From: GWB ¡g.w.bruhn@t-online.de¿
To: Dmitri Rabounski ¡rabounski@ptep-online.com¿
Date: Thu, 06 Mar 2008 17:10:53 +0100
Subject: Crothers' papers in PP
Dear Dr.Rabounski,

please, find attached a html version of my planned paper on "Crothers' views on black holes". Mr. Crothers receives this email too for his information and a possible response, as I hope. I'm going to transfom the paper to Latex and submit it to your Jounal.

Concerning my objections to Crothers views on black holes in GRT I had some discussion with colleagues. They tell me that they don't share my feelings of handling Crothers with gentleness. They say that he has caused his problems himself by not listening to advice he was given. They say that he, since being 50, is responsible for himself, at most the referees of his papers could be hold responsible in some cases for not having better (i.e. more critically) advised their author.

So, whatsoever, please, find attached, my "Discussion of Crothers' views on black holes", where is shown how he goes astray. A typical example you'll find in my Section 4 which mainly deals with Crothers' criticism of a reduction of an isotrope static metric: Here Crothers condemns "the relativists and mathematicians" as a whole for "evidently having failed to understand elementary geometrical facts". Perhaps he should realize the number of scientists who are not willing to follow his dubious views. At this point it was HIS error, and I should recommend him, to be more cautious and not so rash when having found a new 'result' which contradicts well-known results.

Best regards
Gerhard W. Bruhn

PS The paper is posted at
http://www.mathematik.tu-darmstadt.de/ bruhn/CrothersViews.html
and will be updated there in case of changes.

Discussion of S. Crothers' Views on Black Hole Analysis in GRT

Gerhard W. Bruhn, Darmstadt University of Technology

Quotations from Crothers' papers are displayed in **black**. Equation labels of type (n) refer to Crothers' papers.

Abstract
In the last years since 2005 S. Crothers has published a series of papers in the Journal PROGRESS IN PHYSICS (see [3]) which deal with the alleged fact that black holes are not compatible with General Relativity. Crothers views stem from certain dubious ideas on spacetime manifolds, especially in the case of Hilbert/Schwarzschild metrics: His idea is that instead of the 2-sphere of the event horizon there is merely *one single central* point. It will be shown below that this assumption would lead to a curious world where Crothers' "central point" can be approximated in sense of distance by 2-spheres S_r of radius $r > \alpha$. Hence the event horizon cannot be a single point.

1. Crothers' basic views

Crothers bases his objection of Schwarzschild black holes on two statements: He asserts in the Introduction of [1]:

When the required mathematical rigour is applied it is revealed that
1) **$r_o = \alpha$ denotes a point, not a 2-sphere,** and that
2) **$0 < r < \alpha$ is undefined on the Hilbert metric.**

1.1 Objections to claim 1)

We consider the Schwarzschild/Hilbert metric

(1.1) $$ds^2 = -(1 - \alpha/r)\, dt^2 + (1 - \alpha/r)^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

in the spacetime that is accessible for a physical observer, i.e. for $r > \alpha$: Here the metric (1.1) defines sub-manifolds S_r for each pair of fixed values of t and r, the metric of which follows from (1.1) to be

(1.2) $$ds^2 = r^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2)\ .$$

Hence S_r is a 2-sphere with radius r. The set S_α of singularities of the Schwarzschild/Hilbert metric has the metric

(1.3) $$ds^2 = \alpha^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2).$$

and hence is a 2-sphere as well.

The distance between S_r and S_α is given by Crothers' "proper radius" (cf [1, eq. (14)] with $C(r)= r^2$)

(1.4) $$R_p(r) = [r(r-\alpha)]^{1/2} + \alpha \ln\, |(r^{1/2}+(r-\alpha)^{1/2})\, \alpha^{-1/2}|$$

measurable in radial direction between arbitrary associated points of the concentric spheres. Since $R_p(r)$ is continuous at $r=\alpha$ the distance between S_r and S_α tends to 0 for $r \to \alpha$:

(1.5) $$\lim_{r \to \alpha} R_p(r) = R_p(\alpha) = 0\ .$$

Therefore, the set S_α of the metric singularities can be approximated with respect to the distance $R_p(r)$ by concentric 2-spheres of radius $r > \alpha$: Thus,

S_α cannot be a single point.

1.2 Objections to claim 2)

This is not true. The Schwarzschild metric is an admissible metric for $0 < r < \alpha$ as well. Its signature is (+,−,+,+). The variables t and r have exchanged their roles: r has become *timelike* while t is *spacelike* now.

Of course, that region is not accessible for human observers. We can only try to extrapolate the rules that have been found in the accessible part of the world. The method applied here is that of analytic extension which e.g. leads to the Kruskal-Szekeres metric that covers both validity regions of the Schwarzschild metric.

2. Somewhat elementary differential geometry

We shall determine here a subset of the event horizon to show again that it cannot be only one central point:

The metric of an equatorial section $\theta = \pi/2$ through an Euclidean space parametrized by spherical polar coordinates (r, θ, φ)

(2.1) $$ds^2 = dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \quad \Rightarrow \quad ds^2 = dr^2 + r^2 \, d\varphi^2 \, .$$

yields a *plane* with polar coordinates (r, φ), while $\theta = \pi/2$.

A similar equatorial section for the Schwarzschild metric at constant time variable t yields the metric

(2.2) $$ds^2 = (1 - \alpha/r)^{-1} \, dr^2 + r^2 \, d\varphi^2$$

which is no longer plane, i.e. no longer representable in a plane, say z=0. However, instead of the plane z=0 we can define a surface $z = z(r,\varphi)$ over a plane with polar coordinates (r,φ). Due to the spherical symmetry z cannot depend on φ, hence we have to consider a rotational surface $z = z(r)$: The metric of this surface is given by

(2.3) $$ds^2 = (1 + z_r^2) \, dr^2 + r^2 \, d\varphi^2 \, .$$

Comparison with the metric (2.2) yields $z_r = (\alpha/r-\alpha)^{1/2}$, hence

(2.4) $$z(r) = [\alpha(r-\alpha)]^{1/2} \, .$$

This is a rotational surface generated by rotating the parabola $z = [\alpha(r-\alpha)]^{1/2}$ around the z-axis, see the figure of that surface.

We see that $r < \alpha$ is impossible, and $z = 0$ for $r = \alpha$ is the (**red marked**) boundary of the *accessible* world, where $z > 0$.

The boundary (subset of the event horizon) is not a single point.

3. Further comments on Crothers paper [1]

Let us compare the metric usually attributed to Schwarzschild

$$ds^{*2} = (1 - \alpha/r^*)\, dt^2 - (1 - \alpha/r^*)^{-1}\, dr^{*2} - r^{*2}\,(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (6)$$

with Crothers' "new" metric:

$$ds^2 = (C^{½}-\alpha/C^{½})\, dt^2 - (C^{½}/C^{½}-\alpha)\, C'^2/4C dr^2 - C\,(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (7)$$

This metric has a certain blemish: the differential dr can be removed, such that the variable r is completely substituted by the new variable C using C'dr = dC, hence

(3.1) $$ds^2 = (C^{½}-\alpha/C^{½})\, dt^2 - (C^{½}/C^{½}-\alpha)\, 1/4C dC^2 - C\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

What Crothers did not mention in his papers [1] and {2]:
Both metrics, defined by the eqs.(6) and (7)/(3.1) are *equivalent*, i.e. the associated manifolds are *identical*, merely represented by *different* coordinates (t,r*,θ,φ) and (t,C,θ,φ) repectively, associated by the coordinate transform
(3.2) $C = C(r^*) = r^{*2}$ **and** $r^* = r^*(C) = C^{½}$.

So normally there is no reason for considering other than the STANDARD form (6) of the Schwarzschild metric. Other equivalent forms may be of historical interest merely. Crothers' question of correct naming of the different versions of equivalent metrics has become obsolete nowadays. For more see Section 4.

From the coefficients g_{oo} of the metrics (7) and (6) respectively it can be seen directly that the metric (7) becomes singular at $C^{½} = \alpha$, while the metric (6) becomes singular at $r^* = \alpha$.

Crothers defines a value r_o by the equation $C(r_o) = \alpha^2$. From $C(r^*) = r^{*2}$ we obtain $r_o = \alpha$: While the metric (7) is singular at $C = C(r_o) = \alpha^2$ the equivalent metric (6) has its corresponding singularity at $r = r_o = \alpha$.

Crothers doesn't like both coordinates, neither r* nor C; he is interested in a *radial* coordinate with an evident *geometrical* meaning. Therefore he introduces a new variable, a "proper radius" R_p by *radial* integration of the line element ds of (7) (dt=0, dθ=0, dφ=0) starting from the singularity, which after some calculations yields

$$R_p(C) = [C^{½}\,(C^{½}-\alpha)]^{½} + \alpha \ln\,|(C^{¼}+(C^{½}-\alpha)^{½})\,\alpha^{-½}| \qquad (14)$$

The same result would have been attained by radial integration of the line element ds* of (6) starting at its singularity r* = α:

(3.3) $$R_p^*(r^*) = [r^*(r^*-\alpha)]^{½} + \alpha \ln\,|(r^{*½}+(r^*-\alpha)^{½})\,\alpha^{-½}|$$

where $r^* = C^{½}$. We then have $R_p^*(r^*) = R_p(r^{*2})$.

Conclusion The use of the metric (7)/(3.1) instead of the technically simpler Schwarzschild metric (6) is an *unnecessary* complication which cannot yield new results exceeding those attained by use of the Schwarzschild metric.

4. The reasons of Crothers' misunderstandings

Crothers' problems with the analysis of GRT are mainly caused by his misconceptions concerning the role of coordinates. In his paper [2] we read:

The black hole, which arises solely from an incorrect analysis of the Hilbert solution, is based upon a misunderstanding of the significance of the coordinate radius r. This quantity is neither a coordinate nor a radius in the gravitational field and cannot of itself be used directly to determine features of the field from its metric. The appropriate quantities on the metric for the gravitational field are the proper radius and the curvature radius, both of which are functions of r. The variable r is actually a Euclidean parameter which is mapped to non-Euclidean quantities describing the gravitational field, namely, the proper radius and the curvature radius.

Crothers expects a geometrical meaning always being attached to a coordinate. He insinuates that the coordinate r, known from spherical polar coordinates as *radial distance* from the center, should maintain its meaning when appearing in another context, e.g. as the parameter r of the Schwarzschild metric. In [2, Sect.2] we read about an isotropic generalization of the Minkowski line element:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\ , \qquad A,B,C > 0\ , \qquad (2a)$$

where A,B,C are analytic functions. I emphatically remark that *the geometric relations between the components of the metric tensor of (2a) are precisely the same as those of (1).* The standard analysis writes (2a) as,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\ , \qquad (2b)$$

and claims it the most general, which is incorrect. The form of C(r) cannot be pre-empted ...

This renaming method is *somewhat lax* but often used in mathematics, though it could be misunderstood if taken literally: The setting $C := r^2$ means that a *new* meaning is assigned to the variable r. Since r already occurs in eq.(2a), it would be *better* to use a *new* symbol, say r^*, not r, for the new variable: $r^{*2} := C(r)$. As a consequence the terms $A(r)dt^2$ and $B(r)dr^2$ must be rewritten as functions of the new variable r^* by introducing new cofficients $A^*(r^*):=A(r)$ and $B^*(r^*):=B(r)(dr/dr^*)^2$. This yields

$$ds^2 = A^*(r^*)dt^2 - B^*(r^*)dr^{*2} - r^{*2}\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\ , \qquad (2b^*)$$

Then, all *s are removed to obtain

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\ , \qquad (2b)$$

To repeat it: **The terms A, B, r in (2a) and (2b) respectively have different meanings, here precisely specified.** However, the rewriting (2a) as (2b) is legal and justified herewith.

Without loss of generality the coefficient C(r) in eq. (2a) can be assumed as $C(r)=r^2$.

References

[1] S. Crothers, *On the General Solution to Einstein's Vacuum Field and its Implications for Relativistic Degeneracy.* , PROGRESS IN PHYSICS Vol. 1 , April 2005
http://www.ptep-online.com/index_files/2005/PP-01-09.PDF

[2] S. Crothers, *On the Geometry of the General Solution for the Vacuum Field of the Point-Mass,* , PROGRESS IN PHYSICS Vol. 2 , July 2005
http://www.ptep-online.com/index_files/2005/PP-02-01.PDF

[3] S. Crothers, *The Published Papers of Stephen J. Crothers*,
http://www.geocities.com/theometria/papers.html

[4] S.M. Carroll, *Lecture Notes on General Relativity*, http://xxx.lanl.gov/pdf/gr-qc/9712019

Pseudo-Science? Inbox | X

from GWB <g.w.bruhn@t-online.de> hide details Mar 8
to Crothers <thenarmis@yahoo.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Sat, Mar 8, 2008 at 7:11 PM
subject Pseudo-Science?

Dear Mr. Crothers,

recently you complained to be considered as a pseudo-scientist by me.
I derived my opinion from your positioning to Evans' "theories"
which are full with elementary errors which you have agreed to.

Now I've read some of your papers and find my opinion confirmed.
I have offered (private if you prefer) fair discussion on your views
on black holes etc., without any response from you until now.

No response is a response as well.

Regards

G.W. Bruhn

PS Please, find attached a figure where you see a visualization
of a subset of singularities of the Schwarzschild metric. With
the conclusion that this set is **not a single point.**
More on my website
http://www.mathematik.tu-darmstadt.de/~bruhn/CrothersViews.html

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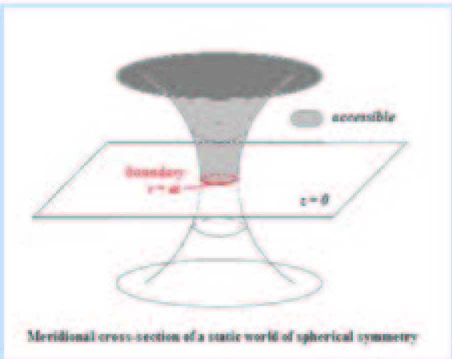

M-Section1.bmp
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Reply Reply to all Forward

from Stephen Crothers <thenarmis@gmail.com> hide details Mar 8
to GWB <g.w.bruhn@t-online.de>,
Dmitri Rabounski <rabounski@ptep-online.com>,
bcc corbis@bigpond.net.au,
date Sat, Mar 8, 2008 at 8:40 PM
subject Re: Pseudo-Science?
mailed-by gmail.com

Prof. G. W. Bruhn,
Sir,

What really motivates you write to me and call me a pseudo-scientist and otherwise insult me? You previously slandered me on your website, for wish I wrote to you. Also, you have never previously invited me to discussion about my work. In addition, you will note that I have written nothing on ECE theory whatsoever. I have only contributed an appendix, by invitation, to a paper by Prof. M. W. Evans and Dr. H. Eckardt, wherein I dealt only with my own work, not ECE theory. Produce evidence of my knowledge of and agreement with ECE theory to support your allegations

Attached is my response to the paper you sent to Progress in Physics, bearing in mind that I received it only about two days ago. I have spend considerable time preparing this response. You have been a bit hasty in issuing your latest insults, so perhaps I was unwise to take your paper seriously in the first place.

If you intend to harass and insult me by email I suggest that you don't bother, as I will not respond to you and will simply block your email. If however you wish to genuinely discuss my work then I will do so. But your track record does not augur well, as far as I can see. It's entirely up to you.

Crothers.

- Show quoted text -

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Reply Reply to all Forward

from Stephen Crothers <thenarmis@gmail.com> hide details Mar 8
to GWB <g.w.bruhn@t-online.de>,
Dmitri Rabounski <rabounski@ptep-online.com>,
bcc Axel Westrenius <corbis@bigpond.net.au>,
Dani Indranu <wings.of.solitude@gmail.com>,
date Sat, Mar 8, 2008 at 8:52 PM
subject Re: Pseudo-Science?
mailed-by gmail.com

Prof. G. W. Bruhn,
Sir,

I just noticed a corruption in the LaTeX file for my response to your paper, from which the pdf was generated. The corrected pdf file is attached. If you plan to discuss things with me (or alternatively attempt to abuse and insult me) then I prefer that you use the intended document.

Crothers.

GWBruhn.pdf
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Reply Reply to all Forward

from GWB <g.w.bruhn@t-online.de> hide details Mar 9
to Stephen Crothers <thenarmis@gmail.com>,
date Sun, Mar 9, 2008 at 9:50 AM
subject Re: Pseudo-Science?
mailed-by t-online.de

Dear Mr Crothers,

Confirmation: I received 2 emails from you sent on Mar 8, 2008. Both pdf files are readable.

My motivation to discuss with you: To find out the truth about your claims. Mathematics
was invented to do so.

History: In the past, when I became aware of your problems with GRT I've sent you two emails offering discussions. You never replied.

Pseudo-science: You are a member of a clearly pseudo-scientific organization: AIAS . See
http://www.aias.us/index.php?goto=showPageByTitle&pageTitle=AIAS_staff
S. Crothers, M.Astronomy (University Western Sydney), Non-Executive Director of the AIAS so don't complain to be considered as a pseudo-scientist - otherwise you have the freedom to criticize some of AIAS claims at least or leave that organization. **That' s the state of today**; perhaps things will change by discussion (as I hope).

Slander(?) on my website concerning you: **Where?** I have once mentioned your name by quoting from Evans' blogsite. And the article on your BH views:
Scientific criticism is no slander!
Remember what you wrote about reputable scientists.

My proposal again: Let's start a fair discussion, without polemics or insinuations.
You must accept that different people have different opinions. That's no reason
to get upset. It's a reason for attempting to decide who is right / which is the truth.

All mathematical errors have to be eliminated, then, at the end we'll see the truth.

It may take me a few days to reply to your attachment *completely*.
Please find attached the beginning. Wait with your comments until I
have read and replied to your text completely, probably on Monday.

Regards

Gerhard W. Bruhn

Discussion Crothers ./. Bruhn, 08.03.2008

Contributions by Crothers in

black, by Bruhn in **blue**.

1.

In the abstract to your paper you assert that I have assumed that there is “merely one single central point”. This is in fact not correct. I assume nothing, because the presence of a single central point is due precisely to the nature of the spherically symmetric metric manifold of a Schwarzschild space.

Then you should read the text I quoted *literally* from your first paper [1]:

"When the required mathematical rigour is applied it is revealed that $r_o = \alpha$ denotes a point, not a 2-sphere"

And I add here from my paper:
The metric of this set S_α of singularities of the metric has the metric

$$ds^2 = \alpha^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)$$

This is the metric of a 2-sphere S_α with radius α.

2.

I shall first amplify upon what is and what is not Schwarzschild’s solution. Here is the metric that is always called “Schwarzschild’s solution” by the proponents of the black hole (using G = c = 1):

$$ds^2 = (1 - {}^{2m}/_{r})\, dt^2 - (1 - {}^{2m}/_{r})^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (1)$$

wherein m is allegedly the mass causing the gravitational field, and wherein r can, by assumption (i.e. without proof), in some way or another, go down to zero. Schwarzschild’s [1] actual solution, for comparison, is

$$ds^2 = (1 - {}^{\alpha}/_{R})\, dt^2 - (1 - {}^{\alpha}/_{R})^{-1}\, dR^2 - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (2)$$

$$R = R(r) = (r^3 + \alpha^3)^{1/3},\ 0 < r < \infty,\ \alpha = \text{const.}$$

Note that (2) is singular only when r =0 (in which case the metric does not actually apply). Schwarzschild did not set α=2m.

Whatever Schwarzschild did is not relevant:
If WE do set $\alpha=2m$ then the metrics given by the eqs. (1) and (2) describe the same manifold. Whether we write r or R is irrelevant for the manifold.

Of course, the substitution $R = R(r) = (r^3 + \alpha^3)^{1/3}$ does NOT transform (2) → (1). Trivially the correct transform is $R(r) = r$.

3.

Comparing (1) to (2), the claim that (1) is valid down to $r = 0$ therein would require that in (2) $R = 0$ by which $r = -\alpha$.

However, the transform $R = R(r) = (r^3 + \alpha^3)^{1/3}$ has nothing to do with the transition from (2) to (1). And in case R=r (and $\alpha=2m$ everything is fine. Where is the problem???

To be continued on MONDAY.

Discussion continued

Inbox X

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GWB Dear Mr. Crothers, please, find attached the continuation of the discussion. ... Mar 12

GWB Loading... Mar 12

GWB to Crothers show details Mar 12 Reply

from **GWB** <g.w.bruhn@t-online.de> hide details Mar 12
to Crothers <thenarmis@yahoo.com>,
date Wed, Mar 12, 2008 at 12:14 AM Reply
subject Discussion continued

Dear Mr. Crothers,

please, find attached the continuation of the discussion.

I have worked through your text and stopped when
I came across a serious problem
contained in your calculations (see No.11).

I think, it would be important to find an escape.
So, please, first have a look at No.11.

Regards
Gerhard W. Bruhn

Discussion Crothers ./. Bruhn, 08.03.2008

Contributions by Crothers in

black, by Bruhn in **blue**.

1.

In the abstract to your paper you assert that I have assumed that there is “merely one single central point”. This is in fact not correct. I assume nothing, because the presence of a single central point is due precisely to the nature of the spherically symmetric metric manifold of a Schwarzschild space.

Then you should read the text I quoted *literally* from your first paper [1]:

"When the required mathematical rigour is applied it is revealed that $r_o = \alpha$ denotes a point, not a 2-sphere"

And I add here from my paper:
The metric of this set S_α of singularities of the Schwarzschild metric has the metric

$$ds^2 = \alpha^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)$$

This is the metric of a 2-sphere S_α with radius α as is well known from differential geometry.

2.

I shall first amplify upon what is and what is not Schwarzschild’s solution. Here is the metric that is always called “Schwarzschild’s solution” by the proponents of the black hole (using G = c = 1):

$$ds^2 = (1 - {}^{2m}/_r)\, dt^2 - (1 - {}^{2m}/_r)^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (1)$$

wherein m is allegedly the mass causing the gravitational field, and wherein r can, by assumption (i.e. without proof), in some way or another, go down to zero. Schwarzschild’s [1] actual solution, for comparison, is

$$ds^2 = (1 - {}^{\alpha}/_R)\, dt^2 - (1 - {}^{\alpha}/_R)^{-1}\, dR^2 - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (2)$$

$$R = R(r) = (r^3 + \alpha^3)^{1/3},\ 0 < r < \infty,\ \alpha = \text{const.}$$

Note that (2) is singular only when r =0 (in which case the metric does not actually apply). Schwarzschild did not set α=2m.

Whatever Schwarzschild did is not relevant:
If WE do set α=2m then the metrics given by the eqs. (1) and (2) describe the same manifold. Whether we write r or R is irrelevant for the manifold.

Of course, the substitution $R = R(r) = (r^3 + \alpha^3)^{1/3}$ does NOT transform (2) → (1). Trivially the correct transform is R(r) = r.

3.

Comparing (1) to (2), the claim that (1) is valid down to r = 0 therein would require that in (2) R = 0 by which r = − α.

However, who does assert that? The metric (1) has TWO different and *disjoint* regions of validity. Coming from outside the **border** of the corresponding region is at r=α. Coordinates have borders of validity in general, see e.g. S.M. Carroll's Lecture Notes on GRT, Chap.2 .

4.

With the foregoing preamble I now address the contents of your paper.

In Section 1.1 of your paper you object to **my claim that r = α denotes a point.**

In your paper [[1]] you wrote that r = α denotes a point. Please, answer the question:

"When the required mathematical rigour is applied it is revealed that r_o = α denotes a point, not a 2-sphere"

Did you write that or not? YES or NO?

If YES, then my objection is justified and correct.

If NO, then explain how that sentence came into your paper [[1]]. And tell me what else S_r and especially S_α are according to your opinion.

See **No.11** also.

5.

You refer to the quantity r in your line-element (1.1) repeatedly as a "radius". Indeed, you repeatedly assert that it is the "radius" of a 2-sphere. In the usual interpretation of Hilbert's corruption of "Schwarzschild's solution" [3, 4, 5], the quantity r therein **has never been properly identified. You do not rightly identify it either.**

That's your *claim* merely. From **differential geometry it is well known** that

$$ds^2 = r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \qquad (r>0 \text{ fixed})$$

yields the line element ds of a sphere S_r of radius r in spherical polar coordinates r,θ,φ. S_r is a *sub-manifold* of the spacetime manifold which belongs to

$$ds^2 = -(1 - {}^{\alpha}/_{r})\, dt^2 + (1 - {}^{\alpha}/_{r})^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2) \qquad (2')$$

to be obtained by setting here dt = dr = 0.

Remark I prefer (2') instead of (2) (with the opposite sign), since (2') yields *real* space-like distances.

6.

In addition to your assertion that it is the "radius" of a 2-sphere, it has been variously called, by the proponents of the black hole, " the radius" [6, 7] of a sphere, the "coordinate radius"[8] or "radial coordinate" [9, 10] or "radial space coordinate" [11], the "areal radius" [8, 12], the "reduced circumference" [13], and a Nobel Laureate has even called it a "a gauge choice, which defines r" [14]. In the particular case of $r = 2GM/c^2$ it is invariably referred to as the "Schwarzschild radius" or the "gravitational radius". However, the irrefutable geometrical fact is that r, in Hilbert's version of the Schwarzschild/Droste line-element, is the radius of Gaussian curvature [15, 16, 17, 18], and as such it does not in fact determine the geodesic radial distance from the centre of spherical symmetry located at an arbitrary point in the related metric manifold.

Indeed, it does not in fact directly determine any distance at all in a spherically symmetric Riemannian metric manifold. It is the radius of Gaussian curvature merely by virtue of its formal geometric relationship to the Gaussian curvature. It must also be emphasized that a geometry is completely determined by the form of its line-element [19]. In other words, proofs must come from the line-element via the intrinsic geometrical relations between the components of the metric tensor specified by the line-element and associated boundary conditions.

For a 2-dimensional spherically symmetric geometric surface described by

$$ds^2 = R_c^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2), \qquad (3)$$

$$R_c = R_c(r),$$

the Riemannian curvature (which depends upon both position and direction) reduces to the Gaussian curvature K (which is independent of direction), given by [15, 16, 20, 21, 22, 23],

$$K = R_{1212}/g\ ,$$

where $R_{ijkm} = g_{in}R^n{}_{jkm}$ is the Riemann tensor of the first kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$. Recall that

$$R^1{}_{212} = \partial\Gamma^1{}_{22}/\partial x^1 - \partial\Gamma^1{}_{21}/\partial x^2 + \Gamma^k{}_{22}\Gamma^1{}_{k1} - \Gamma^k{}_{21}\Gamma^1{}_{k2}$$

$$\Gamma^\alpha{}_{\alpha\beta} = \Gamma^\alpha{}_{\beta\alpha} = \partial/\partial x^\beta\,(\tfrac{1}{2}\ln|g|)$$

$$\Gamma^\alpha{}_{\beta\beta} = -\,1/2g_{\alpha\alpha}\ \partial g_{\beta\beta}/\partial x^\alpha\ ,\ (\alpha \neq \beta)\ ,$$

and all other $\Gamma^\alpha{}_{\beta\gamma}$ vanish. In the above, k,α,β = 1,2, $x^1 = \theta$ and $x^2 = \varphi$, of course. Straightforward calculation gives for expression (3),

$$K = 1/R_c^2,$$

so that R_c is the inverse square root of the Gaussian curvature, i. e. the radius of Gaussian curvature, and so r in Hilbert's “Schwarzschild's solution” is the radius of Gaussian curvature.

OK so far. **I think here we have arrived at an important point to think over again:**

Given a spacetime manifold by its metric (2) or (2'). Then we consider a sub-manifold S_{Rc} by fixing the values of t and r, the latter by $r{:=}R_c$. And your result means that this sub-manifold S_r has constant Gauss curvature $1/r^2$, i.e.

The sub-manifold $S_r = S_{Rc}$ is a sphere of radius $r = R_c$.

This holds especially if we consider the singularity set S_α of the metric (2'), where r=α:

S_α is a sphere with radius r=α.

See **No.11** also.

7.

The geodesic (i.e. proper) radius, R_p, of Schwarzschild's solution, up to a constant of integration, is given by

$$R_p = \int (1-{}^{\alpha}/_{R(r)})^{-1}\, dR(r) + A, \qquad (4)$$

where A is a constant, and for Hilbert's "Schwarzschild's solution", by

$$R_p = \int (1-{}^{\alpha}/_{r})^{-1}\, dr + B, \qquad (4')$$

where B is a constant. Thus the proper radius and the radius of Gaussian curvature are not the same; for the above, in general, $R_p \neq R(r)$ and $R_p \neq r$ respectively.

Of course! I don't know anyone of your opponents who asserted the contrary, i.e. $R_p = R(r)$ or $R_p = r$. Your remark indicates a *misunderstanding* that we should try to remove here.

It's a good opportunity to discuss the influence of a parameter change: You will have realized that both integrals are somehow related. If you have calculated one of the integrals, say (4'), then the other integral, (4), can be evaluated by a mere substitution, by R=R(r). From the *lax* notation (4)/(4'), especially, when the depandancy (r) is removed in (4'), there seems to be no (essential) difference. (4) can be written as

$$R_p = \int (1-{}^{\alpha}/_{R})^{-1}\, dR + A\,.$$

To visualize that let's use a more precise notation instead of (4)/(4'):

$$R_p[a,b] = \int_a^b (1-{}^{\alpha}/_{r})^{-1}\, dr = \int_a^b (1-{}^{\alpha}/_{\rho})^{-1}\, d\rho\,.$$

Then the integral (4) in correct notation would be

$$R_p[R(a),R(b)] = \int_{R(a)}^{R(b)} (1-{}^{\alpha}/_{\rho})^{-1}\, d\rho\,, \qquad (4'')$$

The name of the integration variable (I used the neutral letter ρ here on purpose) is *without influence on the resulting value of the integral*. Instead of ρ you can use whatever you like; however, avoid the danger of a mix-up of notations!

Let's apply this to the calculation of R_p in your paper [[1]]:

Let *first* the line element ds be given by (2'), fixed in time and in radial direction (i.e. θ and φ fixed), hence from (2') $ds^2 = (1-{}^{\alpha}/_{r})\, dr^2$ and by integration of ds between the parameter values r=α and r=r*. Then we obtain

$$R_p[\alpha,r^*] = \int_\alpha^{r^*} (1-{}^\alpha/_r)^{-1}\, dr = [\ (r(r-\alpha))^{½} + \alpha \ln |(r^{½}+(r-\alpha)^{½})|\ ]_\alpha^{r^*}$$

. (4''')

Let's secondly assume that the nobelist XYZ felt attracted by the metric given by

$$ds^2 = -\ (1 - {}^{f(R_o)}/_{f(R)})\ dt^2 + (1 - {}^{f(R_o)}/_{f(R)})^{-1}\ (f\ '(R))^2\ dR^2 + (f(R))^2\ (d\theta^2 + \sin^2\theta\ d\varphi^2)$$

where $\alpha=f(R_o)$ and $r^*=f(R^*)$. So XYZ had to integrate the line element of radial and time-constant direction between $R_o=R(\alpha)$ and $R^*=R(r^*)$, i.e. he had to evaluate the integral

$$R_p = \int_{Ro}^{R^*} (1-{}^{f(R_o)}/_{f(R)})^{-1}\ f\ '(R)\ dR$$

yielding the result

$$R_p = [\ (f(R)(f(R)-f(R_o)))^{½} + f(R_o) \ln |(f(R)^{½}+(f(R)-f(R_o))^{½})|\ ]_{Ro}^{R^*}$$

. (4'''')

It can easily be seen that the results (4''') and (4'''') agree by using the relations $f(R_o)=\alpha$ and $f(R^*)=r^*$.

The reason for this accordance is the *parameter invariance* of the line element ds: In $ds^2 = g_{\alpha\beta}\ dx^\alpha\ dx^\beta$ the metric coefficients $g_{\alpha\beta}$ transform *contragrediently* to the differentials $dx^\alpha\ dx^\beta$.

It does matter which parameters are used for evaluating an invariant quantity like your geodesic radial distance R_p.

Please note for further use (**No.9**): Admissible transformations $f(R^*)=r^*$ are all C^∞-functions with f'>0. The only further condition is $f(R_o)=\alpha$. **No specializations like your eq. (5) are required.**

8.

The radius of Gaussian curvature does not determine the geodesic radial distance from the arbitrary point at the centre of spherical symmetry of the metric manifold [15, 16]. It is a “radius” only in the sense of it being the inverse square root of the Gaussian curvature [15, 16]. It is not a distance in the associated manifold.

Excuse me, but this is wrong. In your paper [[1]], eq.(14) with $C(r)=r^2$, you yourself have derived the formula for the geodesic radial distance, the "proper radius" R_p depending on the radius of Gaussian curvature r : All points of the spacetime manifold with radial parameter value r have the "proper radius"

$$R_p[\alpha,r] = \int_\alpha^r (1-{}^\alpha/_\rho)^{-1}\, d\rho = [\ (\rho(\rho-\alpha))^{½} + \alpha \ln |(\rho^{½}+(\rho-\alpha)^{½})|\ ]_\alpha^r\ .$$

$$= (r(r-\alpha))^{½} + \alpha \ln |(r^{½}+(r-\alpha)^{½})\ \alpha^{-½}|\ .$$

Your calculation of R_p is correct; I have checked it. However, apparently you have problems with the interpretation of your results.

9.

A detailed development of the foregoing, from first principles, is given in [15] and [16], from which you should be able to see that I have only carefully applied to Schwarzschild space the determinations of the pure mathematicians.

Believe me, my knowledge of math is sufficient to know all the traps math has ready for laymen. See one *simple* example below: You cannot identify the upper bound of an integral and its and its integration variable. This is as meaningless as if you write a sum $\Sigma_o^n\ n$. It's a *mixup* of notations (without consequences in *your* case).

Note that in (2) if **α**=0 Minkowski space is recovered:

$$ds^2 = dt^2 - dr^2 - r^2\,(d\theta^2 + \sin^2\theta d\varphi^2)\ , \qquad 0 \le r < \infty.$$

In this case the radius of Gaussian curvature is r and the proper radius is

$$R_p = \int_o^r d\rho = r,$$

so that the radius of Gaussian curvature and the proper radius are identical. It is for this reason that the radii, great circumferences, surface areas and volumes of spheres, etc., in Minkowski space can be determined in terms of the radius of Gaussian curvature. However, in the case of a (pseudo-) Riemannian manifold, such as expressions (1) and (2) above, only great circumferences and surface areas can be determined via the radius of Gaussian curvature. Distances from the centre of spherical symmetry to a geodesic spherical surface in the Riemannian metric manifold described by (1) or (2) can only be determined via the proper radius, except for particular points (if any) in the manifold where the radius of Gaussian curvature and the geodesic radius are identical, and volumes by a triple integral involving a function of the radius of Gaussian curvature. In the case of Schwarzschild's solution (2) (and hence also for (1)), the radius of Gaussian curvature, $R_c = R(r)$, and the proper radius, R_p, are identical only at $R_c \approx 1.467\ \alpha$.

When the radius of Gaussian curvature, R_c, is greater than ≈ 1.467 , $R_p > R_c$, and when the radius of Gaussian curvature is less than ≈ 1.467, $R_p < R_c$.

So what? You have seen above (No. 8.) that we have $R_p = R_p(r)$.
Besides: The Minkowski spherical line element is singular at r=α=0.

The upper and lower bounds on the Gaussian curvature (and hence on the radius of Gaussian curvature) are not arbitrary and so cannot be simply asserted by inspection, but are determined by the proper radius in accordance with the intrinsic geometric structure of the line-element (which completely determines the geometry [19]), manifest in the integral (4). Thus, one cannot merely assume that the radius (formally) of Gaussian curvature for (1) and (2) can vary from zero to infinity, as the proponents of the black hole have always done. Indeed, in the case of (2) (and hence also of (1)), as R_p varies from zero to infinity, the Gaussian curvature varies from $1/\alpha^2$ to zero and so the radius of Gaussian curvature correspondingly varies from α to infinity, as easily determined by evaluation of the constant of integration associated with the indefinite integral (4). Moreover, in the same way, it is easily shown that expressions (1) and (2) can be generalised [18] to all real values, but one, of the variable r, so that both (1) and (2) are particular cases of the general radius of Gaussian curvature, given by

$$R_c = R_c(r) = (|r - r_o|^n + \alpha^n)^{1/n} , \qquad (5)$$

$$r \in \mathbf{R},\ n \in \mathbf{R}^+,\ r \neq r_o,$$

wherein $r \neq r_o$ and n are entirely arbitrary constants. Choosing n=3, r_o =0 and r_o yields Schwarzschild's solution (2). Choosing n=1, $r \neq r_o = \alpha$ and $r > r_o$ yields line-element (1) as determined by Johannes Droste [2] in May 1916, independently of Schwarzschild. Choosing n=1, $r_o = \alpha$ and $r < r_o$ gives $R_c = 2\alpha - r$, with line-element

$$ds^2 = (1 - {}^{\alpha}/_{2\alpha-r})\, dt^2 - (1 - {}^{\alpha}/_{2\alpha-r})^{-1}\, dr^2 - (2\alpha-r)^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2) .$$

Using relations (5) directly, all real values of $r \neq r_o$ are permitted. In any case, however, the related line-element is singular only at the arbitrary parametric point $r=r_o$ on the real line (or half real line, as the case may be), which is the only parametric point on the real line (or half real line, as the case may be) at which the line-element fails (at the point $R_p(r_o)=0\ \forall\ r_o\ \forall\ n$). It is easily shown that the proper radius is given, via (4) and (5), by

$$R_p = R_p(r) = [R_c(r)(R_p(r)-\alpha)]^{1/2} + \alpha \ln |\, [R_c(r)]^{1/2} + [R_c(r) - \alpha]^{1/2}]\alpha^{-1/2} \,| .$$

All these calculations give different forms of **always the same**: different versions of the STANDARD form of the Schwarzschild metric which is given by (2) or (2'). That may be of *historical* interest, since it is shown that and how different authors came to different versions of the metric of a static isotropic spherical symmetric spacetime. However, it seems that for practical reasons under these *equivalent* metrics the STANDARD forms (2) or (2') are most preferable.

Note that $R_p(r_o)=0\ \forall\ r_o$ and $\forall$ n and that $R_c(r_o)= \alpha\ \forall\ r_o$ and $\forall$ n.

Expression (5) above determines an infinite number of equivalent metrics. The Schwarzschild/Droste line element (and Hilbert's corruption thereof) is just a particular case.

All these statements are quite obvious and no miracle, following trivially from the equivalence of the metrics. And a simpler formulation of admissible transformations is possible, see my remark at the end of **No.7**.

Expression (1) above appears in A. Eddington's book [24] as his expression (38.8). In section 43 of his book, Eddington developed his isotropic coordinates for Schwarzschild space, and remarked:

Owing to an identical relation between G11 G22 and G44, the vanishing of this tensor gives only two equations to determine the three unknowns, λ, μ, ν. There exists therefore an infinite series of particular solutions, differing according to the third equation between λ, μ, ν which is at our disposal. The two solutions hitherto considered are obtained by taking μ=0, and λ=ν, respectively. The same series of solutions is obtained in a simpler way by substituting arbitrary functions of r instead of r in (38.8).

Expression (5) above determines an infinite series of equivalent metrics. However, Eddington was too ambitious in asserting that arbitrary functions of r can be substituted into expression (1) above. Only the form given by (5) above is admissible. For example, substituting e^r in place of r in (1) above does not alter the spherical symmetry and does not violate Ric =0, but it is inadmissible - it does not satisfy Einstein's requirement that the line-element be asymptotically Minkowski.

This restriction of admissible coordinates is a *physical* or *geometrical* one. Surely the versions (2),(2') have practical advantages since then r has a *geometrical* meaning which shows that the Schwarzschild metric is asymptotically Minkowskian for large r. However, the substitution $C(r)= 4\pi r^2$ may have some other advantages, giving the surface area sizes of the concentrical spheres S_r. So, from *mathematical* point of view such restrictions are completely unnecessary.

There are no *physical* properties (i.e. invariants) which could not be represented by an arbitrary coordinate system which is *mathematically* admissible.

10.

I developed the corresponding expression to (5) above for the case of Eddington's isotropic coordinates in [25]. Thus, the accusation you make in section 3 of your paper is illegitimate - you incorrectly say:

What Crothers did not realize: Both metrics, defined by eqs.(6) and (7)/(3.1) are equivalent, i.e. the associated manifolds are identical, merely represented *by different coordinates (t, r*, θ, φ) and (t,C, θ, φ) respectively, associated by the coordinate transform (3.2) $C=C(r)=r^2$ and $r^*=r^*(C)=C^{½}$.*

At no time in any of my papers have I ever been guilty of this accusation. My papers clearly testify to this fact. Your accusation has no true basis - it is false.

May be or not: The equivalence of metrics is nowhere mentioned in your papers [[1]] and [[2]]. So I shall modify my text: What Crothers did nowhere mention ...

11.

It is clear from expression (5) above that the quantity r appearing in the infinite number of equivalent metrics determined by it is merely a parameter. It is also plain that since $|r-r_o|$ is a distance on the real line, the arbitrary quantity r_o is an arbitrary point on the real line, and it is associated with the arbitrary point in the Schwarzschild manifold denoted by $R_p(r_o) = 0 \ \forall r_o \ \forall n$, which is in turn associated with the Gaussian curvature $K = 1/\alpha^2$ (for which the radius of Gaussian curvature is $R_c(r_o)=\alpha$, irrespective of the choice of r_o and of n). The quantity $|r-r_o|$ is also easily related to 3-dimensional Euclidean space as the distance between a fixed point r_o and a variable point r on the common radial line through r_o, r and the origin r = 0 of the associated coordinate system. One does not have to locate r_o at the origin of the coordinate system (or at zero on the real line). The quantity $|r - r_o|$ merely reflects the simple geometrical shift of r_o to any point in the parametric space. The equation of a sphere of radius α, with centre C at the extremity of the vector $\mathbf{r}_o$, may be written [26]

$$[\mathbf{r} - \mathbf{r}_o] \bullet [\mathbf{r} - \mathbf{r}_o] = \rho^2,$$

and if $\mathbf{r}$ and $\mathbf{r}_o$ are collinear with the origin of the coordinate system, the vector notation can be dropped, so that $\rho = |r - r_o|$ without any real loss of generality.

Stop! That's a **flaw of thinking**, commited in 2005-2008, more than 100 years after the invention of vector calculus by pure mathematicians (;-)! You refer to the *collinearity* of the vectors $\mathbf{r}$ and $\mathbf{r}_o$. Now, consider a sphere of radius ρ with center $\mathbf{r}_o$. Tell me whether your collinearity assumption is fulfilled for all vectors $\mathbf{r}$ to points on the sphere surface. Let's consider an example in Euclidean $\mathbf{R}^3$ with orthonormal basis **i**,**j**,**k**: Choose $\mathbf{r}_o := \mathbf{i}$ and $\rho := 1$. Then the vector $\mathbf{r} := \mathbf{i}+\mathbf{j}$ points to the surface of our sphere, but is *not collinear* to $\mathbf{r}_o = \mathbf{i}$.

My proposal: We postpone the rest of the discussion until *you have found an escape out of this problem.*

Thanks so far.

Gerhard W. Bruhn

Left for further discussion

The line-element for a 3-dimensional spherically symmetric metric manifold relative to any arbitrary point in the manifold has been known to the pure mathematicians since 1896 [27], and is given by [15, 27],

$$ds^2 = A^2\, dR^2 + R^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2), \qquad (6)$$

$$R = R(r),\ A = A(R).$$

It is easily proved that expression (6) can be conformally represented in Euclidean space [15]. That the arbitrary point denoted by $R_p(r_o) = 0$ has a finite positive surface area but zero volume is odd. This is an oddity of the geometry of the Schwarzschild space, but it is not so startling when one recalls that the indefinite metric associated with Einstein's theory permits such things as null vectors, which are non-zero vectors of zero length (or equivalently, non-zero vectors that are orthogonal to themselves). If such an odd geometry is not suitable for a physical theory, that has no bearing on the strictly geometrical structure of Schwarzschild space and is irrelevant to my work.

Once again, your assertion in the abstract of your paper that I have assumed "one single central point", is incorrect. I assume no such thing, because it is a definite feature of the geometry, as determined by the pure mathematicians, and so I merely apply it. In section 4 of your paper you also say of me:

Crothers expects a geometrical meaning always being attached to a coordinate. He insinuates that the coordinate r, known from spherical polar coordinates as radial distance from the center, should maintain its meaning when appearing in another context, e.g. as the parameter r of the Schwarzschild metric.

This too is not true. I have never said such a thing in any of my papers, and I have never implied it either. I have always asserted that r in Hilbert's corruption of the Schwarzschild/Droste solution is the radius of (Gaussian) curvature and differentiated between it and the geodesic radial distance from the arbitrary point at the centre of spherical symmetry denoted by $R_p(r_o) = 0$ for the arbitrary parametric point r_o, which I just call the proper radius (a rose by any other name is still a rose).

Your assertion in section 1.1 that "S_α cannot be a single point" is also incorrect, owing to the fact that you have not correctly identified the nature of the quantity r in your expression (1.1) and the geometry of the manifold, and your misunderstanding of my published analysis. Similarly, your assertion in section 2 of your paper that "The boundary (subset of the event horizon) is not a single point" is also incorrect. Your assertion in section 3 of your paper that "The use of the metric (7)/(3.1) instead of the technically simpler Schwarzschild metric (6) is an unnecessary complication which cannot yield new results exceeding those attained by use of the Schwarzschild metric” is not correct. My use of a general expression permits the determination of the form of the analytic function that describes the radius of Gaussian curvature (and hence the Gaussian curvature), and that is a new result. Also new is the determination of the correct geometrical structure of the Schwarzschild space, notwithstanding that the pure mathematicians determined the correct geometry long ago. I merely apply that geometry to the Schwarzschild space and show that the usual claims violate the geometry, and so are inadmissible. Accordingly your assertion that “Crothers doesn’t like both coordinates, neither r* nor C; he is interested in a radial coordinate with an evident geometrical meaning. Therefore he introduces a new variable, a ’proper radius’ R_p by radial integration of the line element ds of (7), starting from the singularity ...”, is also incorrect. I introduce nothing new, since the geodesic radial distance from the arbitrary point at the centre of spherical symmetry in the manifold has been determined by the pure mathematicians. That what I call the proper radius rightly determines the geodesic radial distance from an arbitrary point at the centre of spherical symmetry in the manifold is known to some physicists as well [2, 17, 19, 23, 28, 29, 30].

In section 1.2] of your paper you remark that, in the alleged region $0 < r < \alpha$ in relation to your expression (1.1), “the variables t and r have exchanged their roles: r has become timelike while t is spacelike now. To emphasize your claim for the interchange of the characteristics of r4− and t, set r =˜t and t= ˜r.

I have on previous occasions given recipes to my adversaries, by which my arguments can be completely invalidated. None have provided the demonstration. I give you such a simple recipe. All you have to do is prove that R_c in expression (3) above is not the radius of Gaussian curvature, thereby invalidating the pure mathematicians. Alternatively, simply prove that A(R(r))dR in expression (6) above does not describe the elementary geodesic arc of the geodesic emanating from an arbitrary point at the centre of spherical symmetry, cutting the geodesic surface (3) orthogonally, and thereby invalidate the pure mathematicians.

For the foregoing reasons, I must advise the Editorial Board of Progress in Physics that your paper is not suitable for publication. I have appended below your covering email, having noted the contents thereof.

Yours sincerely, Steve Crothers. 8th March 2008

References

[[1]] S. Crothers, *On the General Solution to Einstein's Vacuum Field and its Implications for Relativistic Degeneracy.* , PROGRESS IN PHYSICS Vol. 1 , April 2005
http://www.ptep-online.com/index_files/2005/PP-01-09.PDF

[[2]] S. Crothers, *On the Geometry of the General Solution for the Vacuum Field of the Point-Mass*, , PROGRESS IN PHYSICS Vol. 2 , July 2005
http://www.ptep-online.com/index_files/2005/PP-02-01.PDF

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from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 12
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>, Reply
date Wed, Mar 12, 2008 at 9:28 AM
subject Re: Discussion continued
mailed-by gmail.com

Prof. G. W. Bruhn,

Dear Sir,

I do not think you have understood my work, or in the alternative you are hell-bent on telling me that black is white. It is also apparent that you do not understand as much mathematics as you claim. My account of a 3-d metric manifold is correct, taken directly from the pure mathematicians. I have cited references for you in this regard. You evidently did not check them to verify my exposition. Also, your point (11) is patently false. You do not read what I write, but substitute what you want in order to try to invalidate my argument. My argument is in fact correct. You too have altered my work to suit yourself. That will not do. I regard it as fraud. As for r_o denoting a point, it is so, for it denotes a point in parameter space, corresponding to R_p = 0 in the gravitational manifold. You labour over equivalent metrics, yet I have made it plain that they are equivalent according to my formulation of the expression for the Gaussian curvature, so that your Hilbert's corruption is a particular case, manifest in the form given originally by Droste (and contained in Schwarzschild's original paper). The radius of Gaussian curvature is not a distance in the manifold of the gravitational field. My proof that "r" in Hilbert's corruption of the Schwarzschild/Droste solution is the radius of Gaussian curvature by a formal relation is correct, as you now admit. That is sufficient to vindicate all of my analysis and invalidate all your objections. Therefore, there is realy nothing more to discuss. All else naturally follows from the radius of Gaussian curvature, but you do not seem to be able to see that. According to the recipe I gave you previously, unless you can prove that my proof of the radius of Gaussian curvature is invalid, you have no leg to stand on, and the black hole is history (it was still-born anyway). But you now agree that "r" in Hilbert's corruption is indeed the radius of Gaussian curvature. If you disagreed with that, then you would have been wrong.

Your assertion that I should forget about what Schwarzschild did is outrageous, and dishonest. I will therefore, not comply with your direction.

Consequently, you have effectively admitted the validity of my analysis by admitting the radius of Gaussian curvature, despite your continued plaintive cries to the contrary. It is clear to me that your invitation to discussion is disingenuous.

Yous raithfully,
Stephen J. Crothers.
PS. As for my membership of AIAS, my reasons are none of your business, and I will not permit you to tell me with whom I can and cannot consort. My membership does not imply anything but membership. Again, produce evidence of my knowledge of and agreement with ECE theory to support your accusations. Produce evidence of any writings I have penned on ECE theory. Your accusations on this head are also patently false. I have not said anything or written anything on ECE theory. Your disputes with Prof. M. W. Evans have nothing to do with me whatsoever, and have no bearing whatsoever on my work. Your attempts to discredit me by association are dishonest. I have Jewish friends - does that make me a Zionist? I have Christian friends - does that make me a Christian? I have acquainted politicians - does that make me a politician? In my erstwhile employment in relation to aspects of law and investigation, I met murderers and mafiosi. Does that make me a murderer or a mafioso?

from **GWB** <g.w.bruhn@t-online.de> hide details Mar 12
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Wed, Mar 12, 2008 at 8:40 PM
subject Re: Discussion continued
mailed-by t-online.de

Reply

Dear Mr. Crothers,

I have received your mail with regrets, since I expected at least a bit of understanding from your side, it's a pity.

On your work: You attempted to prove the non-existence of black holes. Such a proof must itself be ***without holes*****.** Already ***one*** gap that cannot be closed would be disastrous for a theory. Then one needs not consider the rest.

Such a hole is discussed in point (11) (attached once more below): I have quoted **your words** *literally* and then told you where I have detected a hole in your proof. Any beginner (you are not!) of vector calculus would understand that your reference to **collinearity** of the *vectors* **r** and $\mathbf{r}_o$ is **wrong**, at least when checking the simple example given.

I asked you to close that gap. You refused by claiming that there is no flaw. Apparently you don't know what to do. Or let me know your opinion about that point *in detail.* Consider my example! You'll recognize that there is **no**

collinearity. I am
always open for your response.

If you cannot close that gap, a main point of your proof of the non-existence of black holes is broken, which devaluates your whole theory.

Regards

Gerhard W. Bruhn

PS. I have noticed that you, though being a *leading* member of AIAS (Non-Executive Director), have no opinion about THE issue propagated by that group. Remarkably, really! That's your problem, not mine.

Stephen Crothers schrieb:

Prof. G. W. Bruhn, Dear Sir, I do not think you have understood my work, or in the alternative you are hell-bent on telling me that black is white. It is also apparent that you do not understand as much mathematics as you claim. My account of a 3-d metric manifold is correct, taken directly from the pure mathematicians. I have cited references for you in this regard. You evidently did not check them to verify my exposition. **Also, your point (11) is patently false. You do not read what I write, but substitute what you want in order to try to invalidate my argument. My argument is in fact correct.** You too have altered my work to suit yourself. That will not do. I regard it as fraud. As for r_o denoting a point, it is so, for it denotes a point in parameter space, corresponding to R_p = 0 in the gravitational manifold. You labour over equivalent metrics, yet I have made it plain that they are equivalent according to my formulation of the expression for the Gaussian curvature, so that your Hilbert's corruption is a particular case, manifest in the form given originally by Droste (and contained in Schwarzschild's original paper). The radius of Gaussian curvature is not a distance in the manifold of the gravitational field. My proof that "r" in Hilbert's corruption of the Schwarzschild/Droste solution is the radius of Gaussian curvature by a formal relation is correct, as you now admit. That is sufficient to vindicate all of my analysis and invalidate all your objections. Therefore, there is realy nothing more to discuss. All else naturally follows from the radius of Gaussian curvature, but you do not seem to be able to see that. According to the recipe I gave you previously, unless you can prove that my proof of the radius of Gaussian curvature is invalid, you have no leg to stand on, and the black hole is history (it was still-born anyway). But you now agree that "r" in Hilbert's corruption is indeed the radius of Gaussian curvature. If you disagreed with that, then you would have been wrong. Your assertion that I should forget about what Schwarzschild did is outrageous, and dishonest. I will therefore, not comply with your direction. Consequently, you have effectively admitted the validity of my analysis by admitting the radius of Gaussian curvature, despite your continued plaintive cries to the contrary. It is clear to me that your invitation to discussion is disingenuous. Yous raithfully,Stephen J. Crothers.PS. As for my membership of AIAS, my reasons are none of your business, and I will not permit you to tell me with whom I can and cannot consort. My membership

does not imply anything but membership. Again, produce evidence of my knowledge of and agreement with ECE theory to support your accusations. Produce evidence of any writings I have penned on ECE theory. Your accusations on this head are also patently false. I have not said anything or written anything on ECE theory. Your disputes with Prof. M. W. Evans have nothing to do with me whatsoever, and have no bearing whatsoever on my work. Your attempts to discredit me by association are dishonest. I have Jewish friends - does that make me a Zionist? I have Christian friends - does that make me a Christian? I have acquainted politicians - does that make me a politician? In my erstwhile employment in relation to aspects of law and investigation, I met murderers and mafiosi. Does that make me a murderer or a
mafioso?

On Wed, Mar 12, 2008 at 12:14 AM, GWB <g.w.bruhn@t-online.de> wrote:
Dear Mr. Crothers,

please, find attached the continuation of the discussion.

I have worked through your text and stopped when
I came across a serious problem
contained in your calculations (see No.11).

I think, it would be important to find an escape.
So, please, first have a look at No.11.

Regards
Gerhard W. Bruhn

Crothers:

It is clear from expression (5)

$$R_c = R_c(r) = (|r - r_o|^n + \alpha^n)^{1/n} , \qquad (5)$$

above that the quantity r appearing in the infinite number of equivalent metrics determined by it is merely a parameter. It is also plain that since $|r-r_o|$ is a distance on the real line, the arbitrary quantity r_o is an arbitrary point on the real line, and it is associated with the arbitrary point in the Schwarzschild manifold denoted by $R_p(r_o) = 0 \ \forall r_o \ \forall n$, which is in turn associated with the Gaussian curvature $K = 1/\alpha^2$ (for which the radius of Gaussian curvature is $R_c(r_o)=\alpha$, irrespective of the choice of r_o and of n).

The quantity $|r-r_o|$ is also easily related to 3-dimensional Euclidean space as the distance between a fixed point r_o and a variable point r on the common radial line through r_o, r and the origin r = 0 of the associated coordinate system. One does not have to locate r_o at the origin of the coordinate system (or at zero on the real line). The quantity $|r - r_o|$ merely reflects the simple geometrical shift of r_o to any point in the parametric space. The equation of a sphere of radius ρ, with centre C at the extremity of the vector $\mathbf{r}_o$, may be written [26]

$$[\mathbf{r} - \mathbf{r}_o] \bullet [\mathbf{r} - \mathbf{r}_o] = \rho^2,$$

and if **r** and $\mathbf{r}_o$ are collinear with the origin of the coordinate system, the vector notation can be dropped, so that $\rho = |r - r_o|$ without any real loss of generality.

Bruhn:

Stop! That's a **flaw of thinking**! You refer to the *collinearity* of the vectors **r** and $\mathbf{r}_o$. Now, consider a sphere of radius ρ with center at the extremity of $\mathbf{r}_o$. Tell me whether your collinearity assumption is fulfilled for all vectors **r** to points on the sphere surface. Let's consider an example in Euclidean $\mathbf{R}^3$ with orthonormal basis **i**,**j**,**k**: Choose $\mathbf{r}_o$:=**i** and ρ:=1. Then the vector **r**:= **i**+**j** points to the surface of our sphere, but is *not collinear* to $\mathbf{r}_o$=**i**.

from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 13
to GWB <g.w.bruhn@t-online.de>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>,
Axel Westrenius <corbis@bigpond.net.au>,
Gianni Giacchetta <geesquared@gmail.com>,
date Thu, Mar 13, 2008 at 9:23 AM
subject Re: Discussion continued
mailed-by gmail.com

Reply

Sir,

You have only succeeded in making yourself look foolish yet again. Collinear means all in a straight line, so r, r_o and the origin of coordinates are all in a straight line. I wrote that r and r_o are collinear, with the origin, in other wrods, are all in a straight line, so the vector notatin can be dropped. Thus, it is you who is wrong, not I. I have already made this plain when referring to the real line, which you apparently also ignored. The expression I have adduced for the radius of Gaussian curvature clearly indicates the collinearity too, as any "beginner" would not fail to see. You did not address what I wrote, and introduced your own arguments, erroneous arguments, in order to try to discredit me. Your claims are fallacious - utter nonsense. I have not refused to answer you. I answered you in my previous email and pointed out therein that you did not read what I wrote and that you altered my work - that is fraud - and so your argument is completely false. There is no "gap" for me to close. You created the gap yourself, with your foolishness and malicious intent, and attempted to attibute your "gap" to me. That is the act of a coward and a liar. I was aware from the outset that you had no intention of rational discussion, your agenda being only to try to discredit me. You have failed in that. Your invitation to discussion was a facade from the beginning. I am under no illusions as your character. I regard you as a criminal, and to be ultimately dealt with as such.

I have no problem with being a member of AIAS. You are the one who has made false accusations and when requested to produce evidence of them, ignored the request. I do not tolerate stand-over men such as you. When I used to work as a detective I encountered many stand-over men. Most end up in gaol, or murdered by their partners in crime. You will not tell me with whom I can and cannot consort. My reasons for membership of AIAS are, I repeat, none of your business, and I have no obligation to explain my reasons to you. Once again, I have read Mien Kempf; does that make me a Nazi? You seem to be the one with problems, in mathematics, honesty and integrity. I repeat that your disupte with Prof. M. W. Evans has nothing whatsoever to do with me, so you are out of order to implicate me in that dispute in any way whatsoever. Your attempts to discredit me owing to association are deceitful and quite pathetic.

Your admission that "r" in the Hilbert corruption is the radius of Gaussian curvature by the formality of being the inverse square root of the Gaussian curvature is sufficient to vindicate ALL my arguments, and therefore to invalidate the black hole nonsense, but you do not yet realise that. In your attempts at refutataion you repeatedly referred to "r" as the "radius" of a 2-sphere. You were completely ignorant of the fact that it is the radius of Gaussian curvature. So much for your claims for mathematical expertise. Your claims about the nature of a spherically symmetric metric manifold are also demonstrably false, and I cited books and papers by the pure mathematicians for you to consult to verify your elementary errors, but you chose to ignore them too. Your claimed knowledge of mathematics does not match your performance. I gave you the recipe for invalidation of my work - just prove that the pure mathematicians are wrong about the Gaussian curvature and the structure of a 3-d spherically symmetric metric manifold. That is all that is required, not the diatribes your produce. You have alread conceded that you were wrong about the radius of Gaussian curvature. You are also wrong about the structure of a 3-d spherically symmetric metric manifold, as the citations to the pure mathematicians I provided clearly testify.

There is no need to condescendingly "pity" me. My work is sound, your attempts at refutation inept.

II will post all our communications on my website, since it is evident that you intend to place certain communications on your website in an attempt to discredit me. Should you alter or omit anything to your advantage then that too will be demonstrated, and so your true character revealed.

I see no further purpose in our communication.

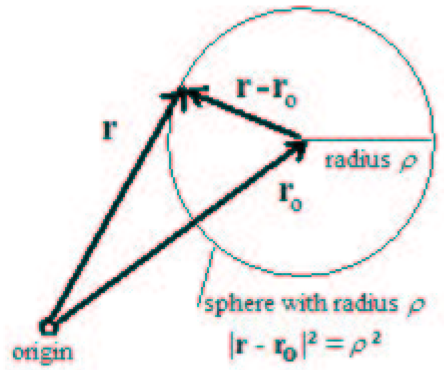
Crothers.

from GWB <g.w.bruhn@t-online.de> hide details Mar 13
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Thu, Mar 13, 2008 at 8:01 PM
subject Re: Discussion continued
mailed-by t-online.de

Reply

Dear Mr. Crothers,

please, focus on the topic of discussion. Anything else is irrelevant.



Crothers: Collinear means all in a straight line, so **r**, **r$_0$** and the origin of coordinates are all in a straight line.

Bruhn: Where do you see collinearity of **r**, **r$_0$** and the origin in that figure of a sphere around the extremity of **r$_0$**?

Your collinearity argument does not apply here.

That's my objection.

Regards

GWB

from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 13
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>,
Axel Westrenius <corbis@bigpond.net.au>,
date Thu, Mar 13, 2008 at 9:05 PM
subject Re: Discussion continued

Reply

mailed-by gmail.com

Sir,

You keep sending arguments that do not rightly report my analysis. I have made it plain that if r, r_o and the origin of coordinates are all in the same straight line, the vector notation can be dropped without loss of generality. That is developed in one my papers, which you cite. It is also clear from the diagram you sent me. Then, if the centre of mass of a body is located at r_o in E3 the radial distance |r - r_o| does not give the distance between the centre of mass and a so-called "test particle at r in Einstein's gravitational field. One does not need to locate the centre of mass at r = 0 in the parametric E3. One can locate it anywhere in the E3. The radial distance |r- r_o| is mapped into a corresponding distance in the gravitational field allegedly for Ric = 0 on a generalisation of Minkowski space (into a 3-d spherically symmetric metric manifold). As the parametric distance |r - r_o| approaches zero the corresponding distance in the gravitational field, R_p, approaches zero. The quantity |r - r_o|, being in E3, is both the radius of Gaussian curvature and the proper distance (the radial distance) - they are identical in E3. |r - r_o| is mapped into the associated radius of Gaussian curvature in the gravitational field (which is actually obtained by the formal relation of it being the inverse square root of the Gaussian curvature). This is not the same as the geodesic radial distance. The geodesic radial distance from the point at the centre of spherical symmetry and the radius of Gaussian (a formal quantity in the gravitational field of Einstein) are in general not the same in Einstein's gravitational field. The location of the mass in parameter space, at r_o, is entirely arbitrary. One needs the functions that map the parametric distance |r - r_o| into the radius of Gaussian curvature (and hence the Gaussian curvature) and the geodesic radial distance. They come from the intrinsic geometry of the line-element and associated boundary conditions (not guessing orasserting by inspection). The radius of Gaussian curvature in Einstein's gravitational field does not directly determine any distance at all. It determines the Gaussian curvature at any point located in a spherically symmetric geodesic surface, according to the pure mathematicians. Geodesic radial distance from the arbitrary point in the gravitational manifold is determined by an integral, as expounded in my papers. That this quantify relates to an arbitrary point at the centre of spherical symmetry in the gravitational manifold follows from the structure of a spherically symmetric 3-d metric manifold, again, as shown by the pure mathematicians. Your objections to this fact are false. I refer you again to the books and papers of the pure mathematicians cited in my assessment of your paper. The arbitrary location of the mass in parametric E3 corresponds to the arbitrary point in the gravitational manifold R_p(r_o) = 0, irrespective of the choice of r_o. In the gravitational field the centre of mass of the body is located at R_p(r_o) = 0. The general expression I obtained for the radius of Gaussian curvature, and hence for the geodesic radial distance, is a function of the parametric distance |r - r_o|, and the resulting line-element is well-defined for all real values of the parameter r except r = r_o, i.e. on the real line but for one point. Thus, r_o denotes a point on the real line, corresponding to the point R_p(r_o) = 0 (but at which the line-element fails). That the line-element might fail at R_p(r_o) =

0 for a general 3-d spherically symmetric metric manifold has long been known to the pure mathematicians. I again refer you to the books and papers cited in my response to your paper. Check them and see if I'm wrong. I can tell you that I have rightly applied what the pure mathematicians say is the case. If you think otherwise, check the pure mathematicians and prove me wrong by proving them wrong.

Crothers.

from **GWB** <g.w.bruhn@t-online.de> hide details Mar 13
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Thu, Mar 13, 2008 at 9:50 PM
subject Re: Discussion continued
mailed-by t-online.de

Reply

Thanks for your response!

IF <the vectors> **r**, **r**_o and the origin of coordinates are all in the same straight line, the vector notation can be dropped without loss of generality.
No question. That's **OK in case "IF"**

But my figure shows that the case "**IF NOT**" occurs **also** on a sphere with center at **r**_o.
You will remember from one of your preceeding emails, on the sphere given by

$$[\mathbf{r} - \mathbf{r}\_o] \bullet [\mathbf{r} - \mathbf{r}\_o] = \rho^2$$

there are points where **r**, **r**_o and the origin **0 are not collinear.**

Thus your above argument does NOT cover ALL possible cases, i.e. it cannot be used in general.
So if you assume the center of mass at the extremity of **r**_o then **you cannot drop the vector notation**
in all possible cases (see my figure).

How do you argue for these points? That's what I mean with "gap".

GWB

from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 13
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu

<wings.of.solitude@gmail.com>,
date Thu, Mar 13, 2008 at 11:32 PM
subject Re: Discussion continued
mailed-by mail.com

Sir,

Of course it is general. The radius of a sphere in E3 centred at r_0 collinear with r and the origin of coordinates is still |r - r_o| and it is only the radial distance that is mapped and which is important. The vectors in your diagram must pass through the collinear situation. One could develop it all in vectors but that adds complications that are not warranted and adds nothing meaningful to the analysis. The parametric radial distance is a real number greater or equal to zero. The corresponding quantities in the gravitational field are real numbers, greater than zero (since at R_p(r_o) = 0 the line-element fails). After all, |r - r_o| = D is the radius of the sphere described by your vectors (|vec(r) - vec(r_o)| = D), where D is the radius of the given sphere. The radius of a given sphere is not changed by using the vectors. The purpose of my writing |r - r_o| rather than just using D >= 0 is to amplify the nature of "r" in Hilbert's corruption, and also in Schwarzschild's solution, and in Droste's solution, and in all the infinite number of equivalent metrics I adduce, so that the relationship between the real valued parameter "r" in the parameter space E3, appearing in all the equivalent line-elements, is emphasized. Strictly speaking there is no real need of "r" explicitly. All one needs to do is map the radial distance D >= 0 from r_o Minkowski space into the gravitational field. That however does not change my analysis. One only then needs to replace |r - r_o| by D throughout all my analysis, since D = |r - r_o|. The important result of this is the lower boundary on "r". In the case of Hilbert's corruption, the lower bound on "r" is r_o = alpha, because in that case r_o = alpha, but that is disguised, and therefore gone completely unrecognised by the proponents of the black hole, who assume that Hilbert's "r" is a radial distance (it isn't), and that it can go down to r = 0 (it can't - the form of the Gaussian curvature function I adduce indicates this, but the alpha terms drop out: it is the dropping out of the alpha terms that disguise the true relationships. If you pick r_o = alpha and assume that r is non-negative, then r cannot go down to zero, because as r approaches r_o from above in parameter space, the parametric radial distance approaches zero, and the geodesic radial distance in the gravitational field approaches zero, and the Gaussian curvature approaches 1/alpha^2, not infinity, so that the radius of Guassian curvature, which is not a distance in the gravitational manifold, approaches alpha. That is the structure of the manifold). With r_o = alpha and "r" non-negative, only "r" appears in Hilbert's corruption (and also in Droste's solution), but that does not mean that r_o = alpha does not still apply as the lower bound on "r" in that case (as Droste correctly emphasized). There is no loss of generality in taking the parametric radii as points on the real line. A radial line is just a real line (or half line if fixed to a point).

Crothers.

from GWB <g.w.bruhn@t-online.de> hide details Mar 14 (10 days ago) Reply
to Crothers <thenarmis@yahoo.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Fri, Mar 14, 2008 at 9:30 PM
subject discussion

Dear Mr. Crothers,

I have received your last email twice. So I'll study it with *double* accuracy.

But let me start with a few words on scientific discussions.

Scientific discussions are necessary for progress of science. Only statements where **no bug can be found** can be accepted as science. So there is a permanent struggle in science between authors and critics: Authors would not like critics, however, authors are critics themselves of other authors. Again, that process of mutually checking between different authors is **essential** for progress in science. You know sayings like ERRARE HUMANUM EST. and Nobody is perfect - and, to modify an old joke - none of us is Mr. Nobody, it's a pity.

I myself have sometimes been criticized by others (that's no good feeling, indeed) - and have detected several flaws in papers and books by other authors, even by my close friends, and they have remained to be my friends nevertheless though they were urged to rewrite an article or a book chapter or withdraw some erroneous statement.

In that sense science is a process of **trial and error**. Flaws of thinking have a long tradition in human history. Think of the sophisms of the ancient Greeks, e.g. of Zeno's Paradox of the Tortoise and Achilles
http://www.mathacademy.com/pr/prime/articles/zeno_tort/
which are by no means useless: they could be considered as a positive contribution to sharpen scientific concepts.

Fallacies often come along disguised as *very tempting* conclusions, especially if someone's personal interests are involved, and it takes a lot of effort to reveal it as fallacies.

So let us discuss your problem further on SINE IRA ET STUDIO to find out the scientific truth:

Let r, r_o denote the lenghts of the vectors $\mathbf{r}$, $\mathbf{r}_o$ respectively and let be $r_o > 0$.
"the point $\mathbf{r}$" means "the point at the extremity of $\mathbf{r}$".

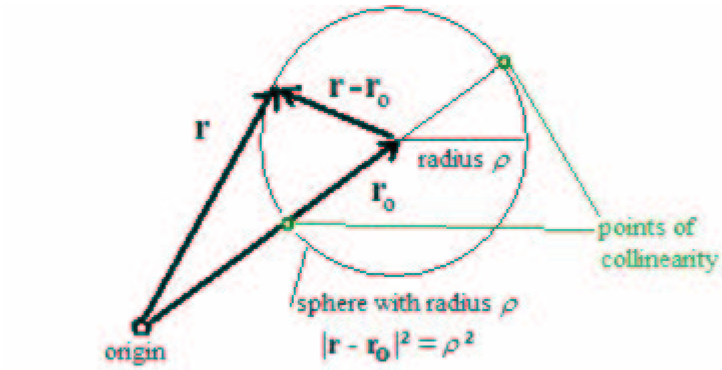
Where we agree (contradict if necessary! with explanation. Check it carefully step by step!):

(1) Then we can drop the vector notation in $|\mathbf{r}-\mathbf{r}_o|$, i.e. have $|\mathbf{r}-\mathbf{r}_o| = |r-r_o|$
if and only if the vectors $\mathbf{r}$, $\mathbf{r}_o$ are (positive) collinear, i.e. $\mathbf{r} = \lambda\, \mathbf{r}_o$ where $\lambda \geq 0$.

(2) The sphere S_ρ of radius ρ€ ($|\mathbf{r}-\mathbf{r}_o|^2 = \rho^2$) contains points **r** where the vectors **r**, $\mathbf{r}_o$ are **NOT** (positive) collinear.

(3) There are **only TWO** points **r** on S_ρ where **r**, $\mathbf{r}_o$ are (positive) collinear:
The points $\mathbf{r} = \mathbf{r}_o (1 \pm \rho/r_o)$ if we assume $r_o > \rho$, see the diagram below.

(4) In **all other** points of S_ρ (in the majority of S_ρ) the equation $|\mathbf{r}-\mathbf{r}_o| = |r-r_o|$ fails to be true.



Crothers: Collinear means all in a straight line, so **r**, $\mathbf{r}_0$ and the origin of coordinates are all in a straight line.

Bruhn: Where do you see collinearity of **r**, $\mathbf{r}_0$ and the origin in that figure of a sphere around the extremity of $\mathbf{r}_0$?

Your collinearity argument does not apply here.

Where we do **NOT** agree:

My **Conclusion** from (1)-(4):
The equation $|\mathbf{r}-\mathbf{r}_o| = |r-r_o|$ is **NOT *generally*** valid on the sphere S_ρ of radius ρ€ ($|\mathbf{r}-\mathbf{r}_o|^2 = \rho^2$). The exceptions are given by (4).

Looking forward with interest to your response.

Regards

Gerhard W. Bruhn

from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 15 (9 days ago) Reply
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>,
date Sat, Mar 15, 2008 at 12:18 AM
subject Re: discussion
mailed-by gmail.com

Dear Sir,

I think you are labouring unnecessarily on a minor point. When vectors r and r_o are collinear, one only need to use the magnitudes of those vectors to determine the radius of a given sphere, i.e. |r - r_o| = D. The radius D of a sphere centred at the extremity of the vector r_o, has a radius |vec(r) - vec(r_o)| = D. The radius D of the given sphere does not change with the vector r, the extremity of which identifies a point in the surface of the given sphere. Your diagram correctly gives the two possibilities for the vector r, in which the case D = |r - r_o| manifests. That is all that is needed. As I have previously remarked, the radius (a scalar, a distance) of a sphere in E3 is mapped into a corresponding distance (a scalar) in the gravitational field (the geodesic radial distance from the point at the centre of spherical symmetry of the gravitational manifold) and into a corresponding Gaussian curvature at all points in the gravitational manifold at that geodesic radial distance from the point at the centre of spherical symmetry. From the Gaussian curvature the radius of Gaussian curvature can be formally obtained and it is this latter quantity that appears explicitly in the line-element for the gravitational field. However, this has misled the proponents of the black hole as they have never identified their "radius" of various names with the Gaussian curvature, as it must be, instead mistaking it for various vague and quite erroneous concepts. In this way the mapping becomes simply that of a distance on the real line into a corresponding geodesic radial distance and a corresponding Gaussian curvature at that distance, in the gravitational manifold. This simplifies the analysis without loss of generality. All that needs to be mapped is a scalar, namely the radius (i.e. the length) of a given sphere in E3, into the corresponding geodesic radial distance and the Gaussian curvature, both scalars in the line-element for the gravitational field. I have plainly stated in my papers that the vector notation on the parameter space is dropped by virtue of the collinearity condition (which must occur at some stage with the motion of the extremity of the vector r about the surface of the given sphere). I clearly said in my relevant paper that a given distance between the centre of mass of a certain mass and a test-particle (a point) in Minkowski space (the parameter space), and hence essentially a distance on the real line, since I apply the simplification due to the collinearity condition, there is a corresponding geodesic radial distance from the point at the centre of spherical symmetry of the manifold of the gravitational field and a corresponding Gaussian curvature from which the radius of Gaussian curvature is formally obtained. Never have I committed the error you seem to think I have. I have never said that |vec(r) - vec(r_o)| = D = |r - r_o| absolutely, which is trivially false, as you rightly note, and which is well-known to me (it is schoolboy analytic geometry). I have only ever availed of the |r - r_o|, under the collinearity condition, which is sufficient for dealing with the whole problem, and I have gone into some detail to expound all this in my papers. The analytic functions in the components of the metric tensor of the gravitational line-element I give therefore as real-valued functions of a real variable (i.e. of |r - r_o|), never as real-valued functions of a vector variable, never as vector-valued functions of a vector variable. In so doing, I reveal the true nature of the variable r, as a parameter in E3, and the nature of the Gaussian curvature and the geodesic radial distance in the gravitational field and their relations via the components of the metric tensor of the line-element for the gravitational field. then, as r approaches r_o (i.e. the distance between the centre of mass of some mass and a test-particle in E3), the geodesic radial distance from the point at the centre of spherical symmetry of the gravitational manifold approaches zero, and the Gaussian curvature approaches 1/alpha^2 (so that the radius of Gaussian curvature approaches alpha). That is the structure of the manifold. The line-element completely determines the geometry. Then I show that Schwarzschild's solution is a particular case of the general case I have adduced, that Droste's solution is a particular case thereof, that Brillouin's solution a particular case, that Hilbert's corruption must rightly be Droste's solution, and I provide an infinite number of equivalent metrics that satisfy the intrinsic geometry of the line-element and the boundary conditions Einstein associated with this alleged configuration of matter. The general form I give for the radius of Gaussian curvature, and hence the associated general line-element, is well-defined for all values of r except r = r_o, where r_o is entirely arbitrary (one can place the parametric centre of mass of the given mass in E3 at any point in E3), and where the quantity n in my generalisation is also entirely arbitrary. In all particular cases (and for the general case itself) , R_p(r_o) = 0 and R_c(r_o) = alpha, irrespective on the choice of r_o and of n. I reiterate that one can place the centre of mass of the given mass in E3 at any point r_o in E3. That centre of mass is always located at R_p(r_o) = 0 in the gravitational manifold, where the Gaussian curvature is positive and finite. This oddity is due to the inescapable fact that the geodesic radial distance from the point at the centre of spherical symmetry of the gravitational field is not the same as the radius of Gaussian curvature, in general, and the radius of Gaussian curvature does not directly give any distance at all in the gravitational field because it is a "radius" only by the formal relation of it being the inverse square root of the Gaussian curvature.

Crothers.

from GWB <g.w.bruhn@t-online.de> hide details Mar 15 (9 days ago) Reply
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Sat, Mar 15, 2008 at 2:37 AM
subject Re: discussion
mailed-by t-online.de

Dear Mr, Crothers,

let me cut the most interesting part of your reply into slices.

You wrote:

(C1) When vectors **r** and **r**_o are collinear, one only need to use the magnitudes of those vectors

to determine the radius of a given sphere, i.e. |r - r_o| = D.
OK

(C2) The radius D of a sphere centred at the extremity of the vector **r**_o, has a radius |**r** - **r**_o| = |vec(r) - vec(r_o)| = D.
OK

(C3) The radius D of the given sphere does not change with the vector r, the extremity of

which identifies a point in the surface of the given sphere.

Caution: This staement is ***ambivalent.*** In general we have $|\mathbf{r} - \mathbf{r}_o| = D$, but **NOT** $|r - r_o| = D$ unless **r** is a point of collinearity. **Otherwise** due to the triangle inequality we have $|r - r_o| = \big|\,|\mathbf{r}| - |\mathbf{r}_o|\,\big| < |\mathbf{r} - \mathbf{r}_o| = D$ (D fixed).

(C4) Your diagram correctly gives the two possibilities for the vector r, in which the case

$D = |r - r_o|$ manifests.

This sentence is not replying my questions. In **case of collinearity I agree** with

$$|\mathbf{r} - \mathbf{r}_o| = |r - r_o|,$$

however, this is true **only for TWO points** on the sphere $|\mathbf{r} - \mathbf{r}_o| = D$ (D fixed).
For the **majority of points r at the sphere** we have $D = |\mathbf{r} - \mathbf{r}_o| > |r - r_o|$. See (C3).

And this is my objection you should reply to.

What about the **points r at the sphere** where $D = |\mathbf{r} - \mathbf{r}_o| > |r - r_o|$?
Then you have **NOT** $|r - r_o| = D$, but $|r - r_o| < D$.

That's the **dark spot** you should enlighten with a satisfying reply.

Regards

GWB

from **Stephen Crothers** <thenarmis@gmail.com> hide details Mar 15 (9 days ago)
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>,
dtalbott@teleport.com,
Jeremy Dunning-Davies <j.dunning-davies@hull.ac.uk>,
walt@holoscience.com,
corbis@bigpond.net.au,
date Sat, Mar 15, 2008 at 5:03 PM
subject Re: discussion
mailed-by gmail.com

Dear Bruhn,

Attached is my reply, containing diagrams.

Crothers.

Dear Bruhn,

You appear to be telling me the radius of a given fixed sphere of radius D in $\mathbf{E}^3$, centred at the extremity of a vector $\mathbf{r}_o$, changes as the extremity of the vector $\mathbf{r}$ moves over the fixed spherical surface. If so, then I say that your claim is utterly and patently false. The radius of a given fixed sphere cannot change since it is fixed and so it does not change if the sphere is described by vectors or not. As the extremity of the vector $\mathbf{r}$ moves over the fixed spherical surface it must sooner or later become collinear with the vector $\mathbf{r}_o$, since both of the vectors $\mathbf{r}$ and $\mathbf{r}_o$ emanate from the origin of the coordinate system. When the vectors are collinear there is, as you admit, no need of the vectors since their magnitudes are sufficient to determine the radius D. It is the radius (the distance) D that is mapped from $\mathbf{E}^3$ into the gravitational field and the arbitrary point at the centre of the parametric sphere (the centre of mass of some mass), located at the extremity of the parametric vector $\mathbf{r}_o$, corresponds to $R_p(r_o) = 0$ in the gravitational field where the Gaussian curvature is $1/\alpha^2$, irrespective of the value of r_o. The counter argument that you adduce has nothing to do with the price of fish.

Consider the directed real line L:

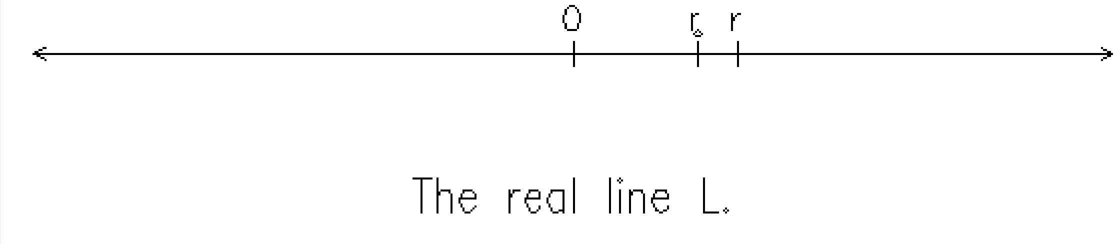


Figure 1

Mark off a point at r_o at distance r_o from the origin 0. Similarly mark off the point r at the distance r from the origin (the directions do not matter, but I chose as shown), where $r_o \neq r$. Then the distance D between r_o and r is given in general by $D = |r - r_o|$, and in the particular case of figure 1 by $(r - r_o)$.

Now consider $\mathbf{E}^3$, but for convenience of diagrams I suppress the 3rd dimension without loss of generality. Take the real line L of figure 1 so marked and place it into $\mathbf{E}^3$ so that the origin of the real line L coincides with the origin of the Cartesian coordinate system for $\mathbf{E}^3$ (the direction of the placement is immaterial) and construct a sphere of radius D centred at r_o on L reaching to r also on L, thus:

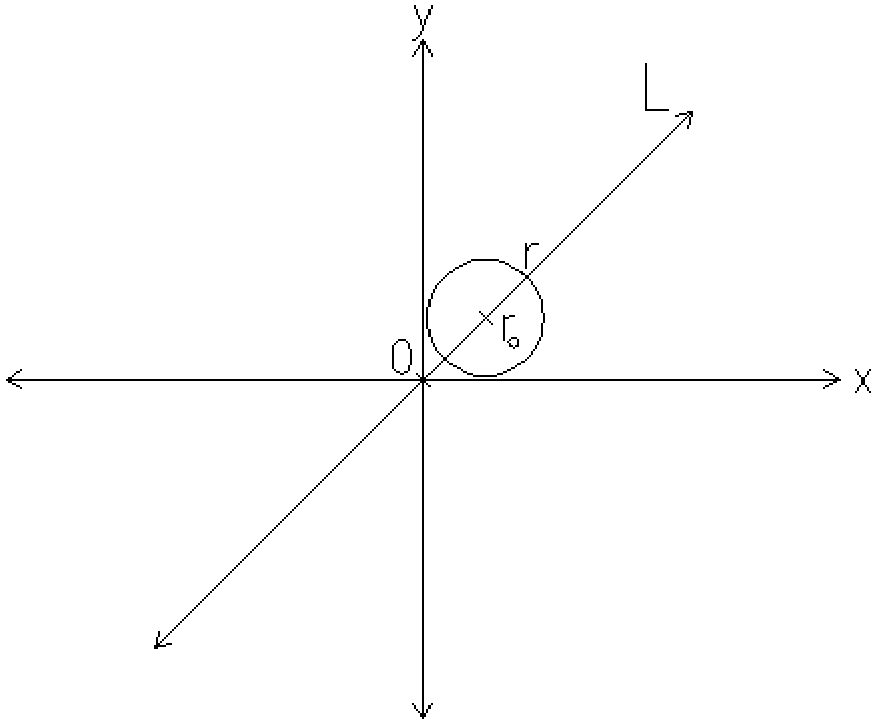


Figure 2

The radius of the sphere in this diagram is then, in general, $D = |r - r_o|$, but in the particular case of figure 2 is $D = (r - r_o)$. This radius is fixed for a fixed sphere of radius D. Introducing vectors for the very same sphere, thus

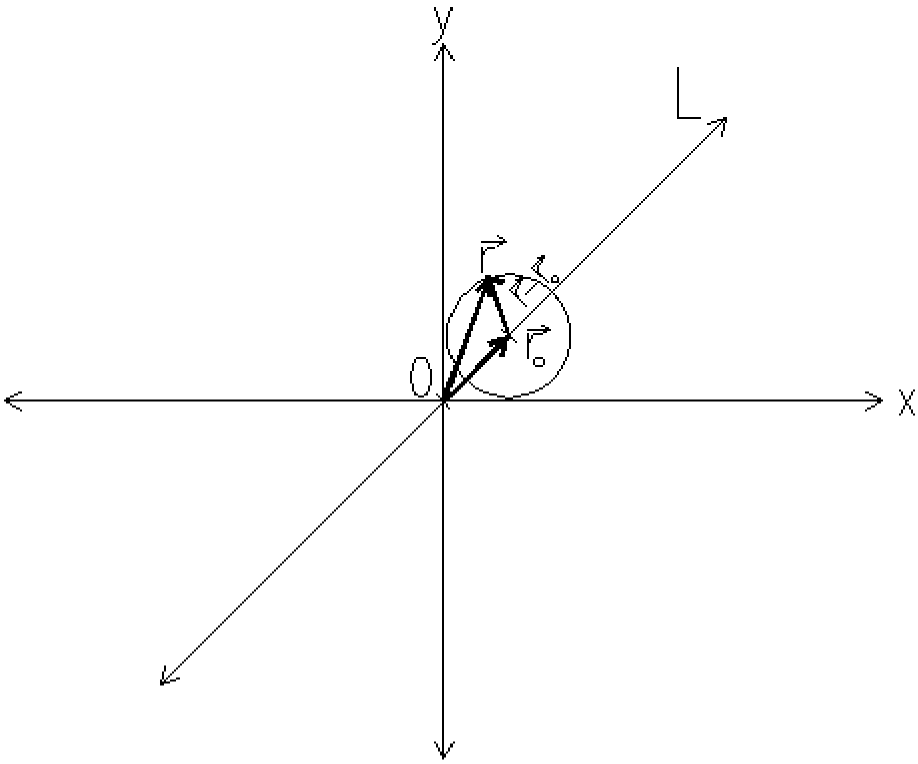


Figure 3

does not change the radius of the fixed sphere at all. The radius of the fixed sphere in figures 2 and 3 is the same. Since the vector **r** in figure 3 can be brought into collinearity with the vector $\mathbf{r}_o$ in the same figure, thus corresponding to the points and distances on the directed real line L of figures 1 and 2, and since any one of the infinite number of radial lines from the centre of the sphere to its surface is equivalent to all the others for the purpose of determining the radius of the sphere, one need only prove things for one radial line to prove it for all equivalent radial lines. The simplest radial line is that in figure 2 (which is just the real line in figure 1), since it involves only scalars, namely, the radial distance D between r and r_o and the distances r and r_o on the real line L as in figures 1 and 2.

Now consider figures 1 and 2. As r approaches r_o therein, the radius of the given sphere approaches zero and so the sphere in figure 3 gets smaller and smaller as well (after all, it's the same sphere). In all cases, as r approaches r_o in figure 2 the radius of the sphere in figures 2 and 3 diminishes but is exactly the same in each case. Clearly the value of r in figures 1 and 2 is not the same as the magnitude of the vector **r** in figure 3. But I have never claimed that they are the same. You introduced the vector complications, I did not. In my papers I dealt only with scalars, as in figures 1 and 2, because the radius of the sphere, i.e. *the distance between the two points* r and r_o on the real line L as in figures 1 and 2, is the important quantity and is the interval precisely on the common radial line through the points 0, r and r_o in figure 2, since the radial line of figure 2 it is the real line L of figure 1. I reiterate: given a distance D (effectively a radial distance) between the centre of mass of some mass and a test-particle in the $\mathbf{E}^3$ associated with Minkowski space, what is the corresponding distance in Einstein's gravitational field, bearing in mind that the line-element for the gravitational field under consideration is a generalisation of the Minkowski line-element in spherical symmetry? As I have previously remarked, I did not have to use the scalar parameter "r" appearing in Minkowski's spherically symmetric line-element. I could have used only the radial distance $D \geq 0$ as described herein and in all my papers. However, I kept the scalar parametric quantity "r" in order to amplify its true geometrical features and to show how and why it manifests in the line-elements of Schwarzschild, Droste, Brillouin, Hilbert's corruption, and in the infinite number of admissible equivalent line-elements that must manifest by a general solution, as noted by Eddington. By this approach I developed the general expression for the radius of Gaussian curvature in terms of the scalars r and r_o and hence correctly determined the components of the metric tensor for the gravitational line-element in accordance with the intrinsic geometry of the line-element that is in fact fixed by the line-element of Minkowski space (I reiterate that a geometry is entirely determined by the *form* of its line-element). One can really just substitute $D \geq 0$ for $|r - r_o|$ in all my relevant papers, and then consider what happens to the mappings from the $\mathbf{E}^3$ of Minkowski space into the corresponding quantities in the gravitational field when the parametric distance between the points r and r_o (i.e. D) in the parametric $\mathbf{E}^3$, denoting therein the positions of a test-particle and the centre of mass of some mass respectively, approaches zero.

Consequently, I feel that you have not understood my analysis, and you have made for yourself a complication that does not appear in my work, and really has no bearing on my work, and then claim that it is a "gap" in my analysis. That is not correct. You introduced the "gap" yourself.

One can see that the black hole involves the misconception that in figures 1,2 and 3, the allegedly infinitely dense point-mass singularity of the alleged black hole is located at r = 0, that is, at the origin of the parametric coordinate system. That is false, because it is located at the arbitrary parametric point r_o in the $\mathbf{E}^3$ of Minkowski space (which is the parameter space). With this misconception the proponents of the black hole, ignorant of the parametric nature of their quantity "r" in Hilbert's corruption; ignorant of the irrefutable geometrical fact that in Hilbert's corruption of the line-element for the gravitational field the quantity "r" is the radius of Gaussian curvature, which does not *directly* determine, in general, any distance at all in the gravitational field; ignorant of the fact that the geodesic radial distance from the point at the centre of the spherically symmetric metric manifold of the gravitational field is not the same as the radius of Gaussian curvature (which is not all that surprising since the gravitational field is non-Euclidean); ignorant of the fundamental geometrical structure of a 3-dimensional spherically symmetric metric manifold. Then to blindly drive their misconceived "r" in Hilbert's corruption down to r = 0 therein (actually down to r = 0 in the parametric situation illustrated in figure 2 above) they develop the Kruskal-Szekeres phantasmagoria, unwittingly constructing thereby a pseudo-Riemannian metric manifold that has nothing to do with the gravitational field whatsoever, amidst the delusion of their misconceptions. Thus, the proponents of the black hole have violated elementary differential geometry. Their claims of black holes are therefore demonstrably false. In fact, their claims of big bangs and expansion of the Universe are founded upon the very same geometrical misconceptions, and are therefore also demonstrably false, as I have developed in my other papers. Accordingly, black holes, big bangs and expansion of the Universe (as allegedly associated with the Friedmann-Robertson-Walker line-element) are inconsistent with Einstein's General Theory of Relativity.

Your arguments are not addressing the core of the problem and are bogged down in minor issues that you have unnecessarily made into mountainous obstructions. Perhaps with this minor issue resolved you will see that the crux of the issue is a mapping of a parametric distance associated with an arbitrary parametric point at the centre of mass of some mass in the parametric $\mathbf{E}^3$ of Minkowski space into corresponding quantities *in the gravitational manifold*.

I reiterate that to prove me wrong is, in principle, very easy - one need only prove that my proof of the Gaussian curvature involving the quantity "r" in the line-element for the gravitational field is false and that the integral I give for the geodesic radial distance from the point at the centre of spherical symmetry is false, proving thereby that the pure mathematicians are incorrect, since I have merely used what the pure mathematicians have expounded. I have provided citations of pure mathematicians for this purpose.

As a retiree you seem to have much time on your hands. However, physics has been for me and still is something I do in my spare time. Currently and for the foreseeable future my spare time is very limited as I must spend most of my time dealing with matters other than physics. Therefore, I can see no purpose in our further correspondence if it involves laborious discussion of minor details you have blown out of all proportion, whether intentionally or unintentionally, rather than the crux of the question, which I have given again above. You are not my only correspondent scientist. I now have too many people to reply to, many educated laymen as well as scientists (but excluding the abusive emails I get from proponents of black holes and such, which I mostly ignore). On top of that I now find that I'm an invited referee of papers, dealing with black holes, for a quite major "mainstream" journal. Apparently some highly placed "mainstreamers" have now studied my papers carefully. It really is only a matter of time (and not much time at that) before the black hole and its cousin, the big bang, are relegated to their rightful places in the dustbin of scientific history.

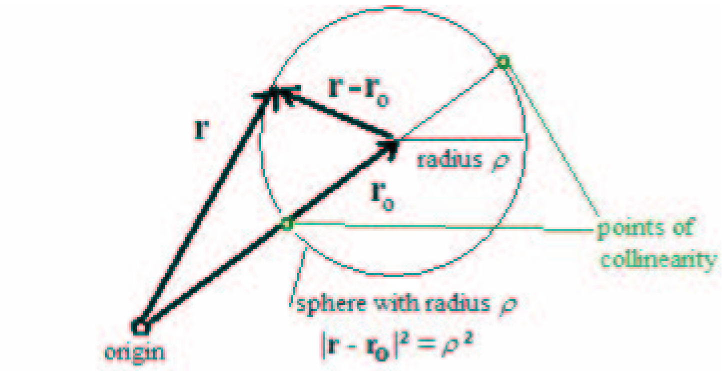
Crothers.

from GWB <g.w.bruhn@t-online.de> hide details Mar 15 (9 days ago) Reply
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Sat, Mar 15, 2008 at 8:18 PM
subject Re: discussion
mailed-by t-online.de

Dear Mr. Crothers,

your long letter is based on a *complete* misunderstanding of my point of view (as repeatedly described in my preceeding emails) already in its first sentence. So, please, read my assessment below carefully and convince yourself.

Regards
GWB



Crothers: Collinear means all in a straight line, so **r**, $\mathbf{r_0}$ and the origin of coordinates are all in a straight line.

Bruhn: Where do you see collinearity of **r**, $\mathbf{r_0}$ and the origin in that figure of a sphere around the extremity of $\mathbf{r_0}$?

Your collinearity argument does not apply here.

Where we do **NOT** agree:
My **Conclusion** from (1)-(4):

The equation $|\mathbf{r}-\mathbf{r}_0| = |r-r_0|$ is **NOT *generally***valid on the sphere S_ρ of radius $\rho \in (|\mathbf{r}-\mathbf{r}_0|^2 = \rho^2)$. The exceptions are given by (4).

Discussion Crothers ./. Bruhn 15.03.2008

Crothers wrote:

You appear to be telling me the radius of a given fixed sphere of radius D in E3, centred at the extremity of a vector ro, changes as the extremity of the vector r moves over the fixed spherical surface.

Bruhn replied:

Of course, NOT!!!
There we are *exactly* at the point of our disagreement: The radius is $|\mathbf{r}-\mathbf{r}_o|$ and fixed by $|\mathbf{r}-\mathbf{r}_o| = D$. However, assumed that the origin is located outside the sphere, then the length-difference $r-r_o$ varies along the sphere surface between the values

$$D-r_o < 0 \qquad \text{for} \qquad \mathbf{r} = \mathbf{r}_- := \mathbf{r}_o\,(1 - {}^D/_{ro})$$

and

$$D+r_o > 0 \qquad \text{for} \qquad \mathbf{r} = \mathbf{r}_+ := \mathbf{r}_o\,(1 + {}^D/_{ro})$$

at the two points $\mathbf{r}_\pm$ of collinearity.

You can easily check this (do that, convince yourself!):

(i) The points $\mathbf{r}_\pm$ both belong to the sphere surface: $|\mathbf{r}_\pm-\mathbf{r}_o| = r_o\,|\pm {}^D/_{ro}| = D$

(ii) $r_+ = |\mathbf{r}_+| = r_o\,(1 + {}^D/_{ro}) = r_o + D$, $\qquad r_- = |\mathbf{r}_-| = r_o\,|1 - {}^D/_{ro}| = r_o - D$,

hence

$$r_+ - r_o = +D > 0\,, \qquad r_- - r_o = -D < 0\,.$$

Result

The difference $r - r_o$ is NOT CONSTANT along the sphere $|\mathbf{r}-\mathbf{r}_o| = D$.
Therefore $|r - r_o|$ cannot agree with $|\mathbf{r}-\mathbf{r}_o| = D$ in general.

Conclusion:

The use of the equation $|r - r_o| = |\mathbf{r}-\mathbf{r}_o|$ along the sphere $|\mathbf{r}-\mathbf{r}_o| = D$ is a clear mistake.

Therefore the rest of your long text does not apply. Sorry!

from Stephen Crothers <thenarmis@gmail.com> hide details Mar 16 (8 days ago) Reply
to GWB <g.w.bruhn@t-online.de>,
bcc Dmitri Rabounski <rabounski@ptep-online.com>,
Dani Indranu <wings.of.solitude@gmail.com>,
Jeremy Dunning-Davies <J.Dunning-Davies@hull.ac.uk>,
dtalbott@teleport.com,
Dennis Holdroyd <holdroyd@googlemail.com>,
Dave Smith <davesmith_au@plasmaresources.com>,
"EMyrone@aol.com" <EMyrone@aol.com>,
date Sun, Mar 16, 2008 at 10:46 AM
subject Re: discussion
mailed-by gmail.com

Dear Prof. Bruhn,

I reiterate that your argument has got nothing to do with my analysis. You have introduced the complication of vectors, not I. I have made it plain, over and over again, that in my analysis, r approaches r_o along the radial line through the points at r, r_o and the origin of the coordinate system. I do not involve any vector the extremity of which moves over the surface of the associated parametric sphere. You have introduced that issue, not I. All that is important is the location of the fixed parametric point r_o and the distance from that point to the moving parametric point r along the radial line through r, r_o and the origin of the parametric coordinate system. Your argument does not involve r, r_o and the origin of the parametric coordinate system all being on a common radial line, and so it is irrelevant. Furthermore, I have already made it plain, over and over again, and as my published papers clearly testify, that I have NEVER claimed that |r - r_o| = |vec(r) - vec(r_o)| "along the sphere |vec(r) - vec(r_o)| = D". I did indicate to you that when vec(r) and vec(r_o) are collinear, then the magnitudes of the vectors can be dropped, as you rightly agreed, and in that case |r - r_o| = |vec(r) - vec(r_o)| = D. I have also made it plain, over and over again, that as r approaches r_o along the aforesaid radial line, so that the distance between the two points approaches zero (and so that the radius of the associated sphere therefore approaches zero), the geodesic radial distance from the point at the centre of spherical symmetry for the gravitational field also approaches zero. To obtain radial "distances" I integrate along the radial line containing the parametric point r_o, at which the centre of mass of some mass is located in the parametric E3, and the parametric point at the test-particle.

You have either not understood my analysis (which is odd, because it is all rather simple), or you have intentionally introduced things to obfuscate. This discussion has degenerated into trivia, and not only that, trivia which is really not part of my analysis, but due to your introduction. Your argument, although mathematically correct, has got nothing to do with my analysis - it is entirely irrelevant. So the mathematical truth of your argument has no bearing on the issue at hand.

Unless you address the core issues, I repeat yet again, our discussion can serve no meaningful purpose, and so must end. Therefore, prove that the radius of Gaussian curvature is not as I have demonstrated (despite your previous admission in earlier correspondence that I am in fact correct) and prove that the integral I use does not give the geodesic radial distance from the point at the centre of spherical symmetry of the gravitational field. All the rest is mere plumbing and of no importance. Stop beating about the proverbial bush and stop avoiding the crux of the matter. I have previously provided you with citations to some pure mathematicians to assist you. Please do not send me any more email unless you prove me wrong by the foregoing. Anything less would be just the usual hot air belched up by the proponents of the black hole and big bang, all of which have demonstrated an ignorance of the most elementary elements of differential geometry, as their scribblings reveal without any shadows of a doubt whatsoever (they don't even understand Gaussian curvature). I am tired of double-talk, deflection to minor or irrelevant issues, disregard for the facts, and refusal to provide proofs. I have provided the proofs of the central issues (following the pure mathematicians), invalidating thereby the claims of the black holers and big bangers, but true to form, the holers and bangers ignore the facts to save their precious nonsense, their egos and their incomes, just as you do. Stop your nonsense and provide the required proofs - that is all that is necessary to invalidate all my work (but admission of the validity of my fundamental geometry ruins the holers and bangers completely).

Crothers.

from **GWB** <g.w.bruhn@t-online.de> hide details Mar 16 (8 days ago) Reply
to Stephen Crothers <thenarmis@gmail.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Sun, Mar 16, 2008 at 9:28 PM
subject Re: discussion
mailed-by t-online.de

Discussion continued on 16.03.2008

Crothers:
You have introduced the complication of vectors, not I.

Bruhn:
Thanks. Remember the equation $[\mathbf{r} - \mathbf{r}_o] \bullet [\mathbf{r} - \mathbf{r}_o] = \rho^2$ in **your** document GWBruhn.pdf from 08.03.2008 where the vector notation was introduced first **by you**. I merely pointed out that the equation $|\mathbf{r} - \mathbf{r}_o| = |r - r_o|$, i.e. dropping the vector notation, is not allowed *in general*. **We agree(!!!)** that dropping of the vector notation requires **collinearity** of the vectors $\mathbf{r}$ and $\mathbf{r}_o$.

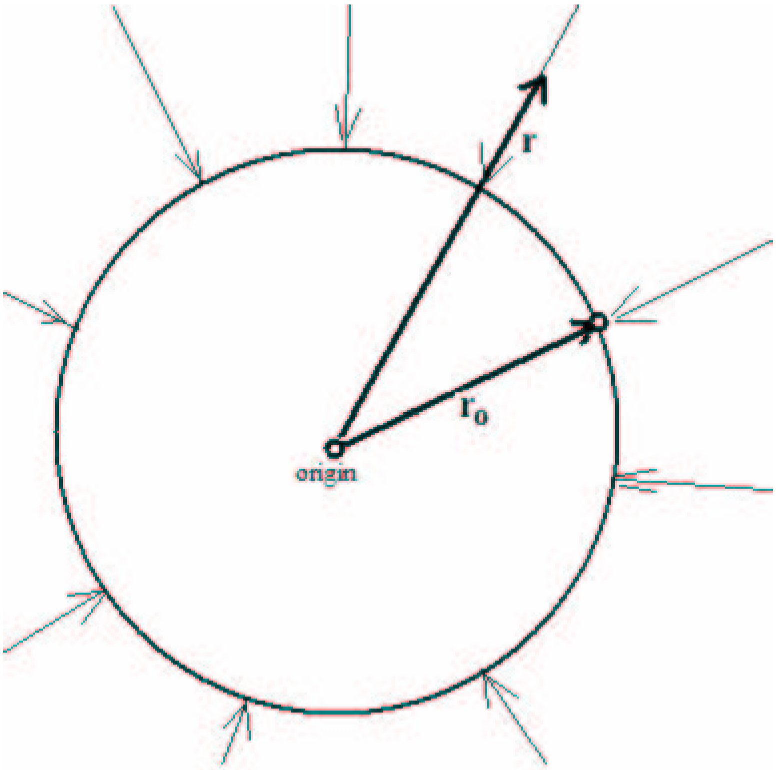
Crothers:

I have made it plain, over and over again, that in my analysis, r approaches r_o along the radial line through the points at r, r_o and the origin of the coordinate system.

Bruhn:
You mean that the **VECTOR** $\mathbf{r}$ approaches the **VECTOR** $\mathbf{r}_o$ along the radial line through the points at $\mathbf{r}$, $\mathbf{r}_o$ and the origin of the coordinate system. This is a **special restriction** we should keep in mind from now on.

Your idea is that instead of the black hole of the "relativists" there is **one** single point $\mathbf{r}_o$ which is not the origin **0** and which is approached when the "relativists" speak of matter approaching the black hole.

Following your imagination the **point at the extremity** of the vector $\mathbf{r}_o$ is approached by matter from all directions. However,
reality is shown by another diagram:



Radially infalling matter approaching a sphere of radius r_0

As can be seen here the matter approaches the center **o** radially from all directions, **but WITHOUT collinearity** of **r** and $\mathbf{r}_o$ in general:

While $|r - r_o|$ decreases to 0 for infalling matter
the vector distance $|\mathbf{r} - \mathbf{r}_o|$ does **not necessarily** tend to 0.

Indeed, under your **special restriction of collinearity** you would have $|\mathbf{r} - \mathbf{r}_o| = |r - r_o| \to 0$,
however, in most cases of approach (as shown by the above figure) there is **NO approach** $|\mathbf{r} - \mathbf{r}_o| \to 0$ when $|r - r_o| \to 0$.

Thus, the "black hole" (whatever it might be) cannot be a **single point situated at the extremity** of some vector $\mathbf{r}_o$.

Regards

Gerhard W. Bruhn

PS Sometimes I get suspicious that your browser cannot distinct in emails between usual and **boldface** letters,
e.g. between **vector** $\mathbf{r}_0$ and scalar $r_0 = |\mathbf{r}_0|$. Then this discussion will never end.
Let me know if so. Then I shall send you hardcopies where that problem does not occur.
No problem!

from **GWB** <g.w.bruhn@t-online.de> hide details Mar 19 (5 days ago) Reply
to Crothers <thenarmis@yahoo.com>,
Prof Bruhn <bruhn@mathematik.tu-darmstadt.de>,
date Wed, Mar 19, 2008 at 6:45 PM
subject Change of email address

Dear Mr. Crothers,
my current email address will be cancelled soon.
For correspondence, please, use the following address
Prof Bruhn <bruhn@mathematik.tu-darmstadt.de>
Regards
G.W. Bruhn

from **Ernest S. Gullible** <bruhn@mathematik.tu-darmstadt.de>
to Crothers <thenarmis@yahoo.com>,
cc Dmitri Rabounski <rabounski@ptep-online.com>,
date Tue, Mar 25, 2008 at 6:00 AM
subject Update

Dear Mr. Crothers,

please have a look at the updated version of my pro BH paper at

http://www.mathematik.tu-darmstadt.de/~bruhn/CrothersViews.html

Your comments are welcome.

Regards

Gerhard W. Bruhn

Discussion of S. Crothers' Views on Black Hole Analysis in GRT

Gerhard W. Bruhn, Darmstadt University of Technology

06.03.2008 , **with updates on 24.03.2008**

Quotations from Crothers' papers are displayed in **black**. Equation labels of type (n) refer to Crothers' papers.

Abstract
In the last years since 2005 S. Crothers has published a series of papers in the Journal PROGRESS IN PHYSICS (see [3]) which deal with the alleged fact that black holes are not compatible with General Relativity. Crothers views stem from certain dubious ideas on spacetime manifolds, especially in the case of Hilbert/Schwarzschild metrics: His idea is that instead of the 2-sphere of the event horizon there is merely *one single central* point. It will be shown below that this assumption would lead to a curious world where Crothers' "central point" can be approximated in sense of distance by 2-spheres S_r of radius $r > \alpha$. Hence the event horizon cannot be a single point. − Concerning the two validity regions of the Schwarzschild metric in contrast to Crothers' claims the fact is remembered that both validity regions of the Schwarzschild metric can be covered by introduction of the Eddington-Finkelstein coordinates.

1. Crothers' basic views

Crothers bases his objection of Schwarzschild black holes on two statements: Concerning the Schwarzschild metric (2.1) below he asserts in the Introduction of [1]:

When the required mathematical rigour is applied it is revealed that
1) **$r_o = \alpha$ denotes a point, not a 2-sphere,** and that
2) **$0 < r < \alpha$ is undefined on the Hilbert metric.**

2. Objections to claim 1)

We consider the Schwarzschild/Hilbert metric

(2.1) $$ds^2 = -(1 - \alpha/r)\, dt^2 + (1 - \alpha/r)^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

in the spacetime that is accessible for a physical observer, i.e. for $r > \alpha$: Here the metric (2.1) defines submanifolds S_r for each pair of fixed values of t and r, the metric of which follows from (2.1) to be

(2.2) $$ds^2 = r^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2)\ .$$

Hence S_r is a 2-sphere with radius r. The set S_α of singularities of the Schwarzschild/Hilbert metric has the metric

(2.3) $$ds^2 = \alpha^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2).$$

and hence is a 2-sphere as well.

The distance between S_r and S_α is given by Crothers' "proper radius" (cf [1, eq. (14)] with $C(r)= r^2$)

(2.4) $$R_p(r) = [r(r-\alpha)]^{1/2} + \alpha \ln |(r^{1/2}+(r-\alpha)^{1/2})\, \alpha^{-1/2}|$$

measurable in radial direction between arbitrary associated points of the concentric spheres. Since $R_p(r)$ is continuous at $r=\alpha$ the distance between S_r and S_α tends to 0 for $r \to \alpha$:

(2.5) $$\lim_{r \to \alpha} R_p(r) = R_p(\alpha) = 0 \ .$$

Therefore, the set S_α of the metric singularities can be approximated with respect to the distance $R_p(r)$ by concentric 2-spheres of radius $r > \alpha$: Thus,

S_α cannot be a single point.

See also **Section 4**.

3. Objections to claim 2)

This claim is not true: As will be shown here the region $r > \alpha$, accessible for human observers, can be extended to the region $r > 0$ by the introduction of simple coordinates well-known as **Eddington-Finkelstein coordinates** (cf. [4, p.184]). The additional part of the world - usually called "black hole" is not directly explorable by human observers. We can only try to extrapolate the rules that have been found in the accessible part of the world.

The special structure of the Schwarzschild metric (2.1) allows a simple extension from the obvervable region $r > \alpha$ to the region $r > 0$ crossing the former border $r = \alpha$.

Before doing so it is advantageous to simplify the notation by an obvious transformation: By applying the substitution $r/\alpha \to r$ we can simplify the Schwarzschild metric (2.1) to

(3.1) $$ds^2 = -(1 - 1/r)\, dt^2 + (1 - 1/r)^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

i.e. in case $\alpha>0$ we are allowed to assume $\alpha=1$ without loss of generality.

Now we rewrite eq. (3.1) to

(3.2) $$ds^2 = (1 - 1/r)\, [\, -dt^2 + (\, r\, dr/r-1)^2] + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \ .$$

Instead of t we introduce a new variable u by

(3.3) $$u = t + r + \ln |r-1| \ ,$$

hence $r\, dr/r-1 = dt - du$ and

(3.4) $$ds^2 = -(1 - 1/r)\, du^2 + du\, dr + dr\, du + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \ ,$$

which metric form is free from singularities in the region $\{(u,r) \mid 0 < r < \infty,\ u \in \mathbf{R}\}$.

The singularities of the Schwarzschild metric (1.1) are spurious merely, i.e. no singularities of spacetime.

Remark Equ. (3.3) is valid for $r < 1$ *as well*, which *generally* yields $du = dt + dr + dr/r-1$. Inserting this in eq. (3.4) leads back to eq. (3.1) as the reader will check immediately. Therefore we have the result:

The metric (3.4) is an extension of each of the two validity regions of the

Schwarzschild metric (3.1) to the other one.

This result can be applied to again calculate the induced metric on the sphere S_α to obtain eq. (2.3) again (with $\alpha=1$).

4. Somewhat elementary differential geometry

We shall determine here a subset of the event horizon to show again that it cannot be only one central point:

The metric of an equatorial section $\theta = \pi/2$ through an Euclidean space parametrized by spherical polar coordinates (r, θ, φ)

(4.1) $$ds^2 = dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \quad \Rightarrow \quad ds^2 = dr^2 + r^2 \, d\varphi^2 \, .$$

yields a *plane* with polar coordinates (r, φ), while $\theta = \pi/2$.

A similar equatorial section for the Schwarzschild metric at constant time variable t yields the metric

(4.2) $$ds^2 = (1 - {}^{\alpha}/_{r})^{-1} \, dr^2 + r^2 \, d\varphi^2$$

which is no longer plane, i.e. no longer representable in a plane, say z=0. However, instead of the plane z=0 we can define a surface $z = z(r,\varphi)$ over a plane with polar coordinates (r,φ). Due to the spherical symmetry z cannot depend on φ, hence we have to consider a rotational surface $z = z(r)$: The metric of this surface is given by

(4.3) $$ds^2 = (1 + z_r^2) \, dr^2 + r^2 \, d\varphi^2 \, .$$

Comparison with the metric (4.2) yields $z_r = ({}^{\alpha}/_{r-\alpha})^{1/2}$, hence

(4.4) $$z(r) = [\alpha(r-\alpha)]^{1/2} \, .$$

This is a rotational surface generated by rotating the parabola $z = [\alpha(r-\alpha)]^{1/2}$ around the z-axis, see the figure of that surface.

We see that $r < \alpha$ is impossible, and $z = 0$ for $r = \alpha$ is the (**red marked**) boundary of the *accessible* world, where $z > 0$.

The boundary (subset of the event horizon) is not a single point.

5. Further comments on Crothers paper [1]

Let us compare the metric usually attributed to Schwarzschild

$$ds^{*2} = (1 - {}^{\alpha}/_{r^*}) \, dt^2 - (1 - {}^{\alpha}/_{r^*})^{-1} \, dr^{*2} - r^{*2} (d\theta^2 + \sin^2\theta \, d\varphi^2) \qquad (6)$$

with Crothers' "new" metric:

$$ds^2 = ({}^{C^{1/2}-\alpha}/_{C^{1/2}}) \, dt^2 - ({}^{C^{1/2}}/_{C^{1/2}-\alpha}) \, {}^{C'^2}/_{4C} dr^2 - C \, (d\theta^2 + \sin^2\theta \, d\varphi^2) \qquad (7)$$

This metric has a certain blemish: the differential dr can be removed, such that the variable r is completely substituted by the new variable C using C'dr = dC, hence

(5.1) $$ds^2 = ({}^{C^{1/2}-\alpha}/_{C^{1/2}}) \, dt^2 - ({}^{C^{1/2}}/_{C^{1/2}-\alpha}) \, {}^{1}/_{4C} dC^2 - C \, (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

What Crothers did not mention in his papers [1] and {2]:
Both metrics, defined by the eqs.(6) and (7)/(5.1) are *equivalent*, i.e. the associated manifolds

are *identical*, merely represented by *different* coordinates (t,r^*,θ,φ) and (t,C,θ,φ) repectively, associated by the coordinate transform

(5.2) $C = C(r^*) = r^{*2}$ and $r^* = r^*(C) = C^{1/2}$.

So normally there is no reason for considering other than the STANDARD form (6) of the Schwarzschild metric. Other equivalent forms may be of historical interest merely. Crothers' question of correct naming of the different versions of equivalent metrics has become obsolete nowadays. For more see Section 4.

From the coefficients g_{oo} of the metrics (7) and (6) respectively it can be seen directly that the metric (7) becomes singular at $C^{1/2} = \alpha$, while the metric (6) becomes singular at $r^* = \alpha$.

Crothers defines a value r_o by the equation $C(r_o) = \alpha^2$. From $C(r^*) = r^{*2}$ we obtain $r_o = \alpha$: While the metric (7) is singular at $C = C(r_o) = \alpha^2$ the equivalent metric (6) has its corresponding singularity at $r = r_o = \alpha$.

Crothers is interested in a *radial* coordinate with an evident *geometrical* meaning. Therefore he introduces a new variable, a "proper radius" R_p by *radial* integration of the line element ds of (7) $(dt=0, d\theta=0, d\varphi=0)$ starting from the singularity, which after some calculations yields

$$R_p(C) = [C^{1/2} (C^{1/2}-\alpha)]^{1/2} + \alpha \ln |(C^{1/4}+(C^{1/2}-\alpha)^{1/2}) \alpha^{-1/2}| \qquad (14)$$

The same result would have been attained by radial integration of the line element ds^* of (6) starting at its singularity $r^* = \alpha$:

$$(5.3) \qquad R_p^*(r^*) = [r^*(r^*-\alpha)]^{1/2} + \alpha \ln |(r^{*1/2}+(r^*-\alpha)^{1/2}) \alpha^{-1/2}|$$

where $r^* = C^{1/2}$. We then have $R_p^*(r^*) = R_p(r^{*2})$.

Conclusion The use of the metric (7)/(5.1) instead of the technically simpler Schwarzschild metric (6) is an *unnecessary* complication which cannot yield new results exceeding those attained by use of the Schwarzschild metric.

5. The reasons of Crothers' misunderstandings

Crothers' problems with the analysis of GRT are mainly caused by his misconceptions concerning the role of coordinates. In his paper [2] we read:

The black hole, which arises solely from an incorrect analysis of the Hilbert solution, is based upon a misunderstanding of the significance of the coordinate radius r. This quantity is neither a coordinate nor a radius in the gravitational field and cannot of itself be used directly to determine features of the field from its metric. The appropriate quantities on the metric for the gravitational field are the proper radius and the curvature radius, both of which are functions of r. The variable r is actually a Euclidean parameter which is mapped to non-Euclidean quantities describing the gravitational field, namely, the proper radius and the curvature radius.

Crothers expects a geometrical meaning always being attached to a coordinate. He insinuates that the coordinate r, known from spherical polar coordinates as *radial distance* from the center, should maintain its meaning when appearing in another context, e.g. as the parameter r of the Schwarzschild metric. In [2, Sect.2] we read about an isotropic generalization of the Minkowski line element:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \qquad (2a)$$
$$A,B,C > 0\,,$$

where A,B,C are analytic functions. I emphatically remark that *the geometric relations between the components of the metric tensor of (2a) are precisely the same as those of (1).* The standard analysis writes (2a) as,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \qquad (2b)$$

and claims it the most general, which is incorrect. The form of C(r) cannot be pre-empted ...

This renaming method is *somewhat lax* but often used in mathematics, though it could be misunderstood if taken literally: The setting $C := r^2$ means that a *new* meaning is assigned to the variable r. Since r already occurs in eq.(2a), it would be *better* to use a *new* symbol, say r^*, not r, for the new variable: $r^{*2} := C(r)$. As a consequence the terms $A(r)dt^2$ and $B(r)dr^2$ must be rewritten as functions of the new variable r^* by introducing new cofficients $A^*(r^*):=A(r)$ and $B^*(r^*):=B(r)(dr/dr^*)^2$. This yields

$$ds^2 = A^*(r^*)dt^2 - B^*(r^*)dr^{*2} - r^{*2} (d\theta^2 + \sin^2\theta \, d\varphi^2) , \qquad (2b^*)$$

Then, all *s are removed to obtain

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) , \qquad (2b)$$

To repeat it: **The terms A, B, r in (2a) and (2b) respectively have different meanings, here precisely specified.** However, the rewriting (2a) as (2b) is perfectly justified herewith.

Without loss of generality the coefficient C(r) in eq. (2a) can be assumed as $C(r)=r^2$.

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[4] S.M. Carroll, *Lecture Notes on General Relativity*, http://xxx.lanl.gov/pdf/gr-qc/9712019

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