frac{1}{r^2}$. However, this is not a general relation, since according to (1.5) and (1.6), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point $O' \in \mathbf{M}^3$ is not the same as the radius of Gaussian curvature of the associated spherically symmetric geodesic surface, but is given by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c(r)$$
$$= \int_0^r \sqrt{B(R_c(r))}\, \frac{dR_c(r)}{dr} dr,$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \leq r < \infty$ in (1.4) that it must follow that in (1.5) and (1.6) $0 \leq R_c(r) < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from (1.5) and (1.6) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the centre of spherical symmetry to some other point in a metric manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the associated radius of Gaussian curvature and the radial distance from the centre of spherical symmetry, owing to the fact that R_p and R_c are identical in that special case.

It should also be noted that in writing expressions (1.4) and (1.5) it is implicit that $O \in \mathbf{E}^3$ is defined as being located at the origin of the coordinate system of (1.4), i.e. O is located where $r = 0$, and by correspondence O' is defined as being located at the origin of the coordinate system of (1.5) and of (1.6), $O' \in \mathbf{M}^3$ is located where $R_p = 0$. Furthermore, since it is well known that a geometry is completely determined by the form of the line element describing it [2], expressions (1.4), (1.5) and (1.6) share the very same fundamental geometry because they are line elements of the same form. Expression (1.6) plays an important rôle in Einstein's gravitational field.

The standard solution in the case of the static vacuum field (i.e. no deformation of the space) allegedly due to a single gravitating body, satisfying Einstein's field equations $R_{\mu\nu} = 0$, is (using $G = c = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1.8)$$

where m is allegedly the mass causing the field, and upon which it is routinely claimed that $2m < r < \infty$ is an exterior region and $0 < r < 2m$ is an interior region. Notwithstanding the inequalities it is routinely allowed that $r = 2m$ and $r = 0$ by which it is also routinely claimed that $r = 2m$ marks a "removable" or "coordinate" singularity and that $r = 0$ marks a "true" or "physical" singularity.

The standard treatment proceeds from simple inspection of (1.8) and the following unproven assumptions:

(a) that r is effectively the radial geodesic distance ($r = 2m$ is even routinely called the "Schwarzschild radius" or the "gravitational radius");

(b) that r can approach zero, even though the line- element (1.8) is singular at $r = 2m$;

(c) that only the first two components of the metric tensor (i.e. g_{00} and g_{11}) are influenced by the quantity $2m$.

With these unstated assumptions, but assumptions nonetheless, it is usual procedure to develop and treat of black holes. However, all three assumptions are demonstrably false at an elementary level.

II. Gaussian curvature

In the usual interpretation of Hilbert's [3, 4, 5] version of Schwarzschild's solution, the quantity r therein has ***never*** been properly identified. The physicists have variously and vaguely called it "*the radius*" of a sphere [6, 7], the "*radius* *of a 2-sphere*" [8], the "*coordinate radius*"[9], the "*radial coordinate*" [10, 11], the "*radial space coordinate*" [12], the "*areal radius*" [9, 13], the "*reduced circumference*" [14], and even "*a gauge choice: it defines the coordinate r*" [15]. In the particular case of $r = 2m = 2GM/c^2$ it is invariably referred to by the physicists as the "*Schwarzschild radius*" or the "*gravitational radius*". However, the irrefutable geometrical fact is that r, in the spatial section of Hilbert's version of the Schwarzschild/Droste line-element, is the radius of Gaussian curvature of a spherically symmetric geodesic surface in the spatial section [1, 16, 17], and as such it ***does not*** in fact determine the geodesic radial distance from the centre of spherical symmetry located at an arbitrary point in the related pseudo-Riemannian metric manifold. It does not in fact directly determine any distance at all in the spherically symmetric metric manifold. It is the radius of Gaussian curvature merely by virtue of its formal geometric relationship to the Gaussian curvature. It must also be emphasized that a geometry is completely determined by the ***form*** of its line-element, a fact that the physicists, with few exceptions [2], have not realised.

It immediately follows from the invalidity of $Ric = 0$ that Einstein's conceptions of the conservation and localisation of gravitational energy are erroneous and that the current search for Einstein's gravitational waves is ill-conceived. Also, the concepts of black holes and their interactions are ill-conceived and the two-body problem has been neither correctly formulated nor solved by means of the General Theory of Relativity.

Recall that Hilbert's corruption of Schwarzschild's solution, erroneously called "Schwarzschild's solution", is (using $c = G = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{2.1}$$

wherein r can, by assumption (i.e. without any proof), in some way or another, go down to zero, and m is allegedly the mass causing the gravitational field. Schwarzschild's [18] actual solution, for comparision, is

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{2.2}$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty,$$

$$\alpha = const.$$

Note that (2.2) is singular only when $r = 0$ (in which case the metric does not actually apply), and that the constant α is indeterminable (Schwarzschild did not assign any value to the constant α for this reason).

For a 2-D spherically symmetric geometric surface [19] determined by

$$ds^2 = R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2.3}$$

$$R_c = R_c(r),$$

the Riemannian curvature (which depends upon both position and direction) reduces to the Gaussian curvature K (which depends only upon position), given by [1, 20, 21, 22, 23],

$$K = \frac{R_{1212}}{g},$$

where $R_{ijkm} = g_{in} R^{n}_{\cdot jkm}$ is the Riemann tensor of the first kind and $g = g_{11} g_{22} = g_{\theta\theta} g_{\varphi\varphi}$ (because the metric tensor is diagonal). Recall that

$$R^{1}_{\cdot 212} = \frac{\partial \Gamma^{1}_{22}}{\partial x^1} - \frac{\partial \Gamma^{1}_{21}}{\partial x^2} + \Gamma^{k}_{22} \Gamma^{1}{}_{k1} - \Gamma^{k}{}_{21} \Gamma^{1}_{k2},$$

$$\Gamma^{\alpha}_{\alpha\beta} = \Gamma^{\alpha}_{\beta\alpha} = \frac{\partial}{\partial x^{\beta}} \left(\frac{1}{2} \ln |g_{\alpha\alpha}| \right),$$

$$\Gamma^{\alpha}_{\beta\beta} = -\frac{1}{2 g_{\alpha\alpha}} \frac{\partial g_{\beta\beta}}{\partial x^{\alpha}}, \quad (\alpha \neq \beta),$$

and all other $\Gamma^{\alpha}_{\beta\gamma}$ vanish. In the above, $k, \alpha, \beta = 1, 2,\ x^1 = \theta$ and $x^2 = \phi$, of course. Straightforward calculation gives for expression (2.3),

$$K = \frac{1}{R_c^2},$$

so that R_c is, in accordance with **Section I** above, the inverse square root of the Gaussian curvature, i. e. the radius of Gaussian curvature, and so r in Hilbert's "Schwarzschild's solution" is the radius of Gaussian curvature. The geodesic (i.e. proper) radius, R_p, of the spatial section of Schwarzschild's solution (2.2), up to a constant of integration, is given by

$$R_p = \int \frac{dR(r)}{\sqrt{1 - \frac{\alpha}{R(r)}}}, \tag{2.4}$$

and for Hilbert's "Schwarzschild's solution" (2.1), by

$$R_p = \int \frac{dr}{\sqrt{1 - \frac{2m}{r}}}.$$

Thus the proper radius and the radius of Gaussian curvature ***are not the same***. The radius of Gaussian curvature does not determine the geodesic radial distance from the arbitrary point at the centre of spherical symmetry of the metric manifold. It is a "radius" only in the sense of it being the inverse square root of the Gaussian curvature.

Note that in (2.2), if $\alpha = 0$ Minkowski space is recovered:

$$ds^2 = dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right),$$

$$0 \leq r < \infty.$$

In this case the radius of Gaussian curvature is r and the proper radius is

$$R_p = \int_0^r dr = r,$$

so that the radius of Gaussian curvature and the proper radius are identical. It is for this reason that in the spacetime of Minkowski the radius of Gaussian curvature of the spherically symmetric geodesic surface in the spatial section can be substituted for the proper radius (i.e. the geodesic radius). However, in the case of a (pseudo-) Riemannian manifold, such as (2.1) and (2.2) above, only the great circumference and the surface area can be directly determined via the radius of Gaussian curvature. Distances from the arbitrary point at the centre of spherical symmetry to a geodesic spherical surface in a Riemannian metric manifold can only be determined via the proper radius, except for particular points (if any) in the manifold where the radius of Gaussian curvature and the geodesic radius happen to be identical, and volumes by a triple integral involving a function of the radius of Gaussian curvature. In the case of Schwarzschild's solution (2.2) (and hence also for (2.1)), the radius of Gaussian curvature, $R_c = R(r)$, and the proper radius, R_p, are identical only at $R_c \approx 1.467\alpha$. When the radius of Gaussian curvature, R_c, is greater than $\approx 1.467\alpha$, $R_p > R_c$, and when the radius of Gaussian curvature is less than $\approx 1.467\alpha$, $R_p < R_c$.

The upper and lower bounds on the Gaussian curvature (and hence on the radius of Gaussian curvature) are not arbitrary, but are determined by the proper radius in accordance with the intrinsic geometric structure of the line-element (which completely determines the geometry), manifest in the integral (2.4). Thus, one cannot merely assume, as the physicists have done, that the radius of Gaussian curvature for (2.1) and (2.2) can vary from zero to infinity. Indeed, in the case of (2.2) (and hence also of (2.1)), as R_p varies from zero to infinity, the Gaussian curvature varies from $1/\alpha^2$ to zero and so the radius of Gaussian curvature correspondingly varies from α to infinity, as easily determined by evaluation of the constant of integration associated with the indefinite integral (2.4). Moreover, in the same way, it is easily shown that expressions (2.1) and (2.2) can be generalised [17] to all real values, but one, of the variable r, so that both (2.1) and (2.2) are particular cases of the general radius of Gaussian curvature, given by

$$R_c = R_c(r) = \left(\left|r - r_0\right|^n + \alpha^n\right)^{\frac{1}{n}}, \tag{2.5}$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_0,$$

wherein r_0 and n are entirely arbitrary constants. Choosing $n = 3$, $r_0 = 0$ and $r > r_0$ yields Schwarzschild's solution (2.2). Choosing $n = 1$, $r_0 = \alpha$ and $r > r_0$ yields line-element (2.1) as determined by Johannes Droste [24] in May 1916, independently of Schwarzschild. Choosing $n = 1$, $r_0 = \alpha$ and $r < r_0$ gives $R_c = 2\alpha - r$, with line-element

$$ds^2 = \left(1 - \frac{\alpha}{2\alpha - r}\right) dt^2 - \left(1 - \frac{\alpha}{2\alpha - r}\right)^{-1} dr^2 - (2\alpha - r)^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

Using relations (2.5) directly, all real values of $r \neq r_0$ are permitted. In any case, however, the related line-element is singular only at the arbitrary parametric point $r = r_0$ on the real line (or half real line, as the case may be), which is the only parametric point on the real line (or half real line, as the case may be) at which the line-element fails (at $R_p(r_0) = 0 \; \forall \; r_0 \; \forall \; n$). Indeed, substituting (2.5) for $R(r)$ in (2.4), and evaluating the constant of integration gives

$$R_p = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right),$$

where $R_c = R_c(r)$ is given by (2.5). Note that in the Standard Model interpretation of (2.1), only g_{00} and g_{11} are modified by the presence of the constant m. However, according to (2.2) and (2.5) ***all*** the components of the metric tensor are modified by the constant α, and since (2.1) is a particular case of (2.5), all the components of the metric tensor of (2.1) are modified by the constant α as well. There is no possibility for the alleged "event horizon" that is claimed to characterise a black hole.

The Kruskal-Szekeres coordinates do not take into account the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of the Schwarzschild manifold. These coordinates thereby violate the geometric form of the line-element, producing a completely separate pseudo-Riemannain manifold that does not form part of the solution space of the Schwarzschild manifold [25], and are consequently invalid. The concept of the Black Hole is therefore invalid.

III. The prohibition of point-mass singularities

The black hole is alleged to contain an infinitely dense singularity. The cosmological singularity of the alleged Big Bang cosmology is, according to many proponents of the Big Bang, also infinitely dense. Yet according to Special Relativity, infinite densities are forbidden because their existence implies that a material object can acquire the speed of light c in vacuo (or equivalently, the existence of infinite energies), thereby violating the very basis of Special Relativity. Since General Relativity cannot violate Special Relativity, General Relativity must thereby also forbid infinite densities. Point-mass singularities are alleged to be infinitely dense objects. Therefore, point-mass singularities are forbidden by the Theory of Relativity.

Let a cuboid rest-mass m_0 have sides of length L_0. Let m_0 have a relative speed $v < c$ in the direction of one of three mutually orthogonal Cartesian axes attached to an observer of rest-mass M_0. According to the observer M_0, the moving mass m is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{3.1}$$

and the volume V thereof is

$$V = L_0^3 \sqrt{1 - \frac{v^2}{c^2}}. \tag{3.2}$$

Thus, the density D is

$$D = \frac{m}{V} = \frac{m_0}{L_0^3 \left(1 - \frac{v^2}{c^2}\right)}, \tag{3.3}$$

and so $v \to c \;\Rightarrow\; D \to \infty$. Since, according to the fundamental postulate of Special Relativity, no material object can acquire the speed c (this would require an infinite energy), infinite densities are forbidden by Special Relativity, and so point-mass singularities are forbidden. Since General Relativity cannot violate Special Relativity, it too must thereby forbid infinite densities and hence forbid point-mass singularities [3, 17, 18]. Point-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

It is nowadays routinely claimed that many black holes have been found. The signatures of the black hole are (a) an infinitely dense 'point-mass' singularity and (b) an event horizon. Nobody has ever found an infinitely dense 'point-mass' singularity and nobody has ever found an event horizon, so nobody has ever assuredly found a black hole. It takes an infinite amount of observer time to verify a black hole event horizon. Nobody has been around and nobody will be around for an infinite amount of time and so no observer can ever verify the presence of an event horizon, and hence a black hole, in principle, and so the notion is irrelevant to physics. All reports of black holes being found are patently false.

IV. *Ric* = 0 is inadmissible

According to Einstein [26], his 'Principle of Equivalence' (equivalence of gravitational and inertial mass) requires that Special Relativity manifest in any freely falling inertial frame located in a sufficiently small region of the gravitational field. Now Special Relativity permits the presence of arbitrarily large (but not infinite) masses in spacetime, which are subject to the mass dilation relation (expression (3.1) above; and hence also to expressions (3.2) and (3.3) as well), and the definition of a relativistic inertial frame requires the *a priori* presence of two masses; the mass of the observer and the mass of the observed (to define relative motion of material bodies). In addition, at any instant the masses defining the freely falling inertial frame (and hence any other masses present therein) can have a speed up to but not including the speed of light in vacuo, by the action of the gravitational field. However, $Ric = R_{\mu\nu} = 0$ precludes, by definition, the presence of any masses and energies in the gravitational field because the energy-momentum tensor $T_{\mu\nu} = 0$ by hypothesis. Therefore, Special Relativity cannot manifest in any "freely falling" inertial frame in the spacetime of $R_{\mu\nu} = 0$. Indeed, a "freely falling" inertial frame cannot even be present since its very definition requires the presence of two masses which are, at any instant, subject to mass dilation under the action of the gravitational field. Similarly the equivalence of gravitational and inertial mass cannot manifest in the absence of matter in the gravitational field. Thus, $R_{\mu\nu} = 0$ violates Einstein's 'Principle of Equivalence' and is therefore inadmissible – it does not describe Einstein's gravitational field. Matter can only be introduced into Einstein's gravitational field via the energy-momentum tensor since it alone is what specifies that which physically causes the curvature of spacetime (i.e. the gravitational field). Clearly, the standard *post hoc* introduction of matter as the physical cause of spacetime curvature, into the so-called "Schwarzschild solution" for $R_{\mu\nu} = 0$, violates the requirements of Einstein's theory because the energy-momentum tensor is set to zero in that case.

V. Gravitational energy cannot be localised

Since $R_{\mu\nu} = 0$ does not describe Einstein's gravitational field, the energy-momentum tensor can never be zero (i.e. if $T_{\mu\nu} = 0$ there is no gravitational field), so Einstein's field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

must be written as [20, 27, 28]

$$\frac{1}{\kappa}G_{\mu\nu} + T_{\mu\nu} = 0, \tag{5.1}$$

wherein the $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Thus, $G_{\mu\nu}/\kappa$ and $T_{\mu\nu}$ *vanish identically*; the total energy is always zero; there is no localisation of gravitational energy (i.e. there are no Einstein gravitational waves). The current international search for Einstein's gravitational waves is destined to detect nothing. Furthermore, according to expression (5.1), Einstein's General Theory of Relativity violates the experimentally established usual conservation of energy and momentum. Thus, if the usual conservation of energy and momentum is valid (there is no experimental data to suggest otherwise), then Einstein's General Theory of Relativity is invalid, and hence the FRW line-element and Big Bang Cosmology are meaningless phantasms.

It is of interest to note that Einstein's pseudo-tensor is frequently utilised as a basis for the localisation of gravitational energy [2, 11, 20, 26, 29, 30]. From the foregoing it is evident that this cannot be correct. This is reaffirmed by the fact that Einstein's pseudo-tensor is mathematically (and hence also physically) meaningless, because it implies the existence of an invariant that has no mathematical existence [28]. Indeed, Einstein's pseudo-tensor, $\sqrt{-g}\, t^{\mu}_{\nu}$, is defined as [2, 11, 20, 26, 28, 29, 30],

$$\sqrt{-g}\, t^{\mu}_{\nu} = \frac{1}{2}\left(\delta^{\mu}_{\nu} L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\nu}\right)$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^{\gamma}_{\alpha\kappa}\Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta}\Gamma^{\kappa}_{\gamma\kappa}\right).$$

Contracting the pseudo-tensor and applying Euler's theorem yields,

$$\sqrt{-g}\, t^{\mu}_{\mu} = L,$$

which is a 1st-order intrinsic differential invariant that depends only upon the components of the metric tensor and their 1st derivatives. However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita [31] proved in 1900 that such invariants ***do not exist***. Consequently, everything built upon Einstein's pseudo-tensor is invalid. Eddington's [30] other objections to the pseudo-tensor are therefore quite well-founded.

Similarly, linearisation of Einstein's field equatins are routinely used to localise his gravitational energy and to obtain a Newtonian approximation of the potential of the gravitational field. But Einstein's field equations cannot be linearised because linearisation implies the existence of a tensor that, except for the trivial case of being precisely zero, ***does not otherwise exist***, as proven by Hermann Weyl [32] in 1944.

Since it has already been proven elsewhere [33] that the so-called "cosmological constant" must be precisely zero, expression (5.1) can contain no other terms.

VI. The two-body problem

Einstein's field equations are non-linear, so the 'Principle of Superposition' cannot apply [23]. Therefore, before one can talk of relativistic binary systems it must first be proven that the two-body system is theoretically well-defined by General Relativity. This can be done in only two ways:

(a) Derivation of an exact solution to Einstein's field equations for the two-body configuration of matter; or

(b) Proof of an existence theorem.

There are no known solutions to Einstein's field equations for the interaction of two (or more) masses, so option (a) has never been fulfilled. No existence theorem has ever been proven, by which Einstein's field equations even admit of latent solutions for such configurations of matter, and so option (b) has never been fulfilled. The "Schwarzschild" black hole is allegedly obtained from a line-element satisfying $Ric = 0$. Ignoring for the moment that $Ric = 0$ violates Einstein's 'Principle of Equivalence', and, for the sake of argument, assuming that black holes are predicted by General Relativity as alleged in relation to expression (2.1), since $Ric = 0$ is a statement that there is no matter in the Universe, one cannot simply insert a second black hole into the spacetime of $Ric = 0$ of a given black hole so that the resulting two black holes (each obtained separately from $Ric = 0$) mutually interact in a mutual spacetime that ***by definition contains no matter***! One cannot simply assert by an analogy with Newton's theory that two black holes can be components of binary systems, collide or merge [34, 35], because the 'Principle of Superposition' does not apply in Einstein's theory. Moreover, General Relativity has to date been unable to account for the simple experimental fact that two fixed bodies will attract one another when released.

Thus, the concepts of black holes, black hole binaries, collisions and mergers are all invalid.

VII. Intrinsic gravitational fields

It is also claimed by the physicists that spacetimes can be intrinsically curved, i.e. that there are gravitational fields that have no material cause. An example is de Sitter's empty spherical Universe, based upon the following field equations [2, 30]:

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{7.1}$$

wherein λ is the so-called "cosmological constant". Now in the case of line-element (1.8) the field equations are:

$$R_{\mu\nu} = 0. \tag{7.2}$$

Curiously, the physicists claim on the one hand that (7.1) is devoid of matter and so has no material cause for the associated alleged gravitational field (i.e. the curvature of spacetime), because the energy-momentum tensor is zero there, yet on the other hand they also claim that (7.2) has a material cause, which they insert *post hoc*, even though the energy-momentum tensor is zero there as well. The interpretations by the physicists of the alleged gravitational fields associated with (7.1) and (7.2) are therefore contradictory. Furthermore, despite the assertions of the physicists for (7.1), there is no experimental evidence whatsoever to support the claim that a gravitational field can exist without a material cause.

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