NOTE: Below are my answers to Professor Paul Davies. He wrote to Professor John K. Webb about my work and his stupid remarks to Webb are in bold type. My replies are in normal text.

My initial reaction is one of skepticism and puzzlement. The textbook version of the Schwarzschild metric has been proved to be the unique spherically symmetric vacuum solution of Einstein's equations. It is not possible to have several contenders. Any metrics which purport to be spherically symmetric vacuum solutions must therefore either be equivalent to the textbook Schwarzschild solution and derivable from it by a simple coordinate transformation, or they are not solutions at all.

Recall Schwarzschild's actual solution:

(1) $$ds^2 = (1 - \alpha/R)dt^2 - (1 - \alpha/R)^{-1}dt^2 - R^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

$$\alpha = 2m, \qquad R = (r^3 + \alpha^3)^{1/3,} \quad 0 < r < \infty.$$

The textbook version is incorrect – that is my point. Obviously there cannot be more than one contender. All solutions must indeed be able to be obtained from Schwarzschild's solution by a transformation of coordinates. Hilbert's solution cannot be so obtained from (1). The Droste/Weyl solution can be so obtained. Satisfying the field equations and far-field flatness are not sufficient for fixing a spacetime. A boundary condition at r = 0 must be applied to fix the value of the metrical coefficient of the angular coordinates of the line-element and thereby, the spacetime. The value of this coefficient selects a particular spacetime from an infinite family of one-parameter, inequivalent spacetimes. In the case of the mass-point this coefficient reduces to α^2 when r = 0. This value is a scalar invariant. Schwarzschild's R^2 reduces to α^2. Any such coefficient function which does not reduce to α^2 at r = 0 is cannot render a solution to the mass-point problem. This condition is not known to the conventional analysis. Hilbert's solution violates this invariant condition. One cannot arbitrarily make this coefficient zero when r = 0. Schwarzschild applied this boundary condition and so obtained the only valid solution. The interior of Hilbert's metric is routinely described with non-static coordinates. This is a non-static solution to a static problem. Contra-hype. Birkhoff's theorem actually says nothing about the range on the radial coordinate. (1) can be easily shown to satisfy that theorem.

It would be tedious for me to check whether your student's (1.1) is a bona fide vacuum solution, but a superficial argument suggests it is not. In the small r limit, the solution reads ds^2 = (r^3/3alpha^3)dt^2 - (3alpah^3/r3)dr^2 etc. This r^3

behavior contrasts with the 1/r behaviour of the textbook Schwarzschild solution, and surely no coordinate transformation can convert the one to the other.

The coordinate radii are not important. What is important is that the proper distance from an event to the singularity goes to zero as the coordinate radius goes to the singularity. Coordinate radii are not well defined quantities. Droste's metric tensor is a transformation from Schwarzschild's, so of course their radial coordinates will behave differently.

I show now that Schwarzschild's solution satisfies the field equations. Take Schwarzschild's metric as

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)d\theta^2 - D(r,\theta)d\phi^2, \quad (2)$$

where

$$A(r) = 1 - \alpha/R,\ B(r) = [A(r)]^{-1}r^4/(r^3 + \alpha^3)^{4/3},\ C(r) = R^2,\ D(r,\theta) = R^2\sin^2\theta,$$

$$R = (r^3 + \alpha^3)^{1/3},\ \ 0 < r < \infty.$$

The metric tensor so denoted must satisfy the vacuum field equations. Therefore, when substituted into the expressions for the stress-energy tensor the metric tensor of Schwarzschild must yield zeros. The expressions for the stress-energy tensor, as derived long ago by Herbert Dingle (I have attached them below as an appendix), for a very general line-element where the coefficients are functions of ALL the coordinates, in the case of Schwarzschild's metric (2) above, reduce to:

$$-8\pi T^1{}_1 = -1/C + C'^2/4BC^2 + A'C'/2ABC,$$

$$-8\pi T^2{}_2 = C''/2BC + A''/2AB - C'^2/4BC^2 - B'C'/4B^2C - A'^2/4A^2B -$$

$$-A'B'/4AB^2 + A'C'/4ABC,$$

$$T^3{}_3 = T^2{}_2.$$

$$-8\pi T^4{}_4 = C''/BC - 1/C - B'C'/2B^2C - C'^2/4BC^2,$$

and $T^i{}_j = 0,\ i \neq j$. If you make the calculations yourself (see appendix) you will verify the above expressions; or just trust my calculation of them. I have not made any errors. If you make the additional calculations using these expressions and Schwarzschild's metric tensor from (2) you will verify that Schwarzschild's solution is 'a bona fide vacuum solution', as I have done myself.

Here are some details (the prime indicates differentiation with respect to r):

$$C' = 2r^2/(r^3 + \alpha^3)^{1/3} \qquad A' = \alpha r^2/(r^3 + \alpha^3)^{4/3}$$

$$-8\pi T^1{}_1 = -1/(r^3 + \alpha^3)^{2/3} + [4r^4/(r^3 + \alpha^3)^{2/3}]\times[1 - \alpha/(r^3 + \alpha^3)^{1/3}] \times(r^3 + \alpha^3)^{4/3}/[4r^4(r^3 + \alpha^3)^{4/3}]$$
$$+ \alpha r^2/(r^3 + \alpha^3)^{4/3} \times 2r^2/(r^3 + \alpha^3)^{1/3} \times (r^3 + \alpha^3)^{4/3}/[2r^4/(r^3 + \alpha^3)^{2/3}]$$

$$= -1/(r^3 + \alpha^3)^{2/3} + 1/(r^3 + \alpha^3)^{2/3} - \alpha/(r^3 + \alpha^3) + \alpha/(r^3 + \alpha^3)$$

$$= 0.$$

In similar fashion it can be verified that all the stress-energy tensor expressions reduce to 0. It is much easier to work with Brillouin's form of Schwarzschild's solution. In that case

$$A(r) = r/(r + \alpha),\ \ B(r) = (r + \alpha)/r,\ \ C(r) = (r + \alpha)^2,\ \ D(r,\theta) = (r + \alpha)^2\sin^2\theta.$$

This form simplifies the calculations. I used Schwarzschild's solution in the calculation above to prove it directly. Using Brillouin's form,

$$A' = \alpha/(r + \alpha)^2,\quad A'' = -2\alpha/(r + \alpha)^3,\quad C' = 2(r + \alpha),\quad C'' = 2,\quad B' = -\alpha/r,$$

$$-8\pi T^2{}_2 = 2r/[2(r + \alpha)^3] - 2\alpha r(r + \alpha)/[2r(r + \alpha)^4] – [4r(r + \alpha)^2]/[4(r + \alpha)^5] +$$

$$+ (\alpha/r^2)[2r^2(r + \alpha)]/[4(r + \alpha)^4] – [\alpha^2 r(r + \alpha)]/[4r^2(r + \alpha)^5] + \alpha^2 r/[4r^2(r + \alpha)^3] +$$

$$+ [\alpha/(r + \alpha)^2]\{2(r + \alpha)/[4(r + \alpha)^2]\}$$

$$= r/(r + \alpha)^3 - \alpha/(r + \alpha)^3 - r/(r + \alpha)^3 + \alpha/[2(r + \alpha)^3] - \alpha^2/[4r(r + \alpha)^3] + \alpha^2/[4r(r + \alpha)^3] +$$

$$+ \alpha/(r + \alpha)^3$$

$$= 0.$$

$$-8\pi T^4{}_4 = 2r/(r + \alpha)^3 – 1/(r + \alpha)^2 + (\alpha/r^2)[2r^2(r + \alpha)]/[2(r + \alpha)^4] – 4r(r + \alpha)^2/[4(r + \alpha)^5]$$

$$= 2r/(r + \alpha)^3 - 1/(r + \alpha)^2 + \alpha/(r + \alpha)^3 - r/(r + \alpha)^3$$

$$= (2r – r - \alpha + \alpha -r) /(r + \alpha)^3$$

$$= 0.$$

Clearly, as I have said, satisfying the field equations is not sufficient to fix a valid solution. Consequently, additional conditions must be met for the fixing of a spacetime. Hilbert's metric does not meet them all and is invalid. The conventional analysis is ignorant of all the necessary conditions. Schwarzschild's solution meets them all and is therefore the only valid solution to the problem. Here is a metric that satisfies the field equations, has a Ricci curvature of 0, is far-field flat, and meets all the standard requirements for a solution:

(3) $$ds^2 = [1 - \alpha/(r-\alpha)]dt^2 - [1 - \alpha/(r-\alpha)]^{-1}dr^2 - (r-\alpha)^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

$$0 < r < \infty.$$

This metric is singular at $r = \alpha$ and at $r = 2\alpha$, but nowhere else. Two horizons? Where is the source of the field? It can be obtained from Hilbert's solution in the same way Hilbert's can be obtained from Schwarzschild's solution - by a simple but erroneous transformation.

So I conclude that either (i) the student has misinterpreted Schwarzschild's paper or, (ii) Schwarzschild made a mistake. I think (i) is more likely, particularly as the student appears to have a hidden agenda, i.e. to prove that black holes do not exist. (And I am bound to ask, what additional "mistakes" invalidate the Reissner-Nordstrom and Kerr solutions, and why can't one recover Schwarzschild as limits from these?)

I have correctly interpreted Schwarzschild's paper. Schwarzschild made no mistakes. His paper is a beautiful piece of mathematical physics. The Reissner-Nordstrom and Kerr solutions reduce to Hilbert's metric for certain values of their parameters. Since Hilbert's metric is incorrect the Reissner-Nordstrom and Kerr metrics are also incorrect. I can provide a detailed demonstration if necessary. I have no hidden agenda. The science goes where it goes. One must draw consistent conclusions from the analysis. The analysis logically discredits the black hole.

I shall now make very plain the error made by Hilbert. Consider again the general metric for the point-mass:

(a) $$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)[d\theta^2 + \sin^2\theta\, d\phi^2],$$

$$A,B,C, > 0 \quad \text{for } r > 0, \qquad 0 < r < \infty.$$

A,B,C are all unknown functions, therefore their forms cannot be pre-empted arbitrarily. Also all one can say about at this stage is ,

(b) $$r \to 0 \quad \Rightarrow \quad A(r) \to A(0), \quad B(r) \to B(0), \quad C(r) \to C(0).$$

These limits cannot be pre-empted either. Do now as Hilbert did, set

(c) $$r^* = \sqrt{C(r)}.$$

Then by (b) it necessarily follows that the lower bound on r^* is,

(d) $$r_o^* = \sqrt{C(0)}.$$

One cannot know the value of $\sqrt{C(0)}$ at this stage. One still doesn't know what $C(r)$ is. It must be somehow determined. Hilbert unfortunately immediately dropped the * on r in (c) converting (a) into

(e) $$ds^2 = M(r)dt^2 - N(r)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

$$0 < r < \infty,$$

from which he obtained his well-known solution. By dropping the * in (c) he effectively set $C(r) = r^2$, which is arbitrary, and took $0 < r < \infty$ directly from (a) into (e) and finally into his solution, in violation of (d). One can see from (c) that the r in (e) is not the same r in (a). Just keep the * on r through the transformation to make this even clearer. The rest of Hilbert is deduction with an incorrect transformation jammed up in the works, resulting in an incorrect metric. It really is that simple. The error is high school level, but it has been carried through since 1916, rather astonishingly.

You should simply ask the student to calculate the curvature scalar R using (1.1) and prove it is zero. I bet it isn't.

Lucky you did not put money on this one. The Ricci curvature is zero.

To simplify the calculations I use Brillouin's form of Schwarzschild's solution. The components of the metric tensor are

$$g_{00} = r/(r+\alpha), \qquad g_{11} = -(r+\alpha)/r, \qquad g_{22} = -(r+\alpha)^2, \qquad g_{33} = -(r+\alpha)^2\sin^2\theta\,, \tag{a}$$

and

$$\sqrt{|g|} = (r+\alpha)^2\sin\theta. \tag{b}$$

The non-zero Christoffel symbols of the second kind are:

$$\Gamma^0_{01} = \alpha/[2r(r+\alpha)] \qquad \Gamma^1_{11} = -\alpha/[2r(r+\alpha)] \qquad \Gamma^2_{21} = 1/(r+\alpha)$$

$$\Gamma^3_{31} = 1/(r+\alpha) \qquad \Gamma^3_{32} = \cot\theta \qquad \Gamma^1_{00} = \alpha r/[2(r+\alpha)^3]$$

$$\Gamma^1_{22} = -r \qquad \Gamma^1_{33} = -r\sin^2\theta \qquad \Gamma^2_{33} = -\sin\theta\cos\theta \tag{c}$$

The Ricci curvature is given by

$$R = g^{\mu\nu}\{\partial^2/\partial x^\mu \partial x^\nu (\ln\sqrt{|g|}) - [\partial/\partial x^\rho (\sqrt{|g|}\, \Gamma^\rho_{\mu\nu})]/\sqrt{|g|} + \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\rho\nu}\}. \tag{d}$$

Now if you put (a), (b) and (c) into (d) you get

$$R = 0;$$

as I have verified. The calculation is:

$R = (r+\alpha)/r\ \{[-1/[(r+\alpha)^2\sin\theta] \times \partial/\partial r[(r+\alpha)^2\sin\theta\ \Gamma^1{}_{00}] + 2\Gamma^0{}_{01}\ \Gamma^1{}_{00}\ \} -$

- $r/(r+\alpha)\ \{\partial^2/\partial r^2 \ln (r+\alpha)^2\sin\theta - 1/[(r+\alpha)^2\sin\theta] \times \partial/\partial r[(r+\alpha)^2\sin\theta\ \Gamma^1{}_{11}] + \Gamma^0{}_{10}\ \Gamma^0{}_{01}$

$+ \Gamma^1{}_{11}\ \Gamma^1{}_{11} + \Gamma^2{}_{12}\ \Gamma^2{}_{21} + \Gamma^3{}_{13}\ \Gamma^3{}_{31}\} -$

$-1/(r+\alpha)^2\ \{\partial^2/\partial\theta^2 \ln (r+\alpha)^2\sin\theta - 1/[(r+\alpha)^2\sin\theta] \times \partial/\partial r[(r+\alpha)^2\sin\theta\ \Gamma^1{}_{22}] + 2\Gamma^2{}_{12}\ \Gamma^1{}_{22} + \Gamma^3{}_{23}\ \Gamma^3{}_{32}\ \}$

- $1/[(r+\alpha)^2\sin^2\theta]\ \{[-1/[(r+\alpha)^2\sin\theta] \times (\partial/\partial r\ [(r+\alpha)^2\sin\theta\ \Gamma^1{}_{33}] + \partial/\partial\theta\ [(r+\alpha)^2\sin\theta\ \Gamma^2{}_{33}]\ )$

$+\ \Gamma^1{}_{33}\Gamma^3{}_{13}\ +\ \Gamma^2{}_{33}\ \Gamma^3{}_{23}\ +\ \Gamma^3{}_{31}\ \Gamma^1{}_{33}\ +\ \Gamma^3{}_{32}\ \Gamma^2{}_{33}\ \}$

$= (r+\alpha)/r\ \{[-1/[(r+\alpha)^2\sin\theta] \times [½\alpha\sin\theta][\alpha/(r+\alpha)^2] + 2\alpha^2 r/[4r(r+\alpha)^4]\} -$

- $r/(r+\alpha)\ \{-2/(r+\alpha)^2 - 1/[(r+\alpha)^2\sin\theta][-½\alpha(r-r-\alpha)\sin\theta\ /r^2] + \alpha^2/[2r^2\ (r+\alpha)^2] + 2/(r+\alpha)^2\} -$

- $1/(r+\alpha)^2\ \{-\csc^2\theta\ + \sin\theta/[(r+\alpha)^2\sin\theta\ ]\ [(r+\alpha)^2 + 2r(r+\alpha)] - r/(r+\alpha) - r/(r+\alpha) + \cot^2\theta\} -$

- $1/[(r+\alpha)^2\sin^2\theta]\ \{\sin^2\theta + 2r^2\sin^2\theta\ /(r+\alpha)\ + 2\cos^2\theta - \sin^2\theta\ - 2r\sin^2\theta\ /(r+\alpha) - 2\cos^2\theta\}$

$= 0 + 0 + 0 + 0 = 0.$

If you want, you can verify the whole calculation; otherwise you can trust me - the result is correct.

Remark

I have now reworked my analysis in general terms, i.e. without reference to any coordinate system in particular. I have obtained correct and complete solutions for the point-mass, the rotating point-mass, the point-charge and the rotating point-charge. The paper that was sent to you was preliminary, and rather vagarious. Not so with my latest writing. I would be happily disposed to discuss matters with you, if you are interested. It would be both interesting and rare. Most seem to want to keep their heads in the sand and tell me to go away, mostly in rather unflattering terms.

Appendix I

Dingle's Equations

Dingle's generalized metric is

$$ds^2 = -A(dx^1)^2 - B(dx^2)^2 - C(dx^3)^2 + D(dx^4)^2,$$

where A, B, C, and D are positive quantities to give a spacetime signature of –2, and can be functions of all the coordinates. The metric does not require spherical symmetry. The components of the generalized energy-momentum tensor are then given by:

$$-8\pi T^1{}_1 = ½[(\partial^2 B/\partial(x^3)^2 + \partial^2 C/\partial(x^2)^2)/BC - (\partial^2 B/\partial(x^4)^2 -$$

$$- \partial^2 D/\partial(x^2)^2)/BD - (\partial^2 C/\partial(x^4)^2 - \partial^2 D/\partial(x^3)^2)/CD)] -$$

$$- ¼\{ [\partial B/\partial x^3 \; \partial C/\partial x^3 + (\partial C/\partial x^2)^2]/BC^2 + [\partial C/\partial x^2 \; \partial B/\partial x^2 +$$

$$+ (\partial B/\partial x^3)^2]/B^2C - [\partial B/\partial x^4 \; \partial D/\partial x^4 - (\partial D/\partial x^2)^2]/BD^2 +$$

$$+ [\partial D/\partial x^2 \; \partial B/\partial x^2 - (\partial B/\partial x^4)^2]/B^2D - [\partial C/\partial x^4 \; \partial D/\partial x^4 -$$

$$- (\partial D/\partial x^3)^2]/CD^2 + [\partial D/\partial x^3 \; \partial C/\partial x^3 - (\partial C/\partial x^4)^2]/DC^2 -$$

$$- [\partial C/\partial x^2 \; \partial D/\partial x^2 + \partial B/\partial x^3 \; \partial D/\partial x^3 - \partial B/\partial x^4 \; \partial C/\partial x^4]/BCD -$$

$$-(\partial B/\partial x^1 \; \partial C/\partial x^1)/ABC - (\partial B/\partial x^1 \; \partial D/\partial x^1)/ABD -$$

$$- (\partial C/\partial x^1 \; \partial D/\partial x^1)/ACD \} + \Lambda$$

$$-8\pi T^2{}_2 = ½[(\partial^2 A/\partial(x^3)^2 + \partial^2 C/\partial(x^1)^2)/AC - (\partial^2 A/\partial(x^4)^2 -$$

$$- \partial^2 D/\partial(x^1)^2)/AD - (\partial^2 C/\partial(x^4)^2 - \partial^2 D/\partial(x^3)^2)/CD)] -$$

$$- ¼\{ [\partial A/\partial x^3 \; \partial C/\partial x^3 + (\partial C/\partial x^1)^2]/AC^2 + [\partial C/\partial x^1 \; \partial A/\partial x^1 +$$

$$+ (\partial A/\partial x^3)^2]/A^2C - [\partial A/\partial x^4 \; \partial D/\partial x^4 - (\partial D/\partial x^1)^2]/AD^2 +$$

$$+ [\partial D/\partial x^1 \; \partial A/\partial x^1 - (\partial A/\partial x^4)^2]/A^2D - [\partial C/\partial x^4 \; \partial D/\partial x^4 -$$

$$- (\partial D/\partial x^3)^2]/CD^2 + [\partial D/\partial x^3 \; \partial C/\partial x^3 - (\partial C/\partial x^4)^2]/C^2D-$$

$$- [\partial C/\partial x^1 \; \partial D/\partial x^1 + \partial A/\partial x^3 \; \partial D/\partial x^3 - \partial A/\partial x^4 \; \partial C/\partial x^4]/ACD -$$

$$-(\partial A/\partial x^2 \; \partial C/\partial x^2)/ABC - (\partial A/\partial x^2 \; \partial D/\partial x^2)/ABD -$$

$$- (\partial C/\partial x^2 \; \partial D/\partial x^2)/BCD \} + \Lambda$$

$$-8\pi T^3{}_3 = ½[(\partial^2 A/\partial(x^2)^2 + \partial^2 B/\partial(x^1)^2)/AB - (\partial^2 A/\partial(x^4)^2 -$$

$$- \partial^2 D/\partial(x^1)^2)/AD - (\partial^2 B/\partial(x^4)^2 - \partial^2 D/\partial(x^2)^2)/BD)] -$$

$$- ¼\{ [\partial A/\partial x^2\ \partial B/\partial x^2 + (\partial B/\partial x^1)^2]/AB^2 + [\partial B/\partial x^1\ \partial A/\partial x^1 +$$

$$+ (\partial A/\partial x^2)^2]/A^2B - [\partial A/\partial x^4\ \partial D/\partial x^4 - (\partial D/\partial x^1)^2]/AD^2 +$$

$$+ [\partial D/\partial x^1\ \partial A/\partial x^1 - (\partial A/\partial x^4)^2]/A^2D - [\partial B/\partial x^4\ \partial D/\partial x^4 -$$

$$- (\partial D/\partial x^2)^2]/BD^2 + [\partial D/\partial x^2\ \partial B/\partial x^2 - (\partial B/\partial x^4)^2]/B^2D-$$

$$- \quad [\partial B/\partial x^1\ \partial D/\partial x^1 + \partial A/\partial x^2\ \partial D/\partial x^2 - \partial A/\partial x^4\ \partial B/\partial x^4]/ABD -$$

$$-(\partial A/\partial x^3\ \partial B/\partial x^3)/ABC - (\partial A/\partial x^3\ \partial D/\partial x^3)/ACD -$$

$$- (\partial B/\partial x^3\ \partial D/\partial x^3)/BCD \} + \Lambda$$

$$-8\pi T^4{}_4 = ½[(\partial^2 A/\partial(x^2)^2 + \partial^2 B/\partial(x^1)^2)/AB + (\partial^2 A/\partial(x^3)^2 +$$

$$+ \partial^2 C/\partial(x^1)^2)/AC + (\partial^2 B/\partial(x^3)^2 + \partial^2 C/\partial(x^2)^2)/BC)] -$$

$$-¼\{[\partial A/\partial x^2\ \partial B/\partial x^2 + (\partial B/\partial x^1)^2]/AB^2 + [\partial B/\partial x^1\ \partial A/\partial x^1 +$$

$$+ (\partial A/\partial x^2)^2]/A^2B + [\partial A/\partial x^3\ \partial C/\partial x^3 + (\partial C/\partial x^1)^2]/AC^2 +$$

$$+ [\partial C/\partial x^1\ \partial A/\partial x^1 + (\partial A/\partial x^3)^2]/A^2C + [\partial B/\partial x^3\ \partial C/\partial x^3 +$$

$$+ (\partial C/\partial x^2)^2]/BC^2 + [\partial C/\partial x^2\ \partial B/\partial x^2 + (\partial B/\partial x^3)^2]/B^2C +$$

$$+ [\partial B/\partial x^1\ \partial C/\partial x^1 + \partial A/\partial x^2\ \partial C/\partial x^2 + \partial A/\partial x^3\ \partial B/\partial x^3]/ABC +$$

$$+ (\partial A/\partial x^4\ \partial B/\partial x^4)/ABD + (\partial A/\partial x^4\ \partial C/\partial x^4)/ACD +$$

$$+ (\partial B/\partial x^4\ \partial C/\partial x^4)/BCD \} + \Lambda$$

$$-8\pi AT^1{}_2 = -8\pi BT^2{}_1 = - ½[(\partial^2 C/\partial x^1\partial x^2)/C + (\partial^2 D/\partial x^1\partial x^2)/D] +$$

$$+ ¼[(\partial C/\partial x^1\ \partial C/\partial x^2)/C^2 + (\partial D/\partial x^1\ \partial D/\partial x^2)/D^2 +$$

$$+ (\partial A/\partial x^2\ \partial C/\partial x^1)/AC + (\partial A/\partial x^2\ \partial D/\partial x^1)/AD +$$

$$+ (\partial B/\partial x^1\ \partial C/\partial x^2)/BC + (\partial B/\partial x^1\ \partial D/\partial x^2)/BD]$$

$$-8\pi A T^1{}_3 = -8\pi C T^3{}_1 = -\tfrac{1}{2}[(\partial^2 B/\partial x^1 \partial x^3)/B + (\partial^2 D/\partial x^1 \partial x^3)/D] +$$

$$+\tfrac{1}{4}[(\partial B/\partial x^1\ \partial B/\partial x^3)/B^2 + (\partial D/\partial x^1\ \partial D/\partial x^3)/D^2 +$$

$$+ (\partial A/\partial x^3\ \partial B/\partial x^1)/AB + (\partial A/\partial x^3\ \partial D/\partial x^1)/AD +$$

$$+ (\partial C/\partial x^1\ \partial B/\partial x^3)/BC + (\partial C/\partial x^1\ \partial D/\partial x^3)/CD]$$

$$-8\pi B T^2{}_3 = -8\pi C T^3{}_2 = -\tfrac{1}{2}[(\partial^2 A/\partial x^2 \partial x^3)/A + (\partial^2 D/\partial x^2 \partial x^3)/D] +$$

$$+ \tfrac{1}{4}[(\partial A/\partial x^2\ \partial A/\partial x^3)/A^2 + (\partial D/\partial x^2\ \partial D/\partial x^3)/D^2 +$$

$$+ (\partial A/\partial x^2\ \partial B/\partial x^3)/AB + (\partial A/\partial x^3\ \partial C/\partial x^2)/AC +$$

$$+ (\partial D/\partial x^2\ \partial B/\partial x^3)/BD + (\partial C/\partial x^2\ \partial D/\partial x^3)/CD]$$

$$-8\pi A T^1{}_4 = 8\pi D T^4{}_1 = -\tfrac{1}{2}[(\partial^2 B/\partial x^1 \partial x^4)/B + (\partial^2 C/\partial x^1 \partial x^4)/C] +$$

$$+ \tfrac{1}{4}[(\partial B/\partial x^1\ \partial B/\partial x^4)/B^2 + (\partial C/\partial x^1\ \partial C/\partial x^4)/C^2 +$$

$$+ (\partial A/\partial x^4\ \partial B/\partial x^1)/AB + (\partial A/\partial x^4\ \partial C/\partial x^1)/AC +$$

$$+ (\partial D/\partial x^1\ \partial B/\partial x^4)/BD + (\partial C/\partial x^4\ \partial D/\partial x^1)/CD]$$

$$-8\pi B T^2{}_4 = 8\pi D T^4{}_2 = -\tfrac{1}{2}[(\partial^2 A/\partial x^2 \partial x^4)/A + (\partial^2 C/\partial x^2 \partial x^4)/C] +$$

$$+ \tfrac{1}{4}[(\partial A/\partial x^2\ \partial A/\partial x^4)/A^2 + (\partial C/\partial x^2\ \partial C/\partial x^4)/C^2 +$$

$$+ (\partial A/\partial x^2\ \partial B/\partial x^4)/AB + (\partial A/\partial x^4\ \partial D/\partial x^2)/AD +$$

$$+ (\partial C/\partial x^2\ \partial B/\partial x^4)/BC + (\partial D/\partial x^2\ \partial C/\partial x^4)/CD]$$

$$-8\pi C T^3{}_4 = 8\pi D T^4{}_3 = -\tfrac{1}{2}[(\partial^2 A/\partial x^3 \partial x^4)/A + (\partial^2 B/\partial x^3 \partial x^4)/B] +$$

$$+ \tfrac{1}{4}[(\partial A/\partial x^3\ \partial A/\partial x^4)/A^2 + (\partial B/\partial x^3\ \partial B/\partial x^4)/B^2 +$$

$$+ (\partial A/\partial x^3\ \partial C/\partial x^4)/AC + (\partial A/\partial x^4\ \partial D/\partial x^3)/AD +$$

$$+ (\partial B/\partial x^3 \, \partial C/\partial x^4)/BC + (\partial B/\partial x^4 \, \partial D/\partial x^3)/BD]$$